Unit 23

DC Circuits

This unit online, with its interactives:
science-playground.com/s/11f23

Flip a light switch and the bulb across the room lights at once. Yet the electrons in the copper wire creep along at a fraction of a millimeter each second: an electron leaving the switch would take hours to reach the bulb. What travels so fast, then? And why does a phone charger get warm, a torch battery give a dimmer light as it runs down, and a street light know when it is dark?

This unit answers those questions with a handful of ideas: current as a flow of charge, the drift of electrons, potential difference and e.m.f. as energy per coulomb, resistance and resistivity, the characteristic graphs of resistors, lamps, diodes and sensors, the potential divider that turns a sensor into a voltage, and the internal resistance hidden inside every cell.

Every unit of resistance in the world carries the name of **Georg Simon Ohm**. His father was a locksmith, and his mother the daughter of a tailor; neither had been to school. His father taught himself, and then gave his sons an excellent education at home: Ohm's brother Martin became a well-known mathematician. Ohm taught at a high school in Cologne, where, with wires he drew himself and a current-measuring needle he built, he found the relationship between the potential difference across a conductor and the current through it, now called **Ohm's law** (1827). His work was received coldly at first; it was years before scientists accepted it, and he became a professor at Munich only near the end of his life.



Figure 23.1 *Georg Simon Ohm (1789–1854).*
Photo: Derivative work: unknown, Public domain

23.1 Electric current

Electric current is the rate of flow of charge: the quantity of charge passing a given point each second.

$$I = \frac{\Delta Q}{\Delta t}$$

where I is the current and ΔQ the charge that flows past the point in time Δt.

Current is measured in **amperes** (A), one of the seven SI base units. Since 2019 the ampere has been fixed by the charge of one electron: the **elementary charge** e is defined to be exactly $1.602\,176\,634 \times 10^{-19}$ C, and one ampere is the current in which $1 / (1.602\,176\,634 \times 10^{-19})$, about 6.24×10^{18}, elementary charges pass a point each second. (Before 2019 the ampere was defined by the force between two parallel wires carrying it: 2×10^{-7} N on each meter of wire 1 m apart.)

In a laboratory, an ampere is quite a large current, so we often use the **milliampere** (1 mA = 10^{-3} A) and the **microampere** (1 μA = 10^{-6} A).

Rearranging the definition,

$$\Delta Q = I\,\Delta t$$

The charge that flows in a certain time is the current multiplied by the time. If a current of 1 A flows for 1 s, the charge that flows is one **coulomb** (C):

One coulomb is the charge that passes a point in one second when the current is one ampere: 1 C = 1 A s.

The coulomb is also a large amount of charge, so we often use μC or even nC. The charge on a single electron is only 1.6×10^{-19} C (in size).

WORKED EXAMPLE

Charge and electrons through a lamp

a. How much charge flows through the filament of a lamp in 1 hour when the current in it is 250 mA?

b. The electron charge is 1.6×10^{-19} C. How many electrons flow through the filament in this time?

Answer

a. $\Delta Q = I\,\Delta t = 250 \times 10^{-3}\ \text{A} \times (60 \times 60)\ \text{s} = \mathbf{900\ C}$

b. $$\text{number of electrons} = \frac{\text{total charge}}{\text{charge of one electron}} = \frac{900\ \text{C}}{1.6 \times 10^{-19}\ \text{C}} = \mathbf{5.6 \times 10^{21}\ \text{electrons}}$$

More than 5 thousand million million million electrons in an hour, through one small lamp.

Check 1

A current of 2.0 mA flows for 5.0 minutes. The charge that passes is:

a 10 mC
b 0.60 C
c 10 C
d 600 C

23.2 Conventional current and drift velocity

Conventional current

"Conventional current" doesn't mean the normal or usual current: a **convention** is an agreement. Long before the electron was discovered (by J. J. Thomson, in 1897), the scientists who worked on electricity agreed to say that current flows **from the positive terminal of a supply, round the circuit, to the negative terminal**.

In metal wires, the moving charges turned out to be electrons, which are negative, and drift the other way: from the negative terminal round to the positive. So **conventional current is in the opposite direction to the flow of electrons**. Nothing is wrong with this: a flow of negative charge one way has the same effect as a flow of positive charge the other way, and every rule in physics is written for conventional current.

Not every current is a flow of electrons. In a solution such as salt water or copper sulphate, or in an ionized gas such as the inside of a fluorescent tube, **positive ions** move in the direction of conventional current, and negative ions the other way.

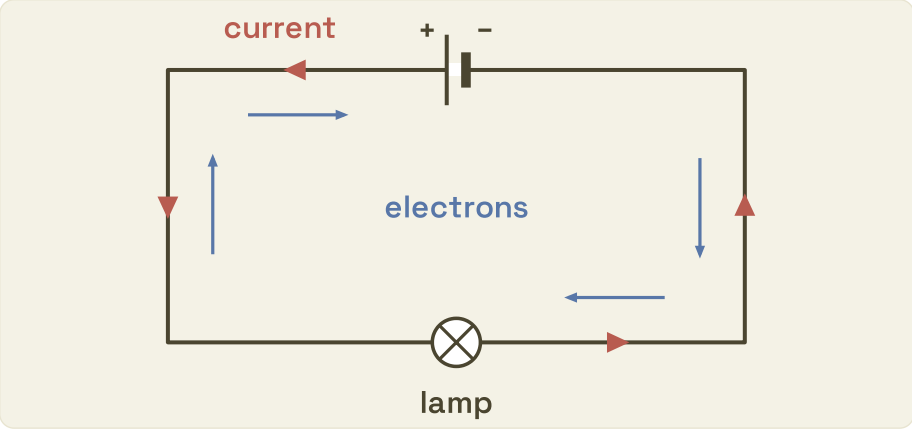


Figure 23.2 *In a wire, conventional current (red) goes from the cell's + terminal round the circuit to its − terminal; the electrons (blue) drift the other way.*

The free electrons in a metal

Connect a battery, a switch and a lamp in series, and close the switch: the lamp lights, as far as we can tell, at once. How fast do the electrons move through the wires?

In a metal atom, the outermost electrons are held only loosely by the nucleus. In a solid metal they leave their atoms altogether and move freely among all the atoms (strictly, the positive **ions** the atoms have become). The metal ions sit in a regular **lattice**,

face the same way
(16.5 V).

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.

Check 19

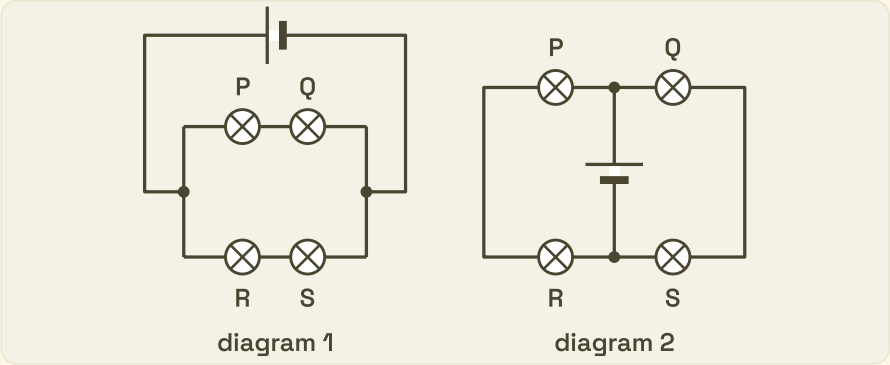


Figure 23.33 *For question 10: four identical lamps P, Q, R and S, in diagram 1 and in diagram 2.*

10. When four identical lamps P, Q, R and S are connected as shown in diagram 1, they have normal brightness. When they are connected as shown in diagram 2, which statement is correct?

a The lamps do not light.
b The lamps are less bright than normal.
c The lamps have normal brightness.
d The lamps are brighter than normal.

Check 20

11. A nichrome wire is 1 m long and 1×10^{-6} m^2 in cross-sectional area. When it is connected to a p.d. of 2 V, a current of 4 A flows in it. The resistivity of this nichrome is:

a 2×10^{-7} Ω m
b 4×10^{-7} Ω m
c 5×10^{-7} Ω m
d 8×10^{-7} Ω m

like islands in a "sea" of **free electrons**, also called delocalized or conduction electrons. In most metals there is roughly **one free electron per atom**.

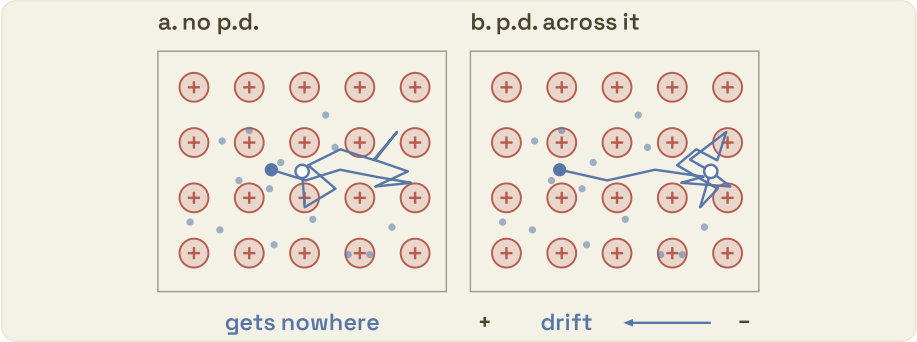


Figure 23.3 *A metal: positive ions in a regular lattice, with free electrons moving among them. a. With no p.d., each electron dashes about at random, bouncing off the vibrating ions: it gets nowhere on average. b. With a p.d. across the wire, the field pushes every electron a little towards the + end between collisions: the random motion now has a slow drift added to it.*

The free electrons are always moving, very fast, at speeds of about **10^6 m/s** (about a three-hundredth of the speed of light), in random directions, colliding with the ions all the time. Because the directions are random, this motion carries no charge along the wire overall. The free electrons are also why metals conduct heat so well.

Connect a cell across the wire, and the p.d. sets up an **electric field** along it. The field pushes every free electron away from the negative terminal and towards the positive terminal. Between collisions each electron gains a little velocity along the wire, so on top of its fast random motion it **drifts** slowly along. The average velocity of this drift is the **mean drift velocity**: it is tiny compared with the random speeds, but it is in one direction, so it carries the current.

The equation I = nAvq

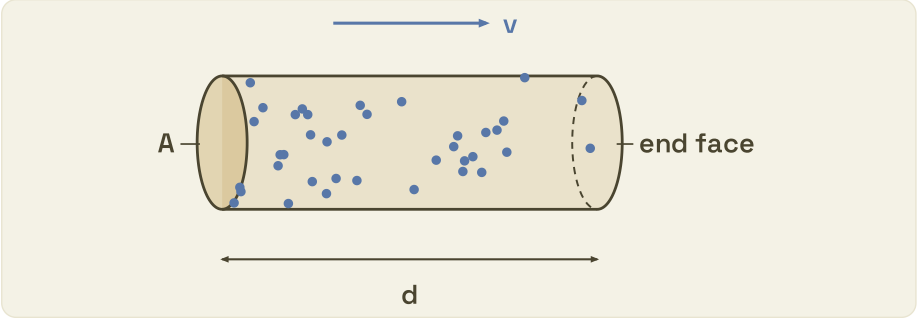


Figure 23.4 *A length d of wire of cross-sectional area A, with n free charge carriers in each cubic meter, each of charge q, drifting at velocity v. In the time t = d/v that carriers take to drift the length d, all the carriers in this length pass through the end face.*

Take a length d of wire of cross-sectional area A (Figure 23.4). It contains **n free charge carriers per unit volume**: n is the **number density** of the charge carriers, the number in each cubic meter (unit m^{-3}).

The carriers in this length. The volume of the length is A × d, so the number of free carriers in it is n × A × d. Each carries a charge q (for an electron, the size of its charge, e), so the charge free to move in this length is

$$Q = nAdq$$

The time to pass the end. The carriers drift at the mean drift velocity v, so in a time

$$t = \frac{d}{v}$$

every one of them passes through the end face of the length.

The current. Current is charge per unit time:

$$I = \frac{Q}{t} = \frac{nAdq}{d/v}$$

and d cancels:

$$\boxed{I = nAvq}$$

For electrons, q = e and I = nAve. The same equation holds for any charge carriers: ions in a solution or in a plasma (an ionized gas), where an ion may carry several times the electron's charge (a Cu^{2+} ion carries $2e = 3.2 \times 10^{-19}$ C).

Rearranged, $v = \frac{I}{nAq}$: for a given current, the carriers drift **faster in a thinner wire** (smaller A) and in a material with **fewer carriers** (smaller n), such as a semiconductor.

WORKED EXAMPLE

Drift in a lamp's filament and its leads

A lamp is connected to a cell by copper leads, and a current of 160 mA flows (Figure 23.5). The diameter of the tungsten filament is 0.025 mm and the diameter of the copper connecting leads is 0.72 mm. Taking the number of charge carriers per cubic meter as 4.0×10^{28} m^{-3} for tungsten and 8.0×10^{28} m^{-3} for copper, calculate the drift velocity of the electrons in a. the filament, b. the connecting leads.

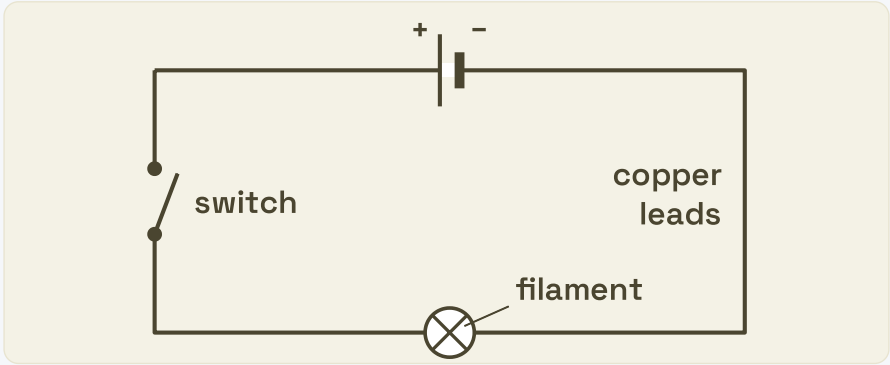


Figure 23.5 *A cell, a switch and a lamp in series, joined by copper leads; the thin tungsten filament is inside the lamp.*

Answer

a. **The filament.** Its radius is 0.5×0.025 mm $= 1.25 \times 10^{-5}$ m, so $A = \pi r^2 = \pi(1.25 \times 10^{-5})^2 = 4.91 \times 10^{-10}$ m^2.

$$v = \frac{I}{nAq} = \frac{160 \times 10^{-3}\ \text{A}}{4.0 \times 10^{28}\ \text{m}^{-3} \times 4.91 \times 10^{-10}\ \text{m}^2 \times 1.60 \times 10^{-19}\ \text{C}} = 0.051\ \text{m/s} = \mathbf{51\ mm/s}$$

b. **The leads.** Radius 0.36 mm $= 3.6 \times 10^{-4}$ m, so $A = \pi(3.6 \times 10^{-4})^2 = 4.07 \times 10^{-7}$ m^2.

$$v = \frac{160 \times 10^{-3}}{8.0 \times 10^{28} \times 4.07 \times 10^{-7} \times 1.60 \times 10^{-19}} = 3.1 \times 10^{-5}\ \text{m/s} = \mathbf{0.031\ mm/s}$$

The same current flows through both, but in the thin filament, with fewer carriers per meter, the electrons drift more than a thousand times faster.

Check 2

The same current flows through a thick copper wire and a thin copper wire joined in series. In the thin wire, the drift velocity of the electrons is:

a smaller
b the same
c larger
d zero

23.3 Potential difference

Why do electrons flow round a circuit at all? A comparison with water flowing in pipes helps.

The water (Figure 23.6a). Water held high in a reservoir has gravitational potential energy. Open the tap, and the difference in water level (the height Δh) makes a pressure difference along the pipe: the water flows, and turns a paddle wheel on its way

Check 17

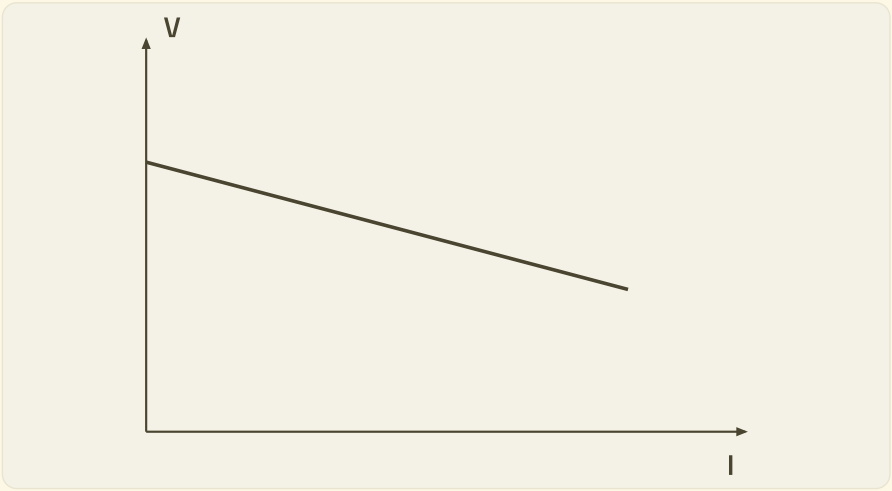


Figure 23.31 *For question 8: the p.d. across a cell against the current in it.*

8. A student investigates the e.m.f. ε and internal resistance r of a cell. The p.d. V across the cell and the current I in it are measured as the load resistance is increased, and plotted on the graph above. How are ε and r found from the graph?

a ε from the area under the graph; r from 1 ÷ (magnitude of the gradient)
b ε from the y-intercept; r from 1 ÷ (magnitude of the gradient)
c ε from the area under the graph; r from the magnitude of the gradient
d ε from the y-intercept; r from the magnitude of the gradient

Check 18

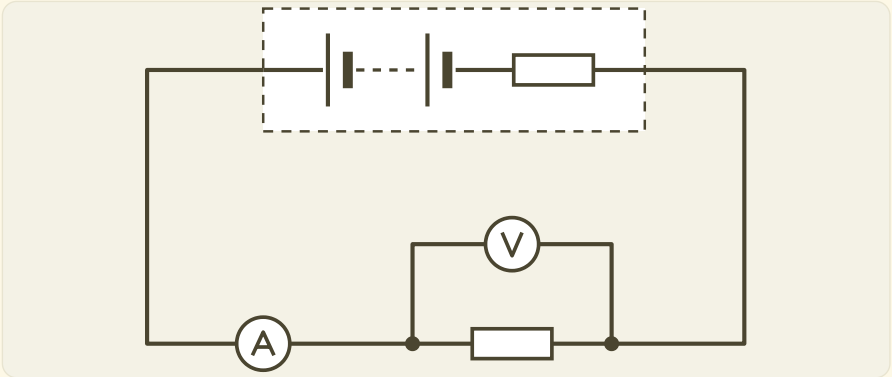


Figure 23.32 *For question 9: a battery (in the dashed box, with its internal resistance) in series with an ammeter and a resistor; a voltmeter across the resistor.*

9. A battery with internal resistance is connected to a fixed resistor, an ammeter and a voltmeter, as shown above. The battery is replaced by one with the same e.m.f. but a greater internal resistance. What happens to the readings on the ammeter and the voltmeter?

a ammeter decreases; voltmeter decreases
b ammeter decreases; voltmeter stays the same
c ammeter stays the same; voltmeter decreases
d ammeter stays the same; voltmeter stays the same

Check 12

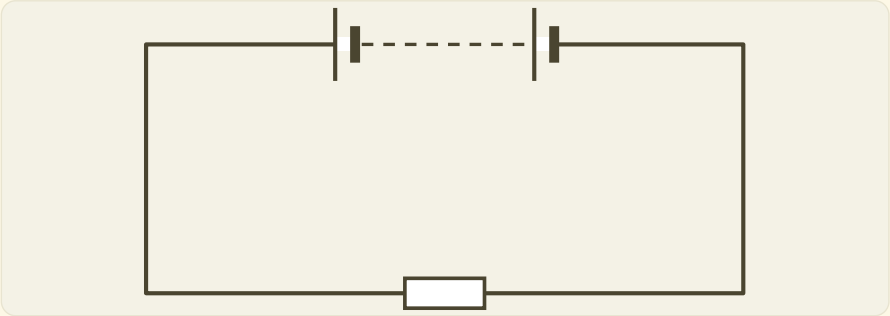


Figure 23.30 *For question 3: a battery of several cells across a resistor.*

3. In the circuit above, the battery converts an amount E of chemical energy to electrical energy when a charge Q passes through the resistor in time t. Which expressions give the e.m.f. of the battery and the current in the resistor?

a e.m.f. EQ; current Q/t
b e.m.f. EQ; current Qt
c e.m.f. E/Q; current Q/t
d e.m.f. E/Q; current Qt

Check 13

4. A lightning strike takes a time t. During the strike, electrons are discharged to the Earth, making a current I. An expression for the total number of electrons discharged to the Earth is:

a $\frac{It}{e}$
b $\frac{I}{te}$
c $\frac{e}{It}$
d $\frac{te}{I}$

Check 14

5. The combined resistance R of two resistors R_1 and R_2 in parallel is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$. Which statement is used in the derivation of this formula?

a The currents through the two resistors are equal.
b The potential difference across each resistor is the same.
c The supply current is split between the two resistors in the same ratio as the ratio of their resistances.
d The total power dissipated is the sum of the powers dissipated in the two resistors separately.

Check 15

6. A cell of e.m.f. ε and internal resistance r is connected to a resistor of resistance R. The current in the circuit is I. An expression for the terminal potential difference V across the cell is:

a V = ε − IR
b V = ε + IR
c V = ε + Ir
d V = ε − Ir

Check 16

7. A motor is used to lift a mass m through a height h in a time t. It draws a current I from the power supply, and the p.d. across the motor is V. A correct expression for the efficiency of the system is:

a $\frac{VI}{mgh}$
b $\frac{mgh}{VI}$
c $\frac{VIt}{mgh}$
d $\frac{mgh}{VIt}$

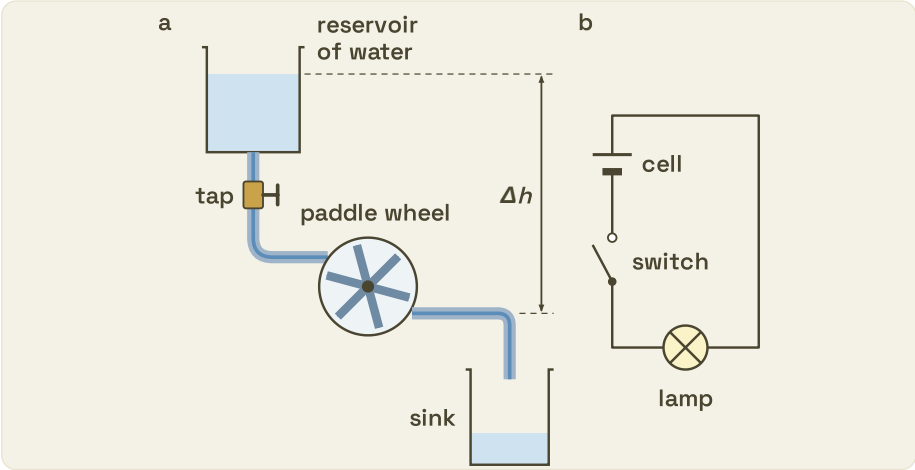


Figure 23.6 *Water and an electric circuit. a. Water held a height Δh above the outflow pipe runs, when the tap is opened, through the pipe, turns a paddle wheel and pours into the sink. b. When the switch is closed, the cell drives charge round the circuit and through the lamp. The reservoir's height is like the cell's p.d., the tap like the switch, and the paddle wheel, which takes the water's energy, like the lamp.*

down to the sink. The water's gravitational potential energy is turned into the kinetic energy of the turning wheel. If the wheel drove a pump, the water could be lifted back up from the sink to the reservoir, and go round again.

The circuit (Figure 23.6b). A chemical reaction in the cell makes an electrical **potential difference** (symbol V) between its terminals. When the switch is closed, the electric field this sets up pushes the electrons round the circuit and through the lamp. The chemical energy stored in the cell is turned into the vibrational kinetic energy of the atoms in the filament: its temperature rises, and it glows.

In every circuit, electrical energy is converted into other forms: a lamp turns it into thermal energy and light, an electric motor into mechanical energy, a loudspeaker into sound.

These energy conversions are the basis of the definition of potential difference:

The potential difference (p.d.) between two points in a circuit is the electrical energy converted into other forms (thermal, light, mechanical…) when unit charge passes from one point to the other.

$$\text{potential difference} = \frac{\text{electrical energy converted into other forms}}{\text{charge passing}}$$

or, since the energy converted is the work W done by the charge:

$$V = \frac{W}{Q}$$

The unit of p.d. is therefore the joule per coulomb (J/C), called the **volt** (V):

A p.d. of 1 volt exists between two points in a circuit if 1 J of energy is converted when 1 C of charge passes between them: 1 V = 1 J/C.

Because p.d. is measured in volts, it is often called the **voltage** between two points.

WORKED EXAMPLE

An electric kettle

The element of an electric kettle takes a current of 12.5 A and produces 540 kJ of thermal energy in three minutes.

a. How much charge passes through the element in these three minutes?

b. What is the potential difference across the ends of the element?

Answer

a. $\Delta Q = I\,\Delta t = 12.5\text{ A} \times (3.0 \times 60)\text{ s} = \mathbf{2250\text{ C}}$

b. $$V = \frac{W}{Q} = \frac{540 \times 10^3\text{ J}}{2250\text{ C}} = \mathbf{240\text{ V}}$$

Each coulomb through the element gives it 240 J: close to Nepal's mains voltage of 230 V.

23.4 Potential difference and electromotive force

Current only measures the **rate** at which charged particles flow round a circuit. It tells us nothing about the energy they carry. The quantity that describes the energy is the potential difference, symbol V, measured in volts.

The p.d. across a component is measured with a **voltmeter** connected **across** the component, **in parallel** with it. (An ammeter goes **in series**, so that the current being measured passes through it.)

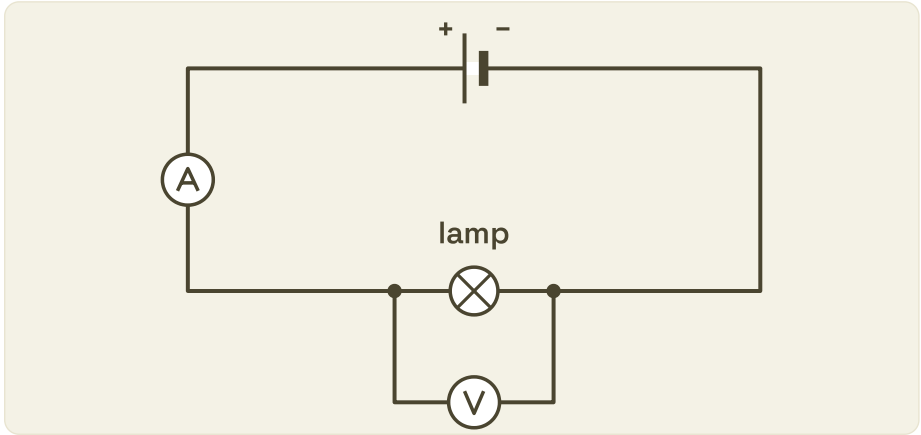


Figure 23.7 *An ammeter A in series with a lamp, so that the current through the lamp passes through it; a voltmeter V across the lamp, in parallel with it.*

Potential difference is the electrical work done **by** each coulomb of charge as it flows **through a component** such as a lamp, a motor or a resistor; the component turns that electrical energy into heat, light or other forms:

$$\text{potential difference, } V\text{ (V)} = \frac{\text{electrical work done by the charge, } W\text{ (J)}}{\text{charge flow, } Q\text{ (C)}} \qquad V = \frac{W}{Q}, \quad 1\text{ V} = 1\text{ J/C}$$

P.d. can't describe what happens in a **source** of electrical energy: a cell, a generator or a mains power supply. These do the opposite: they transfer other forms of energy (chemical, kinetic, light in a solar cell) **into** electrical energy. For them we define a second quantity, the **electromotive force** (e.m.f.), symbol E:

The electromotive force of a source is the electrical work done **on** each coulomb of charge as it flows through the source (a cell, generator or power supply unit): the energy converted into electrical energy per unit charge.

$$\text{e.m.f., } E\text{ (V)} = \frac{\text{electrical work done on the charge, } W\text{ (J)}}{\text{charge flow, } Q\text{ (C)}} \qquad E = \frac{W}{Q}$$

Both e.m.f. and p.d. are measured in volts, with a voltmeter. Despite its name, e.m.f. is not a force: it is energy per charge.

Conservation of energy, round a loop. In a series circuit, the energy each coulomb gains in the sources must all be given up in the components by the time it gets back round. So, round any complete loop:

$$\text{sum of the e.m.f.s} = \text{sum of the p.d.s}$$

Exercise

Choose the correct option

Check 10

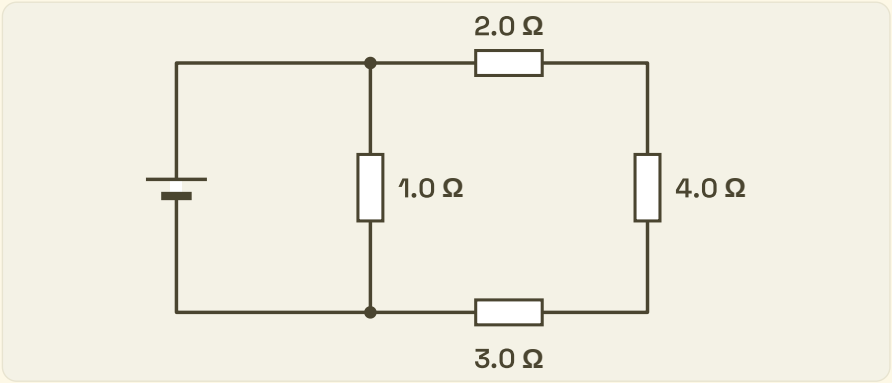


Figure 23.29 *For question 1.*

1. Four resistors of resistances 1.0 Ω, 2.0 Ω, 3.0 Ω and 4.0 Ω are connected together as shown in the figure above. A battery is connected across the 1.0 Ω resistor. If I_0 is the battery current and I_1 is the current in the 4.0 Ω resistor, the ratio $\frac{I_0}{I_1}$ is:

a 0.1 b 0.5
c 1.5 d 10

Check 11

2. Four different types of lamps produce the same brightness. The power consumption and the current marked on the packaging of each are shown in the table. Which type of lamp has the greatest efficiency?

Type of lamp	Power (W)	Current (A)
filament	60	0.25
fluorescent	12	0.05
halogen	48	4
LED	6	0.5

a filament b fluorescent
c halogen d LED

Numerical problems

12. a. The copper wire supplying the sockets in your house has a cross-sectional area of 2.5 mm². A hair dryer draws a current of 8.2 A. What is the mean drift velocity of the electrons in the wire? Take the number density of free electrons in copper as $8.49 \times 10^{28}\text{ m}^{-3}$.

b. A 1 molar solution of copper sulphate contains 6.02×10^{26} Cu^{2+} ions per cubic meter. A beaker of this solution has two electrodes placed in it, through which a current of 30 mA is passed. What is the mean drift velocity of the copper ions? Take the cross-sectional area of the solution as 100 cm², and assume the copper ions carry all the current.

13. A tungsten lamp filament of diameter 0.046 mm carries a current of 0.50 A. What is the mean drift velocity of the electrons in the filament? Take the number density of free electrons in tungsten as $1.26 \times 10^{29}\text{ m}^{-3}$.

14. A copper wire of diameter 0.65 mm carries a current of 0.25 A. There are 8.5×10^{28} charge carriers in each cubic meter of copper; the charge on each carrier (an electron) is 1.6×10^{-19} C. Calculate the drift speed of the charge carriers.

15. a. State what physical quantity is represented by each term in the equation I = nAvq.

DID YOU KNOW?

The kilowatt-hour on your bill

Nepal Electricity Authority bills energy in **units**: one unit is a **kilowatt-hour** (kWh), the energy used by 1 kW for 1 hour, $1000\ \text{W} \times 3600\ \text{s} = 3.6 \times 10^6$ J. A 1000 W heater run for 3 hours uses 3 units.

IN SHORT

Key ideas of Unit 23

- **Current** $I = \dfrac{\Delta Q}{\Delta t}$, in amperes; 1 C = 1 A s. Conventional current flows from + to −, opposite to the electrons.
- **Drift:** $I = nAvq$. Free electrons drift at a fraction of a millimeter per second; the field that drives them travels at nearly the speed of light.
- **P.d.** $V = \dfrac{W}{Q}$ (electrical energy → other forms, in a component); **e.m.f.** $E = \dfrac{W}{Q}$ (other energy → electrical, in a source). Round a loop, the e.m.f.s equal the sum of the p.d.s.
- **Resistance** $R = \dfrac{V}{I}$, in ohms. **Ohm's law:** I ∝ V for a metal at constant temperature. Filament lamps, diodes, thermistors and LDRs are non-ohmic.
- **Resistivity** $R = \dfrac{\rho l}{A}$, in Ω m. A strain gauge: ΔR ∝ ΔL.
- **Series:** $R = R_1 + R_2$. **Parallel:** $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$.
- **Potential divider:** $\dfrac{V_1}{V_2} = \dfrac{R_1}{R_2}$, $V_1 = \dfrac{R_1}{R_1 + R_2} V_{\text{in}}$; with a thermistor or LDR it becomes a sensor.
- **Internal resistance:** $E = V + Ir$, $V = E - Ir$, $I = \dfrac{E}{R + r}$; on a V–I graph, intercept E, gradient −r.
- **Power** $P = VI = I^2R = \dfrac{V^2}{R}$, in watts; energy W = VIt.

CAREFUL

E.m.f. or p.d.?

Both are joules per coulomb. Ask which way the energy goes. In a **source**, other energy becomes electrical energy: that is **e.m.f.** In a **component**, electrical energy becomes other forms: that is **p.d.**

A 6 V battery driving a lamp and a resistor in series might give 4 V across the lamp and 2 V across the resistor: 4 + 2 = 6.

Check 3

A cell of e.m.f. 1.5 V drives a charge of 20 C round a circuit. The energy it converts into electrical energy is:

a 13 J
b 0.075 J
c 21.5 J
d 30 J

23.5 Resistance

When current flows through a material, the material gets in the way of the flow. On a microscopic level, the drifting electrons collide with the **vibrating positive ion cores** of the metal lattice. In each collision the electron gives up some of the energy it gained from the field, and the ions vibrate more: the wire heats up. This is why a current always heats a wire (except a superconductor).

Resistance is the opposition of a component to the flow of electric current through it. Its symbol is R and its unit is the **ohm** (Ω).

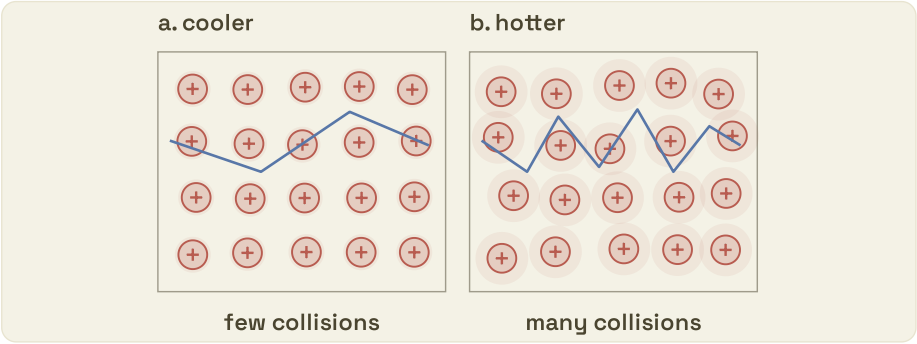


Figure 23.8 *Electrons drifting through a metal lattice. a. Cooler: the ions vibrate a little. b. Hotter: they vibrate more, filling more of the space, and get in the way of the electrons more often: the resistance is higher.*

The same picture explains why **the resistance of a metal increases as its temperature rises**. In a hotter metal the ion cores vibrate with larger amplitude and get in the way of the drifting "gas" of electrons more often.

Components with a very high resistance let almost no current through them: they are **insulators**. And some materials, cooled to very low temperatures, have **zero resistance**: they are **superconductors**. A current set going in a superconducting loop keeps flowing for years with no cell at all. The magnets of MRI scanners in hospitals are superconducting coils, cooled by liquid helium to about −269 °C.

23.6 Current–potential difference characteristics and Ohm's law

Many properties of a component can be found from a graph of the current through it against the p.d. across it. This graph is called the component's **electrical characteristic**, or its **I–V characteristic**. It is measured for both directions of current, which some books call forward and reverse bias, by connecting the component one way round and then the other: negative values of V and I mean the current is the other way.

The simplest characteristic is that of a fixed resistor: a straight line through the origin. The graph can be drawn with **either variable on either axis**.

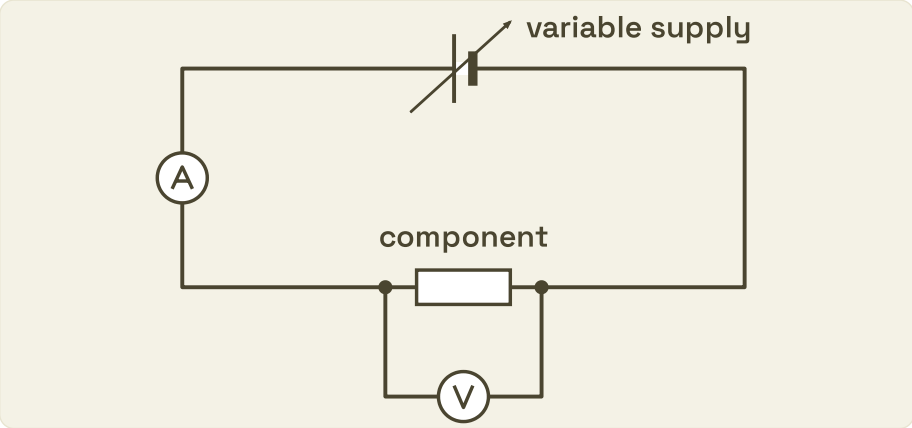


Figure 23.9 Measuring a characteristic: a variable supply drives a current through the component; an ammeter in series measures the current, and a voltmeter across the component the p.d. Reversing the supply gives the negative half of the graph.

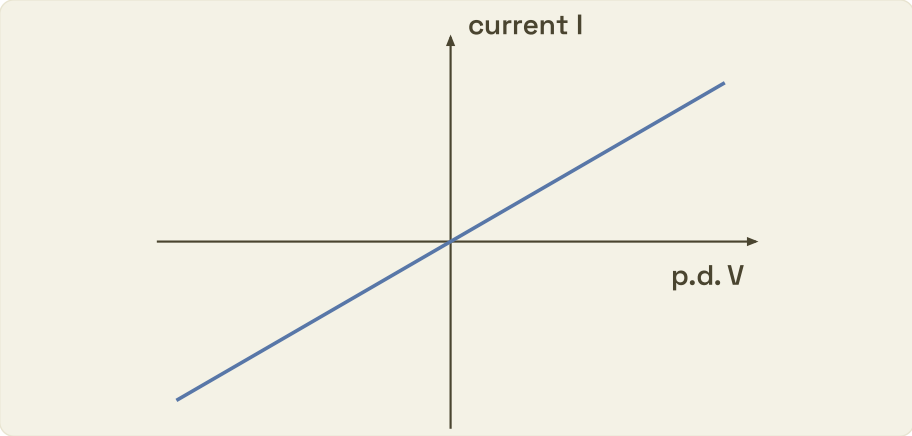


Figure 23.10 The characteristic of a fixed resistor: current against p.d., a straight line through the origin, running into the negative quadrant when the p.d. and the current are reversed.

23.6.1 Ohm's law

In 1827, Georg Ohm measured the current through metal wires at different p.d.s, keeping the temperature constant. He found that the current I through a wire was **directly proportional** to the p.d. V across it, as long as the temperature (and other physical conditions) stayed the same: the characteristic is a straight line through the origin.

Ohm's law: the current through a metal conductor is directly proportional to the potential difference across it, provided its temperature and other physical conditions remain constant.

$$I \propto V$$

Ohm's law holds only for some conductors, and only in some conditions: it is a special case, true of metal wires at constant temperature. The idea of resistance, however, applies to every component. The resistance of a component is **defined** as the ratio of the p.d. across it to the current through it:

$$R = \frac{V}{I} \qquad \text{or} \qquad V = IR$$

The ohm: a component has a resistance of one ohm if a p.d. of 1 V across it drives a current of 1 A through it: 1 Ω = 1 V/A.

This definition holds for **every** component, whether it obeys Ohm's law or not, at each point of its characteristic. It says that for a given p.d., a small current means a large resistance; and that for a given current, a large resistance needs a large p.d. to drive

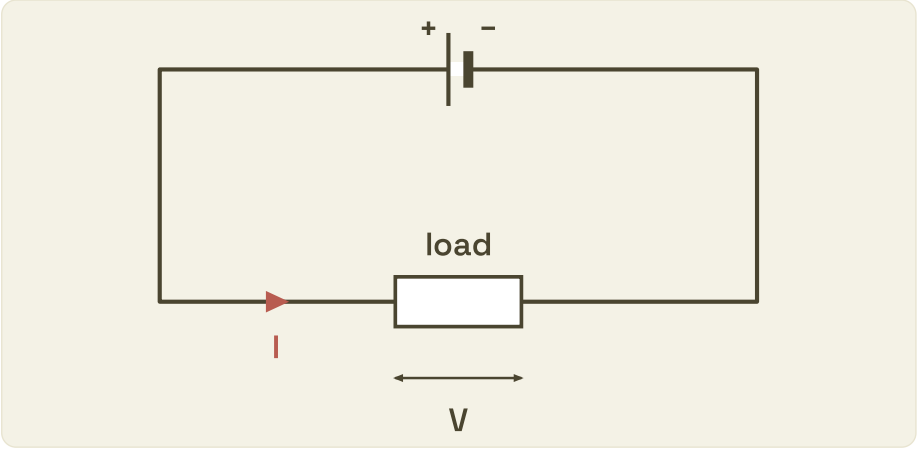


Figure 23.28 A steady current I through a load with a p.d. V across it. In a time t a charge Q = It passes through, and energy W = QV is dissipated in the load.

From the definition of p.d., the energy dissipated is

$$W = QV$$

where W is the energy dissipated in a time t, Q is the charge that flows in time t, and V is the p.d. across the load. The current is steady, so Q = It, and therefore

$$W = VIt$$

Power is the rate of dissipation (or transfer) of energy, $P = \frac{W}{t}$. Since V and I are constant:

$$P = \frac{W}{t} = \frac{VIt}{t} \quad \Rightarrow \quad \boxed{P = VI}$$

For a resistor, V = IR, so two more forms follow:

$$P = (IR)I = I^2R \qquad \text{and} \qquad P = V\left(\frac{V}{R}\right) = \frac{V^2}{R}$$

The unit of power is the **watt** (W): 1 W = 1 J/s. W comes out in joules and P in watts when V, I and t are in volts, amperes and seconds. (Don't confuse W for energy, in the equations, with W for watts, the unit.)

Which form to use? I^2R when the current is known (resistors in **series** share a current, so the larger resistance gets the larger power); V^2/R when the p.d. is known (resistors in **parallel** share a p.d., so the **smaller** resistance gets the larger power).

WORKED EXAMPLE

The safe current through a resistor

A manufacturer says that the maximum safe power for a range of resistors is 250 mW. What is the maximum safe current for a 47 kΩ resistor from this range?

Answer

$P = I^2R$, so

$$I^2 = \frac{P}{R} = \frac{250 \times 10^{-3}\ \text{W}}{47\,000\ \Omega} = 5.32 \times 10^{-6}\ \text{A}^2 \quad \Rightarrow \quad I = \mathbf{2.3\ mA}$$

YOUR TURN

A heater's element

A 1000 W heater runs on 230 V mains. What current does it take, and what is the resistance of its element when hot?

Answer

$I = \frac{P}{V} = \frac{1000}{230} = \mathbf{4.3\ A}$; $R = \frac{V^2}{P} = \frac{230^2}{1000} = \mathbf{53\ \Omega}$.

$$V = E - Ir \qquad \text{or} \qquad V = -rI + E$$

which is the equation of a straight line, y = mx + c, with a **negative gradient**. Plot the terminal p.d. V against the current I, for different values of the load resistance R:

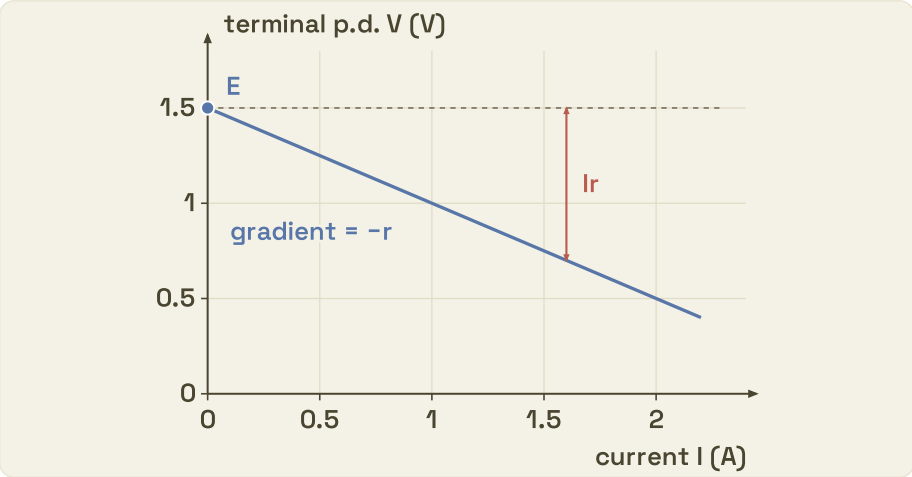


Figure 23.27 *The terminal p.d. of a cell against the current drawn from it: a straight line sloping down. Its intercept on the V-axis (at I = 0, open circuit) is the e.m.f. E; its gradient is −r. The "lost volts" Ir grow with the current.*

- the **intercept on the V-axis** (where I = 0) is the **e.m.f.** E;
- the **gradient** is **−r**: its magnitude is the internal resistance.

The difference E − V = Ir is sometimes called the **lost volts**: the p.d. used up inside the cell.

Putting V = IR for the external resistor into E = V + Ir gives another useful form, which finds the current:

$$E = IR + Ir \quad \Rightarrow \quad \boxed{I = \frac{E}{R + r}}$$

WORKED EXAMPLE

A torch

A torch lamp of resistance 6.4 Ω runs from a battery whose e.m.f. is 4.5 V and whose internal resistance is 0.4 Ω. Find the current through the lamp.

Answer

$$I = \frac{E}{R + r} = \frac{4.5\ \text{V}}{6.4\ \Omega + 0.4\ \Omega} = \frac{4.5}{6.8} = \mathbf{0.66\ A}$$

The terminal p.d. is then $V = IR = 0.66 \times 6.4 = 4.2$ V: 0.3 V is lost inside the battery. As a battery runs down, its internal resistance grows, the lost volts grow, and the torch dims.

Check 9

A cell of e.m.f. 2.0 V and internal resistance 0.50 Ω drives a current of 1.0 A. Its terminal p.d. is:

a 2.5 V b 2.0 V
c 1.5 V d 0.50 V

23.12 Energy and power in electrical circuits

Consider a simple circuit in which a steady current I flows through a **load**: a motor, a heater, or simply a resistor. As the charge flows through the load, it loses potential energy, which the load turns into other forms (it **dissipates** the energy). The energy dissipated equals the potential energy lost by the charge as it moves through the p.d. between the load's input and output terminals.

it. What Ohm's law adds is that, for an ohmic conductor, R is the **same** at every point.

If the characteristic is drawn with **current on the x-axis** and p.d. on the y-axis (a V–I graph), then for a component that obeys Ohm's law **the gradient of the line is the resistance**. A high resistance gives a steep line; a low resistance, a shallow one. (On an I–V graph, the gradient is 1/R, so a high resistance gives a shallow line: always check which way round the axes are.)

Fixed resistors and metal wires at constant temperature obey Ohm's law over their whole range of current: their characteristics are straight lines through the origin. They are called **ohmic conductors**.

Check 4

On a graph of p.d. (y-axis) against current (x-axis), the line for resistor X is steeper than the line for resistor Y. Then:

a X has the greater resistance b Y has the greater resistance
c they have the same resistance d neither obeys Ohm's law

23.6.2 Other electrical characteristics

The filament lamp. A tungsten filament lamp turns electrical energy into light and heat. As the current increases, the electrons collide with the tungsten ions more often and give them more energy: the ions vibrate with larger amplitude, the temperature rises (to over 2000 °C), and so does the resistance. A higher current means a higher temperature, which means a higher resistance. The characteristic **bends**: as V increases, the ratio V/I, the resistance, increases.

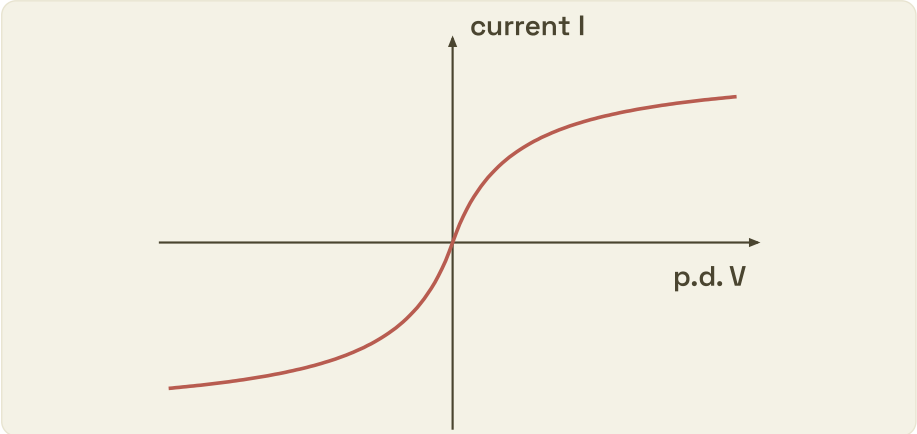


Figure 23.11 *The characteristic of a filament lamp. At low current the filament is cool and the graph is nearly straight; as it heats up its resistance rises and the curve flattens: each extra volt adds less current. The same shape appears with the current reversed.*

Components whose characteristics are not straight lines through the origin are **non-ohmic conductors**. Their resistance V/I depends on the current.

Fixed resistors and filament lamps behave the same with the current in either direction: their characteristics have the same shape for positive and negative p.d.

The semiconductor diode doesn't. Diodes act as **one-way valves** for current: they let it flow in one direction and block it in the other. They are found in nearly every electronic circuit, and in mains power supplies (chargers and adapters) they turn alternating current (a.c.) into direct current (d.c.).

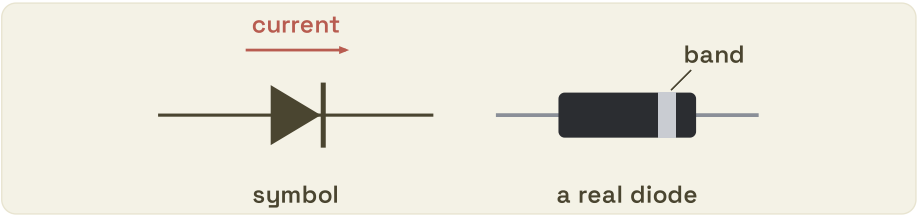


Figure 23.12 *A diode's symbol: the triangle points the way conventional current can flow. The real diode has a band at the same end as the symbol's bar.*

A diode conducts only in **forward bias**, in the direction the arrow of its symbol points. On the component itself, a colored ring marks the end the current comes out of (the end with the bar in the symbol). In **reverse bias** it doesn't conduct: its resistance is very high (effectively infinite). In forward bias it hardly conducts at all until the p.d. reaches about 0.6 V (for silicon); after that, its resistance becomes very low and the current rises steeply. The characteristic of a diode is usually drawn as current (y-axis) against p.d. (x-axis):

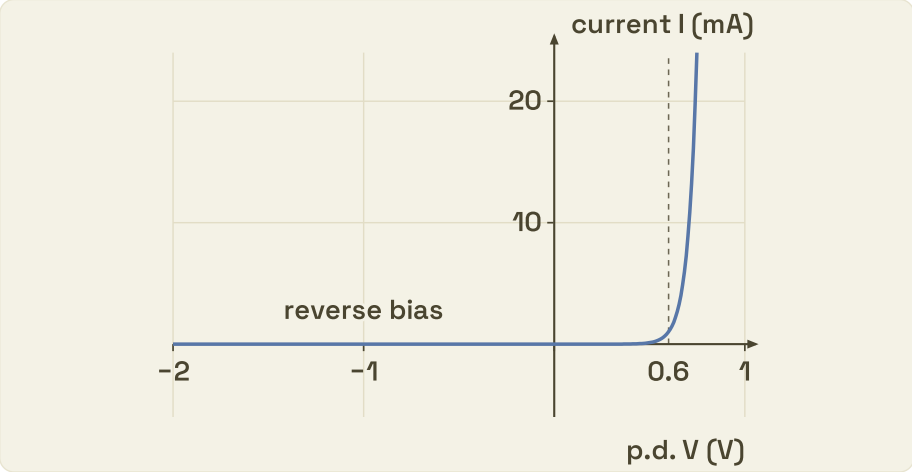


Figure 23.13 *The I–V characteristic of a silicon diode. In reverse bias, practically no current flows. In forward bias, the current stays nearly zero until about 0.6 V and then rises very steeply.*

Check 5

A component's resistance V/I rises as the current through it increases. It could be:

a a fixed resistor at constant temperature
b a filament lamp
c a wire obeying Ohm's law
d a superconductor

23.6.3 Thermistors

A **thermistor** (from *thermal resistor*) is a component whose resistance changes a lot with temperature. It is made from the oxides of metals, in many shapes and sizes, so that its resistance changes much more than an ordinary resistor's.

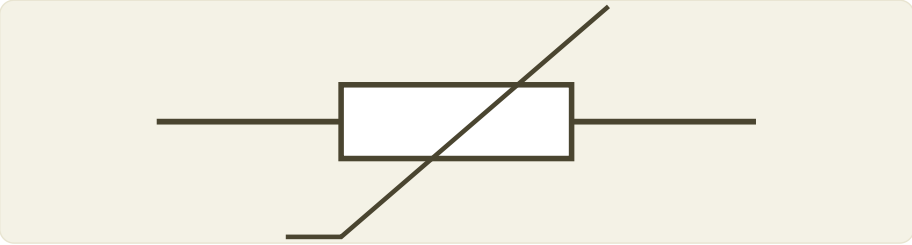


Figure 23.14 *The circuit symbol of a thermistor.*

Most thermistors are **negative temperature coefficient** (NTC) thermistors: their **resistance decreases as their temperature increases**. (In a semiconductor, more charge carriers are freed as it warms, which outweighs the extra vibration of the lattice.) The change is non-linear: a graph of resistance against temperature is a curve.

The resistance changes by a large amount with temperature, but the change is non-linear: the curve is steep at low temperatures and flattens as the temperature rises.

Thermistors are used to **sense temperature** and to control devices: the change in the thermistor's resistance changes a current or a voltage in a circuit, which can switch something on or off. They are found in:

- thermal cut-outs that switch a device off before it overheats (a hair dryer, a phone's battery charger);

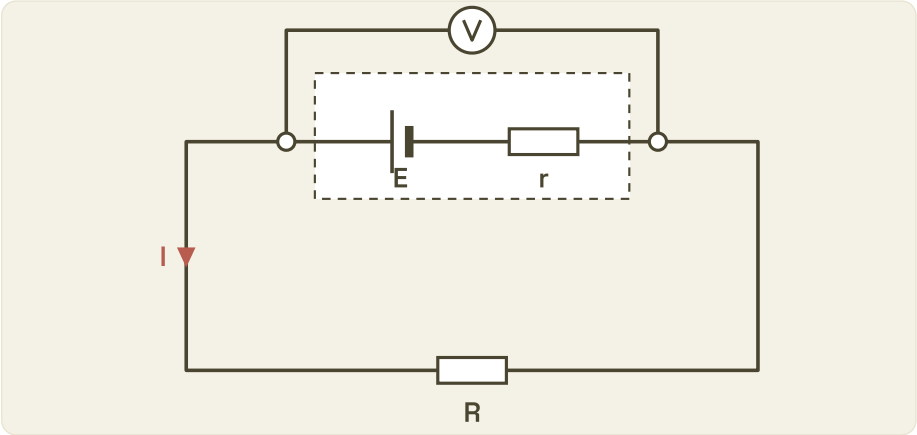


Figure 23.25 *A real cell, drawn as an ideal cell of e.m.f. E in series with its internal resistance r (in the dashed box), driving a current I through an external load R. The voltmeter across the terminals reads the terminal p.d. V = IR.*

IN SHORT

Points to remember

- **E.m.f.** is the work done by a source in moving unit charge round a complete circuit (other energy → electrical).
- **P.d.** is the work done by unit charge passing through a component (electrical energy → other forms).

Finding E and r

The internal resistance can't be measured directly: it is "inside" the supply. It is found from the supply's characteristic, with a circuit like Figure 23.26.

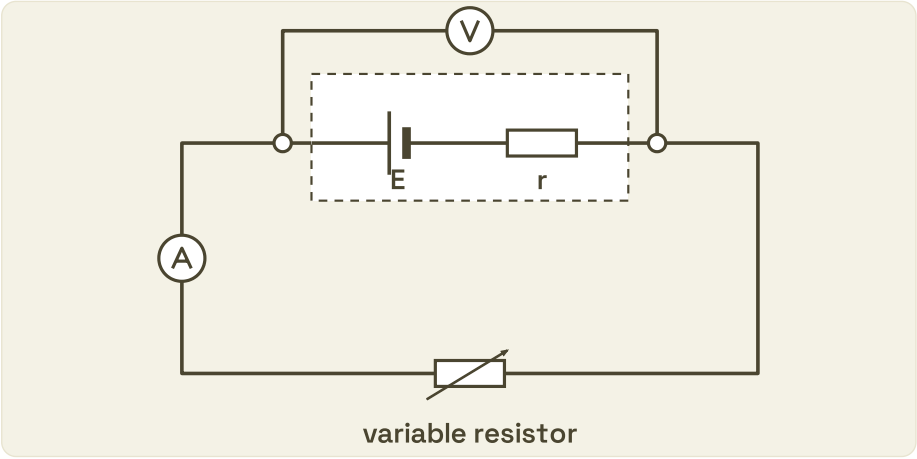


Figure 23.26 *Finding a cell's e.m.f. and internal resistance: the cell (with its internal resistance r) drives a current through an ammeter and a variable resistor; a voltmeter across the cell's terminals reads the terminal p.d. V. Changing the variable resistor changes the current I.*

By conservation of energy, the e.m.f. equals the sum of the p.d.s round the circuit. There are two: the p.d. V across the external variable resistor (the terminal p.d., which the voltmeter reads) and the p.d. across the internal resistance, which can't be measured directly but is equal to Ir. So:

$$E = V + Ir$$

The current I is measured by the ammeter, and E and r are constants, so this can be rewritten as

A street light that switches on at dusk uses the opposite arrangement: with the fixed resistor and the LDR swapped (or the output taken across the LDR), the output rises as it gets **darker**, and switches the lamp on when it passes a set value.

WORKED EXAMPLE

A light sensor

A light sensor consists of a 6.0 V battery, an 1800 Ω resistor and a light-dependent resistor in series. When the LDR is in darkness, the p.d. across the resistor is 1.2 V.

a. Calculate the resistance of the LDR when it is in darkness.

b. In the light, the LDR's resistance falls to 2400 Ω. Calculate the p.d. across the LDR.

Answer

a. The p.d. across the LDR is $6.0 - 1.2 = 4.8$ V. The current, the same in both, is found from the resistor:

$$I = \frac{V}{R} = \frac{1.2\ \text{V}}{1800\ \Omega} = 0.667\ \text{mA}$$

So the LDR's resistance is

$$R = \frac{4.8\ \text{V}}{0.667 \times 10^{-3}\ \text{A}} = \mathbf{7200\ \Omega}$$

(Or by the ratio: 4.8 V : 1.2 V = 4 : 1, so the LDR is $4 \times 1800 = 7200\ \Omega$.)

b. The ratio of the resistances is $\frac{2400\ \Omega}{1800\ \Omega} = 1.33$, so the p.d.s must be in the same ratio, adding up to 6.0 V:

$$V_{\text{LDR}} = \frac{2400}{2400 + 1800} \times 6.0\ \text{V} = \mathbf{3.4\ V}$$

and 2.6 V across the resistor ($3.4 \div 2.6 = 1.33$).

Check 8

In a potential divider of a fixed resistor and an NTC thermistor across a 9 V battery, the output is taken across the **thermistor**. As the thermistor is warmed, the output:

a rises

b falls

c stays at 4.5 V

d stays at 9 V

23.11 Internal resistance and electromotive force

Measure the p.d. across the terminals of a cell, its **terminal p.d.**, and it depends on the current being drawn from it. When **no current** is drawn (the cell on **open circuit**), the terminal p.d. has its largest value: this is the cell's **e.m.f.** As the current increases, the terminal p.d. falls. Why?

The chemicals inside a cell oppose the flow of current through it: the cell has an **internal resistance**, r. When the cell drives a current through an external circuit (a **load**), part of its e.m.f. drives the current through the load, and the rest drives the same current through its own internal resistance. The internal resistance behaves as if it were a resistor **in series** with an ideal cell and the external circuit.

The chemical action inside a cell produces a surplus of electrons at one terminal (the negative) and a shortage at the other (the positive). It can keep electrons flowing, a current, through any circuit connected across its terminals, for as long as the chemical action lasts. The cell does work in moving charge round the circuit:

Electromotive force (e.m.f.), symbol E, is the electrical work done by a source in moving unit charge round a complete circuit:

$$\text{e.m.f.} = \frac{\text{work done by the source}}{\text{charge}} \qquad E = \frac{W}{Q}$$

In a circuit, the current carries energy from a store, such as a battery, to the components, which pass it on to the surroundings: a lamp, for example, by light and by heating.

Real power supplies, batteries and laboratory power packs, always have some internal resistance r. As the current flows through the supply, its internal resistance has a p.d. across it, and some electrical energy is turned into thermal energy **inside the supply**. That is one reason a phone or tablet warms up when it is used, or charged, for a long time.

The internal resistance of most batteries and power supplies is very low: a typical AA cell has an internal resistance of about 0.2 Ω, so it can usually be ignored in a circuit of several ohms. It matters when the external resistance is small, or the current large: a car's starter motor draws over 100 A, and its battery must have an internal resistance of only a few milliohms.

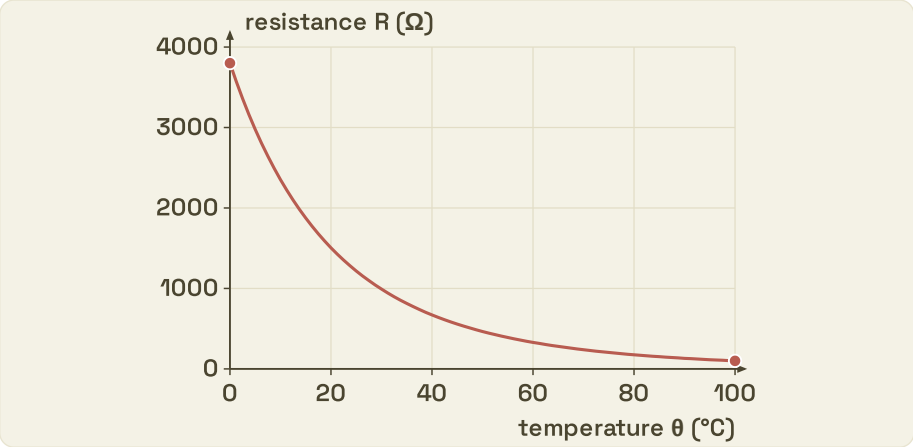


Figure 23.15 *The resistance of a typical NTC thermistor against temperature: about 3800 Ω at 0 °C, falling steeply at first, then more slowly, to about 100 Ω at 100 °C. A big change, but not a straight line.*

- room heaters, water heaters, rice cookers and refrigerators that hold a temperature;
- digital thermometers, including the ones used to check a fever;
- car engines' management circuits.

23.6.4 Strain gauge

A **strain gauge** is a length of very fine wire, folded back and forth many times, sealed in a small rectangle of thin plastic. It is glued very firmly to the material under test: a beam of a bridge, the wing of an aircraft, the deck of a weighing scale.

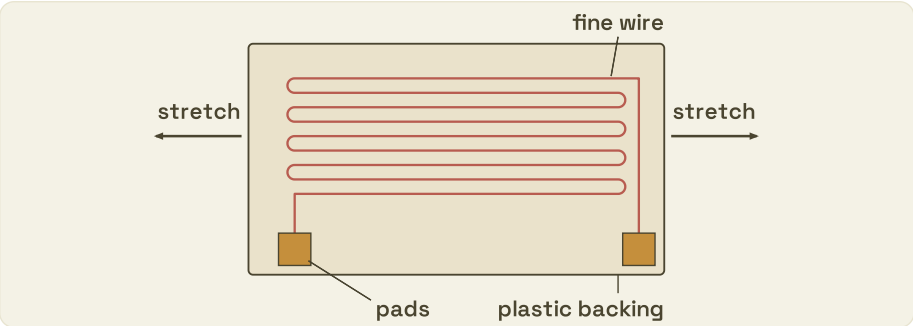


Figure 23.16 *A strain gauge: a long fine wire folded into a zigzag of parallel strands, sealed in thin plastic, with two connecting pads. Stretched along the strands, each strand gets slightly longer and thinner, and the resistance rises.*

When the material is stretched (it experiences a strain), the gauge and its wire are stretched too. The wire gets slightly **longer** and its cross-sectional area slightly **smaller**: both changes **increase its resistance**. The folded strands make the total length of wire large, so a small strain gives a measurable change.

How large is the change? The resistance of a wire of length L and uniform cross-sectional area A is (Section 23.7)

$$R = \frac{\rho L}{A}$$

where ρ is the resistivity of the wire's material. When the wire extends by a small amount ΔL, take the change in its area as negligible. The new resistance is

$$R + \Delta R = \frac{\rho(L + \Delta L)}{A}$$

Subtracting the first equation from the second:

$$\Delta R = \frac{\rho \Delta L}{A} \qquad \text{so} \qquad \Delta R \propto \Delta L$$

The strain is proportional to the extension ΔL, so the **change in resistance is proportional to the strain**. Measuring ΔR measures how much the material has stretched. (In a real gauge the area does shrink a little, which adds to the change; the result is still proportional to the strain.)

DID YOU KNOW?

Weighing with a strain gauge

Most digital weighing scales, from a shop's kitchen scale to the scale that weighs trucks at a customs point, work by strain gauges glued to a metal beam (a **load cell**). The load bends the beam by a fraction of a millimeter, the gauges stretch, and their resistance changes by well under one percent; the electronics turn that into kilograms.

23.6.5 Light-dependent resistor (LDR)

A **light-dependent resistor** (LDR) is made by laying a film of a semiconductor, such as **cadmium sulphide**, between two metal electrodes. Light falling on the semiconductor frees electrons in it, which can then carry current: **the brighter the light, the lower the resistance**. Typically, its resistance is about 1 MΩ in moonlight and about 100 Ω in sunlight.

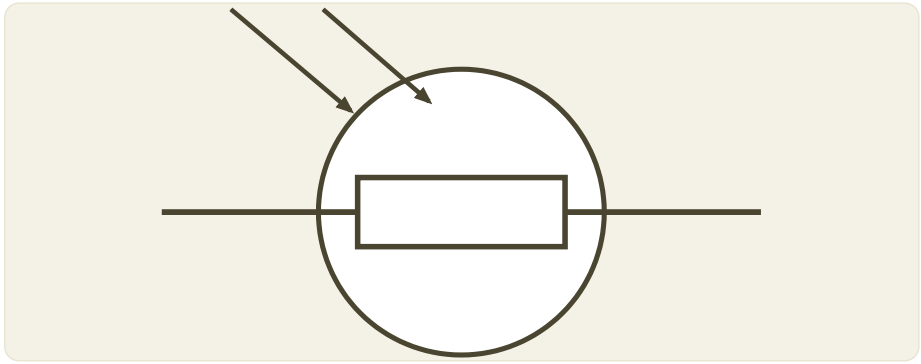


Figure 23.17 *The circuit symbol of an LDR: a resistor in a circle, with two arrows for light falling on it.*

At a steady light intensity the resistance of an LDR is constant, but it changes with the intensity, and **not linearly**: a graph of resistance against light intensity is a curve, falling steeply in dim light and flattening in bright light (Figure 23.35, in the exercise).

23.6.6 The temperature characteristic of an NTC thermistor

The resistance of most materials changes slightly with temperature. A thermistor is made so that the change is large (Figure 23.15). For an NTC thermistor, as the temperature θ rises, the resistance R falls: by a large amount, but not in a straight line.

23.7 Resistivity

How do we compare how well different materials conduct? How do we choose a material for the pins inside a mains plug, or for the insulators that hold up a high-voltage line? The resistance of a wire depends on its **length** and its **cross-sectional area**, as well as on the **material** it is made of.

- **Length.** The resistance R increases with the length l: there are more ion cores in the way of the electrons. If the length doubles, the resistance doubles: $R \propto l$.
- **Area.** The resistance decreases as the cross-sectional area A increases: there are more pathways side by side for the electrons. If the area doubles, the resistance halves: $R \propto \frac{1}{A}$.

Combining both proportionalities:

$$R \propto \frac{l}{A}$$

Replacing the proportionality sign by a constant of proportionality ρ:

- the p.d.s across them **add up** to the input: $V_{in} = V_1 + V_2$.

By V = IR, $V_1 = IR_1$ and $V_2 = IR_2$. Dividing V_1 by V_2:

$$\frac{V_1}{V_2} = \frac{IR_1}{IR_2} \quad \Rightarrow \quad \boxed{\frac{V_1}{V_2} = \frac{R_1}{R_2}}$$

The p.d. is divided in the ratio of the resistances: the larger resistance takes the larger share.

And since $I = \frac{V_{in}}{R_1 + R_2}$,

$$\boxed{V_1 = \frac{R_1}{R_1 + R_2} V_{in}} \qquad V_2 = \frac{R_2}{R_1 + R_2} V_{in}$$

WORKED EXAMPLE

A potential divider

A potential divider consists of two resistors, of 3.0 Ω and 6.0 Ω, in series with a battery of 18 V. Calculate, for each resistor, a. the p.d. across it, b. the current in it.

Answer

a. The two resistors carry the same current, and their p.d.s add up to the battery's 18 V. The total resistance is 3.0 + 6.0 = 9.0 Ω, so:

$$V_{3\,\Omega} = \frac{3.0\ \Omega}{9.0\ \Omega} \times 18\ \text{V} = \mathbf{6.0\ V}, \qquad V_{6\,\Omega} = \frac{6.0\ \Omega}{9.0\ \Omega} \times 18\ \text{V} = \mathbf{12\ V}$$

(Check: 6 + 12 = 18 V, in the ratio 3 : 6.)

b. The current is the same in both, since they are in series:

$$I = \frac{V}{R} = \frac{18\ \text{V}}{9.0\ \Omega} = \mathbf{2.0\ A}$$

23.10.2 Potential dividers as sensors

The most useful potential dividers are **sensor circuits**. Replace R_2 by a component whose resistance changes with a physical quantity, such as temperature or light intensity, and the p.d. across the fixed resistor R_1 changes with that quantity too. The voltmeter across R_1 can then be **calibrated** in degrees or in lux: the circuit becomes an electronic thermometer or an electronic light meter, or its output can switch a heater, a fan or a lamp on and off.

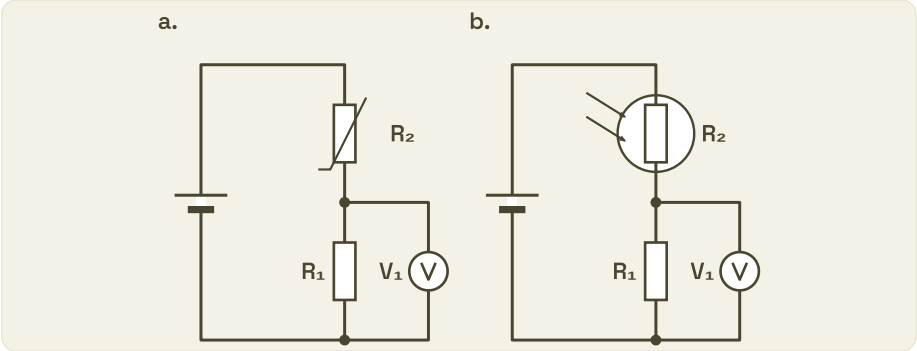


Figure 23.24 *Two sensor circuits. a. An NTC thermistor as R_2: as the temperature rises, its resistance falls, and the output V_1 across the fixed resistor R_1 rises. b. An LDR as R_2: as the light gets brighter, its resistance falls, and V_1 rises.*

An electronic thermometer. With an NTC thermistor as R_2: as the temperature rises, the thermistor's resistance falls, so it takes a smaller share of the p.d., and the p.d. V_1 across the fixed resistor **rises**. Increasing temperature gives an increasing output. If the voltmeter were connected across the thermistor instead, the opposite would happen: the output would fall as the temperature rises. Most uses want the output to rise with temperature, so the voltmeter is usually connected across the fixed resistor.

A light sensor. An LDR is made of a semiconductor: as light shines on it, electrons are freed and can carry current, so its resistance falls. In the dark an LDR's resistance can be of the order of megohms, while in bright light it can drop to a few hundred ohms. With an LDR as R_2, as the light gets brighter, the output V_1 across the fixed resistor **rises**.

WORKED EXAMPLE

Four equal resistors

Four resistors, each of resistance 1.5 Ω, are connected as shown. Calculate their combined resistance.

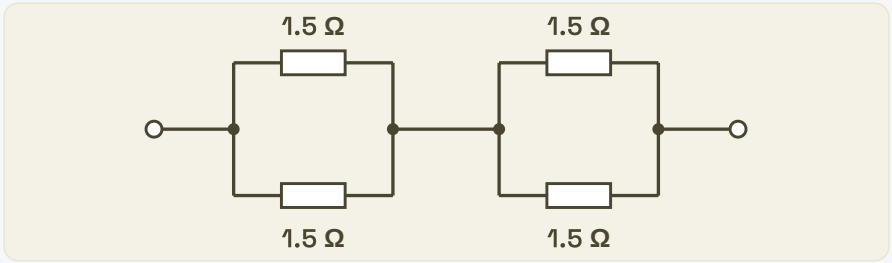


Figure 23.22 Two pairs of 1.5 Ω resistors: each pair in parallel, and the two pairs in series.

Answer

Each pair in parallel:

$$\frac{1}{R} = \frac{1}{1.5} + \frac{1}{1.5} = \frac{2}{1.5} \quad \Rightarrow \quad R = 0.75\ \Omega$$

The two pairs in series: $0.75 + 0.75 = \mathbf{1.5\ \Omega}$, the same as one of the resistors.

Check 7

A 6 Ω and a 3 Ω resistor are joined in parallel. Their combined resistance is:

a 9 Ω
b 4.5 Ω
c 2 Ω
d 0.5 Ω

23.10 Potential dividers

A **potential divider** uses two resistances in series to divide an input p.d. into smaller parts. It can give a fixed or a varying output p.d., which becomes the input to another circuit, or be used to measure a p.d. or a resistance. The volume knob of a radio is a potential divider.

23.10.1 The potential divider circuit

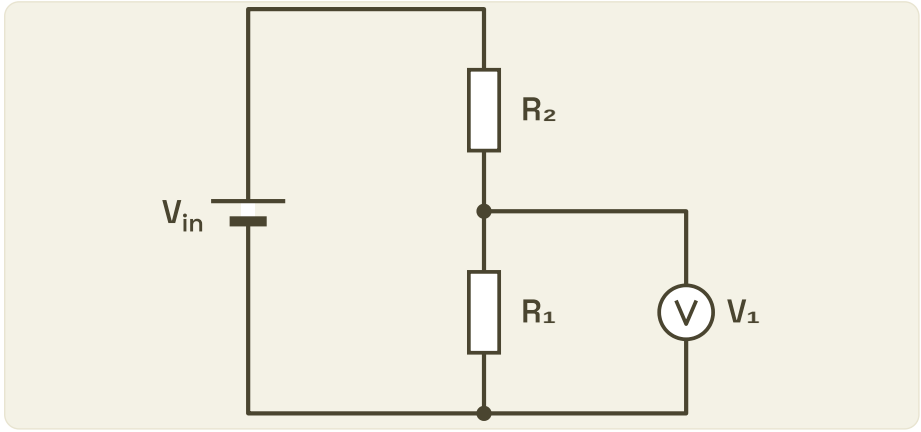


Figure 23.23 A potential divider: R_1 and R_2 in series across an input p.d. V_{in}. A voltmeter of very high resistance measures V_1, the p.d. across R_1.

In the circuit of Figure 23.23, the voltmeter has a very high resistance, so the tiny current through it can be ignored. Then:

- R_1 and R_2 are **in series**, so the total resistance is $R = R_1 + R_2$, and the **same current** I passes through both;

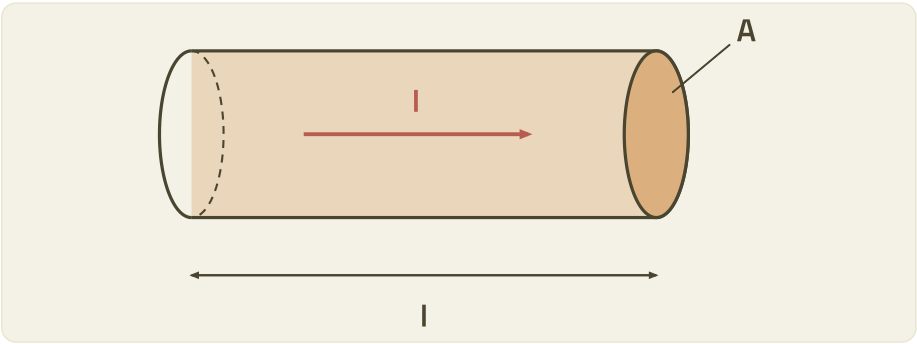


Figure 23.18 A uniform conductor of length l and cross-sectional area A, with current flowing along it.

ρ (the Greek letter rho) is the **resistivity** of the material. It gives the resistance of the **material itself**, whatever the size and shape of the piece. Rearranged, it defines resistivity:

$$\rho = \frac{RA}{l}$$

with R in ohms, A in square meters and l in meters, so **the unit of resistivity is the ohm meter** (Ω m).

The resistivity of a material is numerically the resistance of a piece of it 1 m long with a cross-sectional area of 1 m^2 (a 1 m cube, with the current between opposite faces).

The resistivity of a material depends on its inside: in particular, on the **number of free charge carriers** per cubic meter and on how easily they move through the structure (their **mobility**). The arrangement of the atoms, any impurities, and the temperature all affect the mobility.

Material	Resistivity at 20 °C (Ω m)	
silver	1.6×10^{-8}	best conductor
copper	1.68×10^{-8}	wires and cables
aluminium	2.7×10^{-8}	overhead power lines
tungsten	5.6×10^{-8}	lamp filaments
nichrome	about 1.1×10^{-6}	heater elements
carbon (graphite)	about 3×10^{-5}	resistors
silicon (pure)	about 2×10^{3}	semiconductor
glass	about 10^{10}–10^{14}	insulators
ABS plastic	about 1×10^{15}	mains plugs

Good conductors have very low resistivities; good insulators, such as ABS plastic (the material most mains plugs are made of), have extremely high ones: there is a factor of about 10^{23} between copper and ABS. **Superconductors** have zero resistivity below a certain **critical temperature**.

A water picture. A hose pipe packed with sand is like a material of high resistivity: push water through it, and only a trickle comes out. An empty hose pipe, under the same pressure, lets the water rush through: a low resistivity. But how much flows also depends on the hose's length and width, just as a wire's resistance depends on its length and area as well as its resistivity.

Resistivity and temperature.

- The resistivity of **metals increases** with temperature (the ions vibrate more).
- The resistivity of many **semiconductors**, such as silicon and germanium, **decreases** as they warm: more charge carriers are freed, which outweighs the extra vibration. NTC thermistors work this way.
- A **superconductor** behaves like a metal above its critical temperature, its resistivity falling as it cools; at the critical temperature its resistivity drops suddenly to zero.

WORKED EXAMPLE

A carbon chip

A carbon chip of resistivity $3.0 \times 10^{-5}\ \Omega\ \text{m}$ has the dimensions shown. What is its resistance for a current in the direction shown?

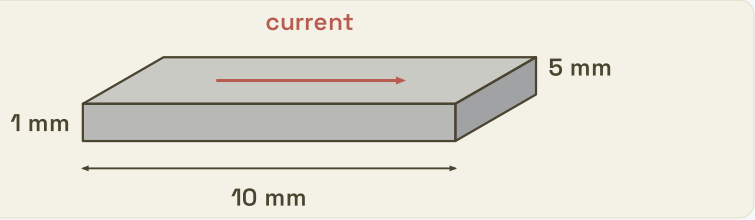


Figure 23.19 *A carbon chip 10 mm long, 5 mm wide and 1 mm thick, with the current flowing along its 10 mm length.*

Answer

The length along the current is $l = 10\ \text{mm} = 10 \times 10^{-3}\ \text{m}$. The cross-section the current passes through is $A = 5\ \text{mm} \times 1\ \text{mm} = 5 \times 10^{-3}\ \text{m} \times 1 \times 10^{-3}\ \text{m} = 5 \times 10^{-6}\ \text{m}^2$.

$$R = \frac{\rho l}{A} = \frac{3.0 \times 10^{-5}\ \Omega\,\text{m} \times 10 \times 10^{-3}\ \text{m}}{5 \times 10^{-6}\ \text{m}^2} = \mathbf{0.060\ \Omega}$$

YOUR TURN

The same chip, the other way

What would the chip's resistance be for a current through its 1 mm thickness (from its top face to its bottom face)?

Answer

Now $l = 1\ \text{mm} = 1 \times 10^{-3}\ \text{m}$ and the area is $10\ \text{mm} \times 5\ \text{mm} = 5 \times 10^{-5}\ \text{m}^2$:

$R = \dfrac{3.0 \times 10^{-5} \times 1 \times 10^{-3}}{5 \times 10^{-5}} = 6.0 \times 10^{-4}\ \Omega = \mathbf{0.60\ m\Omega}$, a hundred times smaller: a tenth of the length and ten times the area.

Check 6

A wire is stretched to twice its length, its volume staying the same. Its resistance becomes:

a twice as large
b the same
c half as large
d four times as large

23.8 Resistors in series

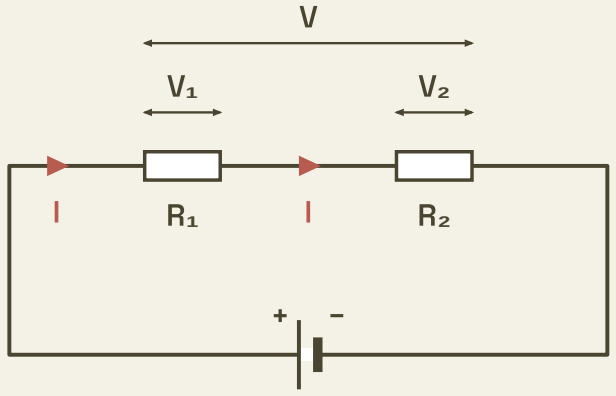


Figure 23.20 *Two resistors R_1 and R_2 in series, with the same current I through both. The p.d.s across them, V_1 and V_2, add up to the p.d. V across the pair.*

Resistors are in **series** when they are joined one after another with **no junction** between them, so that the **same current** flows through all of them. Let I be the common current.

The p.d.s add. By conservation of energy, the energy each coulomb gives up in R_1 and in R_2 adds up to the energy it gives up across the pair:

$$V = V_1 + V_2$$

Each p.d. By V = IR,

$$V_1 = IR_1 \qquad \text{and} \qquad V_2 = IR_2$$

The single resistor. One resistor R that would carry the same current I at the same p.d. V has V = IR. So

$$IR = IR_1 + IR_2 \quad \Rightarrow \quad \boxed{R = R_1 + R_2}$$

For more than two resistors in series, simply add all of them: $R = R_1 + R_2 + R_3 + \ldots$ Adding resistors in series **increases** the total resistance.

23.9 Resistors in parallel

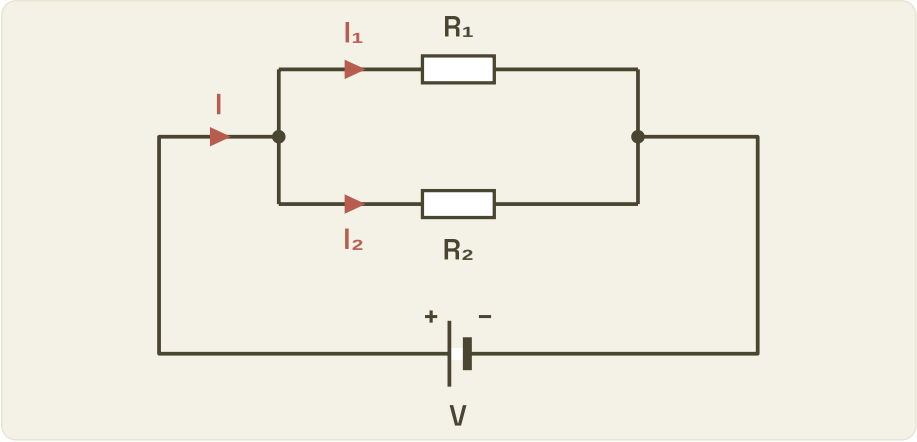


Figure 23.21 *Two resistors R_1 and R_2 in parallel across a p.d. V. The current I splits at the junction into I_1 and I_2, which join again on the other side.*

In a **parallel** circuit, the current **splits**: each resistor is connected across the same two points, so each has the **same p.d.** V across it.

The currents add. By conservation of charge, the current flowing into a junction equals the current flowing out:

$$I = I_1 + I_2$$

Each current. By Ohm's law, $I_1 = \dfrac{V}{R_1}$, $I_2 = \dfrac{V}{R_2}$, and for the single equivalent resistor $I = \dfrac{V}{R}$. So

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

Dividing by V:

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

For two resistors this gives $R = \dfrac{R_1 R_2}{R_1 + R_2}$ (product over sum). The formula shows that the total resistance of resistors in parallel is **smaller than the smallest** of them: each new branch is a new path for the current.

With these two rules, any group of resistors connected in series or in parallel can be replaced by a single resistor, which greatly simplifies a circuit.