Unit 24

Answers: Nuclear Physics

Quick checks

1 c The push between two charges grows as $1/r^2$. If the gold's positive charge were spread through the whole atom, an alpha particle could never get close to all of it at once, and the push would stay small (Thomson's atom in the widget). Packed into a tiny nucleus, the charge can be approached very closely, and the push becomes enormous. Electrons are about 7300 times lighter than an alpha particle: they can no more turn it back than a mosquito can stop a bus.

2 b $d = \dfrac{2kZe^2}{E_K}$: d is inversely proportional to E_K. Twice the energy, half the distance. (Fast enough alpha particles reach the nucleus itself, which is how Rutherford later saw the first signs of the strong nuclear force, section 24.9.)

3 c $Z = 26$ protons; $A = 56$ nucleons; so $N = 56 - 26 = 30$ neutrons. The lower number is always the protons.

4 b The two chlorines have the same Z (17) and different A: isotopes. Argon-40 and calcium-40, like tritium and helium-3, share A: isobars. Carbon-14 and oxygen-16 both have 8 neutrons: isotones.

5 b $\Delta m = \dfrac{\Delta E}{c^2} = \dfrac{9.0 \times 10^{13}}{9.0 \times 10^{16}} = 10^{-3}$ kg = 1 g. About that much mass vanished in the Hiroshima explosion of 1945 (roughly 6×10^{13} J).

6 b The photon must create **two** particles, each with a rest energy of 0.51 MeV, so it needs at least 1.02 MeV. The nucleus only takes up momentum; it gives no energy. Photons above 1.02 MeV do produce matter, every day, in hospital machines and in cosmic rays.

7 c The strong force acts on all nucleons equally, is attractive at nuclear distances, and dies away beyond a few femtometers. The electric force holds the electrons in an atom; gravity has an infinite range but is far too weak.

8 b Moving towards the peak at $A \approx 60$ from either side means moving up the curve: each nucleon ends up more tightly bound, and the extra binding energy is given out. No protons are destroyed; the numbers of protons and neutrons are conserved in both.

9 c At $k = 1$, each fission leads to exactly one more, so the rate of fission, and the power, stays constant. Each fission gives out 2 or 3 neutrons, but the reactor's control rods and its other materials absorb the extras so that, on average, exactly one causes a new fission.

10 c All nuclei are positive and repel each other. Only very fast nuclei, as in a gas at millions of kelvin, can come within about 10^{-15} m of each other, where the strong force takes over and pulls them together. The strong force itself doesn't depend on temperature.

11 b The scattering showed a small, heavy, charged nucleus, but nothing about what it is made of. The proton was named in 1920 and the neutron found only in 1932.

12 d The nucleus **repels** the alpha particles, so both must be pushed **away** from it (down, in the diagrams), and the one aimed **closer** must be turned through the **larger** angle.
A and C: the closer alpha particle swings round the nucleus, as if pulled towards it: that would be attraction.
B: the farther alpha particle is turned more than the closer one.
D: both are pushed away, and the closer one is turned back through more than 90°. Correct.

13 a Most went straight through or nearly so; about 1 in 8000 was turned through more than 90°. That "few" was the surprise that revealed the nucleus.

14 a Isotopes have the same number of protons: X and Y both have 15 (they are phosphorus-31 and phosphorus-32). Y and Z share their nucleon number (isobars); X and Z share their neutron number (isotones).

15 c The curve rises steeply, peaks near $A = 60$, and falls slowly. The most stable nuclide is at the **peak**; **fusion** happens among the light nuclei on the steep left side; **fission** among the heavy nuclei on the right.
A has fission and fusion swapped and puts the most stable nuclide at the heaviest end.
B points to helium-4's spike, not the peak, as the most stable nuclide.
D is upside down.

16 c The most stable is the one with the greatest binding energy per nucleon, the nearest to the peak: calcium-40 at 8.55 MeV, against magnesium-24 at 8.26, mercury-202 at 7.90 and helium-4 at 7.07 MeV.

17 d Binding energy $= 0.0990 \times 931.5 = 92.2$ MeV (the trap in option c); per nucleon, $92.2 \div 12 =$ **7.68 MeV**.

25. Calcium-40 has 20 protons and $40 - 20 = 20$ neutrons.
a. Mass of the separated nucleons $= 20 \times 1.00728 + 20 \times 1.00867 = 20.1456 + 20.1734 = 40.3190$ u.
Mass defect $\Delta m = 40.3190 - 39.9516 =$ **0.3674 u**.
b. Binding energy $= 0.3674 \times 931.5 =$ **342 MeV** (8.56 MeV per nucleon).

26. a. Two protons and two neutrons: $2 \times 1.00728 + 2 \times 1.00867 = 2.01456 + 2.01734 = 4.03190$ u.
$\Delta m = 4.03190 - 4.00151 =$ **0.03039 u**.
b. Binding energy $= 0.03039 \times 931.5 = 28.31$ MeV. Per nucleon: $28.31 \div 4 =$ **7.08 MeV**.

27. a. Overall, in the proton–proton chain:

$$4\,{}^{1}_{1} \rightarrow {}^{4}_{2} + 2\,{}^{0}_{+1} + 2\nu + 2\gamma \qquad (26.7)$$

built from the steps ${}^{1}_{1} + {}^{1}_{1} \rightarrow {}^{2}_{1} + {}^{0}_{+1} + \nu$; ${}^{2}_{1} + {}^{1}_{1} \rightarrow {}^{3}_{2} + \gamma$; ${}^{3}_{2} + {}^{3}_{2} \rightarrow {}^{4}_{2} + 2\,{}^{1}_{1}$.
b. The nuclei must get within about 10^{-15} m of each other, the range of the strong nuclear force. Because they are all positive, they repel strongly, so:

- **a very high temperature**, about 10^{7} K (1.5×10^{7} K in the Sun's core), so that the nuclei move fast enough to overcome the repulsion;
- **a very high density** (about 1.5×10^{5} kg/m^3 in the Sun's core), so that collisions are frequent enough for many fusions per second;
- these conditions held for long enough: in the Sun, by its own gravity.

28.

$$m = \frac{E}{c^2} = \frac{5.0 \times 10^{13}}{(3.0 \times 10^{8})^2} = \frac{5.0 \times 10^{13}}{9.0 \times 10^{16}} = \mathbf{5.6 \times 10^{-4}\ kg}$$

about half a gram.

29. a. The α particle is ${}^{4}_{2}$: $A = 4$, $Z = 2$.
For X: $220 = A + 4$, so $\mathbf{A = 216}$; $86 = Z + 2$, so $\mathbf{Z = 84}$. Element 84 is polonium: ${}^{216}_{84}$.
b. Mass lost $= 220.011\,394 - (216.001\,915 + 4.002\,603) = 220.011\,394 - 220.004\,518 = 0.006\,876$ u.
Energy $= 0.006\,876 \times 931.5 = \mathbf{6.40\ MeV} = 6.40 \times 1.60 \times 10^{-13} = \mathbf{1.03 \times 10^{-12}\ J}$.
Almost all of it goes to the alpha particle as kinetic energy (6.29 MeV), the heavy polonium recoiling with the rest.

30. a. Upper numbers: $2 + 1 = A$, so $A = 3$: helium-3, ${}^{3}_{2}$.

b. Mass lost $= (2.01355 + 1.00728) - 3.01493 = 3.02083 - 3.01493 = 0.00590$ u.

Energy $= 0.00590 \times 931.5 = \mathbf{5.50\ MeV}$ (8.8×10^{-13} J): the second step of the proton–proton chain.

31. Mass before $= 7.014\,727 + 0.000\,549 = 7.015\,276$ u. Mass after $= 7.014\,356$ u.
Mass lost $= 7.015\,276 - 7.014\,356 = 0.000\,920$ u.
Energy released $= 0.000\,920 \times 931.5 = \mathbf{0.857\ MeV}$ (1.37×10^{-13} J), carried off almost entirely by the neutrino.

32.

a. 1. $N = A - Z = 9 - 4 =$ **5 neutrons**.

2. Separated nucleons: $4 \times 1.00728 + 5 \times 1.00867 = 4.02912 + 5.04335 = 9.07247$ u. Mass defect $= 9.07247 - 9.00999 =$ **0.06248 u** (58.2 MeV).

b. 1. Four protons and four neutrons: $4 \times 1.00728 + 4 \times 1.00867 = 8.06380$ u. Mass defect $= 8.06380 - 8.00311 =$ **0.06069 u**.

2. $0.06069 \times 931.5 =$ **56.5 MeV**.

3. $56.5 \div 8 =$ **7.07 MeV per nucleon**.

c. 1. Two alpha particles: $2 \times 28.31 =$ **56.62 MeV**.

2. Energy change = binding energy after − binding energy before $= 56.53 - 56.62 =$ **−0.09 MeV**. The binding energy goes **down**: beryllium-8 is less tightly bound than two separate alpha particles, so about 0.09 MeV must be **supplied**; no energy is given out.

That is why beryllium-8 falls apart into two alpha particles in about 10^{-16} s, and why stars can only make carbon by catching a third alpha particle in that brief moment (the "triple-alpha" process). Compare beryllium-9: one extra neutron makes it stable.

33.

a. 1. Before: $2 \times 2.01355 = 4.02710$ u. After: $3.01493 + 1.00867 = 4.02360$ u. Change in mass $= 4.02710 - 4.02360 =$ **0.00350 u = 3.5×10^{-3} u**.

2. $0.00350 \times 931.5 =$ **3.26 MeV**.

b. 1. Before: $2.01355 + 3.01550 = 5.02905$ u. After: $4.00151 + 1.00867 = 5.01018$ u. Change = **0.01887 u ≈ 1.9×10^{-2} u**.

2. $0.01887 \times 931.5 =$ **17.6 MeV**: the reaction planned for fusion power stations.

c. 1. Before: $2.01355 + 3.01493 = 5.02848$ u. After: $4.00151 + 1.00728 = 5.00879$ u. Change = **0.01969 u ≈ 0.02 u**.

2. $0.01969 \times 931.5 =$ **18.3 MeV**.

d. 1. The fourth reaction: before 4.02710 u; after $3.01550 + 1.00728 = 4.02278$ u; change 0.00432 u, so $0.00432 \times 931.5 = 4.02$ MeV.
Total: $3.26 + 17.6 + 18.3 + 4.02 =$ **43.2 MeV ≈ 43 MeV**. (Check: 6 deuterons in, 2 ^{4}He + 2p + 2n out; the tritium and helium-3 made in the first two reactions are used up in the next two.)

2. Energy $= 6.0 \times 10^{23} \times 43$ MeV $= 2.58 \times 10^{25}$ MeV $= 2.58 \times 10^{25} \times 1.60 \times 10^{-13}$ J = **4.1×10^{12} J**.

3. $m = \dfrac{E}{c^2} = \dfrac{4.1 \times 10^{12}}{9.0 \times 10^{16}} =$ **4.6×10^{-5} kg**, about 0.05 g out of the 12 g of deuterium used ($6 \times 6.0 \times 10^{23}$ deuterons, 6 mol × 2 g).

34. 1. $E = mc^2 = 3.1 \times 10^{-28} \times (3.0 \times 10^8)^2 = \mathbf{2.8 \times 10^{-11}\ J}$ (about 174 MeV).

2. $n = \dfrac{1.0}{2.79 \times 10^{-11}} = 3.6 \times 10^{10}$ nuclei. The data are given to 2 significant figures, so the answer is **3.6×10^{10} nuclei**: 36 billion fissions for one joule.

35. 1. The nucleus has Z protons and $(A - Z)$ neutrons:

$$\Delta m = Z m_p + (A - Z)\, m_n - M$$

2. Binding energy = $\Delta m\ c^2$, shared among A nucleons:

$$E = \frac{[Z m_p + (A - Z)\, m_n - M]\, c^2}{A}$$

36. a. 92 protons and 235 − 92 = 143 neutrons.
Mass of the separated nucleons = 92 × 1.00728 + 143 × 1.00867 = 92.66976 + 144.23981 = 236.90957 u.
Mass defect = 236.90957 − 234.993 = 1.91657 u.
Binding energy = 1.91657 × 931.5 = 1785 MeV. Per nucleon: 1785 ÷ 235 = **7.60 MeV**.

b. 1. Uranium-235 is at the **far right** of the graph, on the slowly falling part, below the peak. The fission fragments (with A of roughly 90 to 145) are in the **middle**, **higher up** the curve, nearer the peak.

2. The fragments have a **greater binding energy per nucleon** than the uranium nucleus (about 8.5 MeV against 7.6 MeV). The number of nucleons stays the same, so the total binding energy of the products is greater than that of the uranium: the nucleons are more tightly bound, the total mass has decreased, and the difference, about 200 MeV, is given out, mostly as the kinetic energy of the fragments.

37. a. Reading the graph: 1. U-235, **7.6 MeV**; 2. Sr-90, **8.7 MeV**; 3. Xe-144, **8.2 MeV**.

b. Binding energy before: 235 × 7.6 = 1786 MeV.

After: 144 × 8.2 + 90 × 8.7 = 1181 + 783 = 1964 MeV.
Energy released ≈ 1964 − 1786 ≈ **180 MeV** (the masses give 176 MeV).
(Check the equation: upper numbers 235 + 1 = 144 + 90 + 2; lower numbers 92 + 0 = 54 + 38: xenon is element 54.)

38. a. Nucleon numbers: $236 = 144 + A + 3$, so $\mathbf{A = 89}$. Proton numbers: $92 = 56 + Z + 0$, so $\mathbf{Z = 36}$: krypton-89, $^{89}_{36}$.

b. 1. Mass after $= 143.892\,211 + 88.898\,071 + 3 \times 1.008\,67 = 235.816\,292$ u.

Change in mass $= 235.995\,061 - 235.816\,292 = \mathbf{0.178\,769\ u}$.
2. Energy $= 0.178\,769 \times 931.5 = \mathbf{167\ MeV \approx 170\ MeV}$.

c. 1. The fuel must supply $\dfrac{1500}{0.40} = 3750$ MW $= 3.75 \times 10^9$ J/s.
In one day: $3.75 \times 10^9 \times 86\,400 = 3.24 \times 10^{14}$ J.
Mass used: $\dfrac{3.24 \times 10^{14}}{6.8 \times 10^{13}} = \mathbf{4.8\ kg}$, less than 5 kg a day.
2. Only 5 % of the fuel is fissionable, so the fuel needed is $\dfrac{4.8}{0.05} = \mathbf{95\ kg\ a\ day}$. A coal power station of the same output burns about 12 000 tonnes of coal a day.

39. a. Binding energy $= 1.9151 \times 931.5 = 1784$ MeV $= 1784 \times 1.60 \times 10^{-13} = \mathbf{2.85 \times 10^{-10}\ J}$.

b. $1784 \div 235 = \mathbf{7.59\ MeV\ per\ nucleon}$.

40. a. **Nuclear fusion**: two light nuclei join to form a heavier one.

b. Total binding energy before: $2 \times (2 \times 1.11) = 4.44$ MeV.

After: $3 \times 2.58 = 7.74$ MeV (the neutron adds nothing).
Energy released $\approx 7.74 - 4.44 = \mathbf{3.3\ MeV}$, as from the masses in question 33a.

41. Energy per helium nucleus $= 26.71 \times 1.60 \times 10^{-13} = 4.27 \times 10^{-12}$ J.
Helium nuclei made per second $= \dfrac{4 \times 10^{26}}{4.27 \times 10^{-12}} = 9.36 \times 10^{37}$.
Hydrogen used $= 4 \times 9.36 \times 10^{37} \times 1.67 \times 10^{-27}$ kg $= 6.25 \times 10^{11}$ kg/s.
Hydrogen available $= 1.99 \times 10^{30} \times 0.73 \times 0.10 = 1.45 \times 10^{29}$ kg.
$t = \dfrac{1.45 \times 10^{29}}{6.25 \times 10^{11}} = 2.3 \times 10^{17}$ s $\approx \mathbf{7 \times 10^9\ years}$ (7 billion years).

42. Total binding energy before: $40 \times 8.551 + 4 \times 7.073 = 342.04 + 28.29 = 370.33$ MeV.
After: $44 \times 8.533 = 375.45$ MeV.
Energy released $= 375.45 - 370.33 = \mathbf{5.12\ MeV}$.
Titanium-44 has a slightly **lower** binding energy per nucleon than calcium-40, yet energy is still released: it is the **total** binding energy that counts, and the helium-4's nucleons gain a lot (from 7.07 to 8.53 MeV each).