Unit 22

Capacitors

This unit online, with its interactives:
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Press the shutter of a camera in a dark room, and the flash fires: a burst of light thousands of times more powerful than the camera's small battery could ever drive directly. Under the cover of every ceiling fan in Nepal sits a small cylinder that gives the motor its first push. Inside your phone, a few thousand tiny parts keep its voltage steady while the processor gulps current in bursts, and its touch screen knows where your finger is because your finger changes how much charge the screen can hold.

All of these are **capacitors**: two conductors kept close together but apart, which store electric charge, and with it energy, for a moment. In this unit you will see how a capacitor stores charge, what its capacitance is and how to calculate it for a pair of plates, why an insulator between the plates helps, how capacitors combine, and how much energy a charged capacitor holds.

22.1 Introduction

A **capacitor** is a component of an electric circuit that **stores electric charge** for a while. Inside, it separates charge: one side becomes positive, the other equally negative.

Capacitors come in many shapes and sizes, from specks smaller than a grain of rice to cans as big as a water bottle, but every one is built the same way: **two metal plates separated by an insulator**. The insulator is called the **dielectric**: air, paper, plastic film, ceramic, mica or a thin layer of metal oxide.

The **circuit symbol** shows the two plates edge-on, two short parallel lines with a gap between them, with a wire joined to the middle of each.

You can make a capacitor at home from two sheets of aluminium foil with a sheet of newspaper between them.

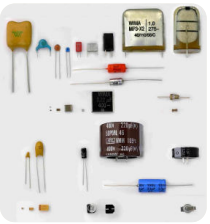


Figure 22.1 *Capacitors of many kinds: ceramic discs and chips, plastic film blocks, electrolytic cylinders (the brown one is 220 µF for up to 400 V) and, at the right, a small supercapacitor of 1.0 F. Each has its capacitance and the highest safe voltage printed on it, or written in a short code.*

Photo: Fabian ~ (Fabian R at de.wikipedia), CC BY-SA 3.0

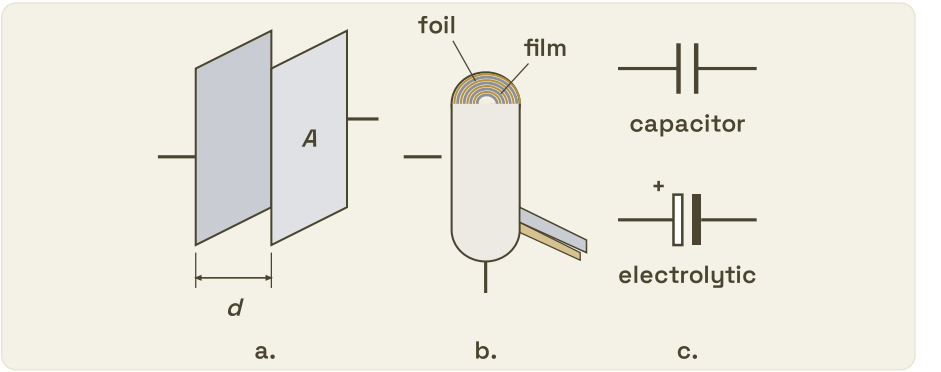


Figure 22.2 *Every capacitor is two conductors separated by an insulator. a. Two flat plates, each of area A, a distance d apart. b. To get a large area into a small can, two long strips of metal foil with a thin insulating film between them are rolled up together. c. The circuit symbol: two short parallel lines with a gap, the plates seen edge-on. An electrolytic capacitor, which must be connected the right way round, has its + side marked.*

A STORY FROM SCIENCE

A jar that bit

In 1745, the German clergyman Ewald von Kleist pushed a nail into a medicine bottle of water and charged it from a friction machine. When he touched the nail with his other hand he got a shock that threw him back. A few months later, Pieter van Musschenbroek at Leiden in the Netherlands did the same by accident and wrote that he would not do it again for the kingdom of France. The **Leyden jar**, a glass jar coated with metal foil inside and outside, was the first capacitor: two conductors (the foils) separated by an insulator (the glass). Benjamin Franklin used rows of them, which he called a *battery*, the first use of that word in electricity.

Check 1

The two plates of a capacitor are separated by:

a a thin wire, so that charge can flow between them
b an insulator, so that charge cannot flow between them
c a semiconductor
d nothing at all: they touch

22.2 Storing charge

Connect a capacitor to a cell (Figure 22.3). For a **very short time**, a current flows:

1. The cell's positive terminal pulls electrons off plate A, and its negative terminal pushes the same number of electrons onto plate B. No electron crosses the gap: the insulator stops them.
2. Plate A, having lost electrons, has a **net positive charge**, +Q. Plate B, having gained them, has a **net negative charge**, −Q.
3. Why equal? The current is the same at every point of the series loop, and charge is conserved: every electron that leaves A arrives, by way of the cell, on B. So the two plates always carry **equal and opposite charges**.
4. The separated charges make a **potential difference** across the plates, which pushes back against the cell. As more charge piles up, this p.d. grows, and the current gets smaller.
5. When the p.d. across the plates equals the cell's e.m.f. E, nothing pushes the electrons any more: the current stops, and the capacitor is **fully charged**.

The total charge on the capacitor is still zero: $+Q + (-Q) = 0$. A capacitor doesn't store charge the way a tank stores water; it **separates** charge into +Q on one plate and −Q on the other. When we say "a capacitor stores a charge Q", we mean the size of the charge on either plate.

Take the charged capacitor out of the circuit, and the charges stay where they are, held by their attraction across the gap. Join its plates through a lamp, and the extra electrons on B flow round to A: the lamp glows for a moment while the capacitor **discharges**.

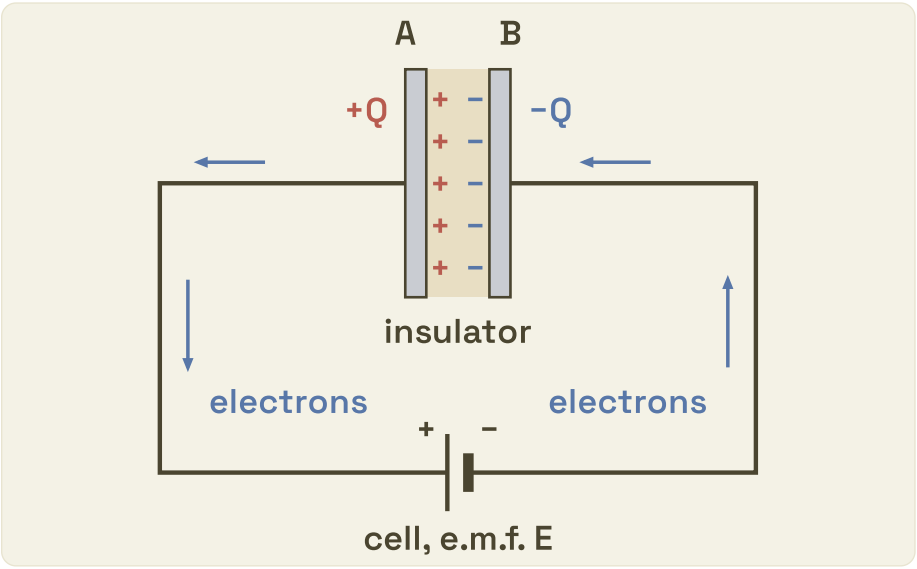


Figure 22.3 *A capacitor connected to a cell of e.m.f. E. For a very short time electrons flow off plate A towards the cell's positive terminal, and the same number flow from the cell's negative terminal onto plate B. A is left with a charge +Q, B with −Q. No electron crosses the insulator between the plates.*

Check 2

A capacitor is fully charged from a 9 V battery, with +20 μC on one plate. The total charge on the capacitor is:

a 40 μC
b 20 μC
c zero
d 180 μC

22.3 Capacitance

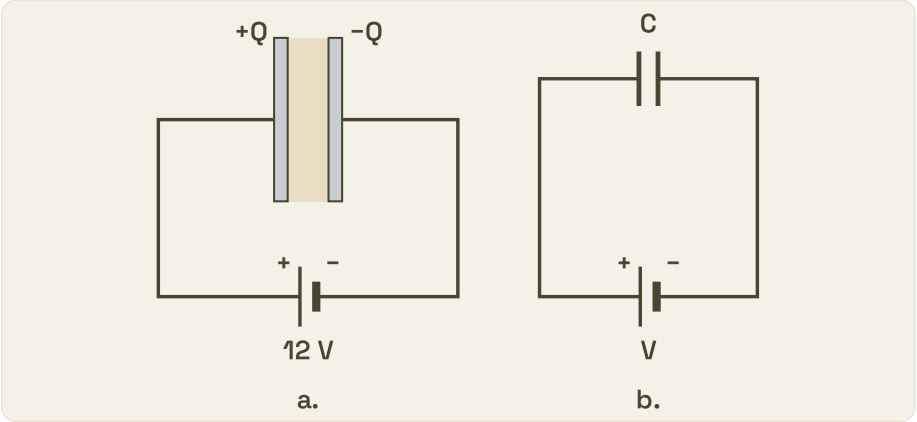


Figure 22.4 *a. A capacitor joined to a 12 V battery: the plate joined to the + terminal gets +Q, the plate joined to the − terminal gets −Q. b. The same circuit drawn with symbols.*

Join a capacitor to a battery with wires (Figure 22.4). The plates quickly charge, one positive and one equally negative. Each terminal of the battery and the plate joined to it are at the **same potential** (a wire with no current through it has no p.d. across it), so once the current has stopped, the **full battery voltage** is across the capacitor.

Now repeat with batteries of different voltages. For a given capacitor, the charge Q on each plate is found to be **directly proportional** to the p.d. V between the plates: twice the voltage, twice the charge.

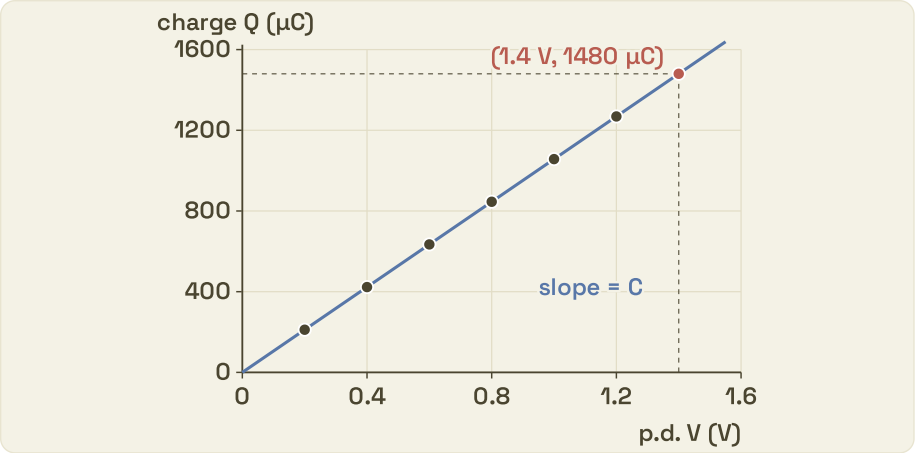


Figure 22.5 *Measurements of the charge stored by one capacitor at different p.d.s. The points lie on a straight line through the origin, so $Q \propto V$. Its slope is the capacitance: at 1.4 V the capacitor holds 1480 μC, so $C = 1480\ \mu C \div 1.4\ V \approx 1060\ \mu F$.*

The ratio Q/V is therefore fixed for each capacitor. It is the capacitor's **capacitance**, C:

$$Q = CV \qquad \text{or} \qquad C = \frac{Q}{V} = \frac{\text{charge stored on one plate}}{\text{p.d. between the plates}}$$

The capacitance of a capacitor is the charge stored (on either plate) per unit potential difference across it. It measures how much charge the capacitor stores for each volt, and so, in everyday words, its ability to store charge. A capacitor with a large capacitance stores a lot of charge at a low voltage.

Notice what it does not depend on: Q and V change together, but their ratio stays the same. Capacitance depends only on how the capacitor is built (Section 22.4). Commercial capacitors have it printed on them, "470 μF", beside the highest voltage they can safely take.

The farad

The SI unit of capacitance is the **coulomb per volt**, called the **farad** (F), after Michael Faraday:

A capacitor has a capacitance of one farad if a charge of one coulomb on its plates gives a p.d. of one volt between them.

In base units: $1\ \text{F} = \dfrac{1\ \text{C}}{1\ \text{V}}$, with $1\ \text{C} = 1\ \text{A s}$ and $1\ \text{V} = 1\ \text{J/C} = 1\ \text{kg m}^2\ \text{s}^{-3}\ \text{A}^{-1}$, so

$$1\ \text{F} = \frac{\text{A s}}{\text{kg m}^2\ \text{s}^{-3}\ \text{A}^{-1}} = \text{A}^2\ \text{s}^4\ \text{kg}^{-1}\ \text{m}^{-2}$$

A coulomb is a lot of charge, so a farad is a huge capacitance. Most capacitors are measured in smaller units:

Unit	Symbol	Value	A typical use
millifarad	mF	10^{-3} F	a large smoothing capacitor in a power supply
microfarad	μF	10^{-6} F	a ceiling fan's capacitor (2–4 μF), a camera flash (a few hundred μF)
nanofarad	nF	10^{-9} F	the foil-and-newspaper capacitor above
picofarad	pF	10^{-12} F	the capacitors that tune a radio

DID YOU KNOW?

How big is a farad?

A capacitor of 1 F made of two flat plates 1 mm apart in air would need plates of about 113 km² each, a little over twice the area of Kathmandu Metropolitan City. Real capacitors of several farads exist, the **supercapacitors**, and fit in your hand: their "plates" are porous carbon with an enormous inside surface, and their "gap" is only a few molecules wide (Section 22.6).

WORKED EXAMPLE

Capacitance from charge and p.d.

A pair of parallel plates stores 2.5×10^{-6} C of charge at a potential difference of 15 V. Calculate the capacitance.

Answer

$$C = \frac{Q}{V} = \frac{2.5 \times 10^{-6}\ \text{C}}{15\ \text{V}} = 1.7 \times 10^{-7}\ \text{F} = \mathbf{170\ nF}$$

YOUR TURN

Charge on a fan capacitor

A ceiling fan's capacitor is marked 2.5 μF. What charge is on each of its plates when the p.d. across it is 300 V?

Answer

$Q = CV = 2.5 \times 10^{-6} \times 300 = 7.5 \times 10^{-4}$ C = **0.75 mC**.

Check 3

A capacitor holds 60 μC at 12 V. If the p.d. across it is raised to 18 V, its capacitance:

a rises to 1.5 times
b falls to two thirds
c stays 5 μF, and its charge rises to 90 μC
d stays 5 μF, and its charge stays 60 μC

22.4 Parallel-plate capacitor

The simplest capacitor is two flat, parallel metal plates separated by an insulator: a **parallel-plate capacitor**. The insulator can be a vacuum, air or another gas, or a solid such as plastic.

Its capacitance depends on three things:

- the **area** A of the plates (more room for charge to spread out);
- their **distance apart** d (closer plates hold charge more strongly);
- the insulator between them, through a property called its **permittivity**, ε.

Permittivity

Put an insulator in an electric field and its molecules are pulled slightly out of shape or turned: it is **polarized** (Section 22.5). The polarized insulator sets up a field of its own that partly cancels the applied field. **Permittivity** measures how strongly a material does this: the higher its permittivity, the more it weakens the field due to a given charge, so the more charge the plates can hold at a given p.d.

Permittivities are given relative to the permittivity of free space (a vacuum), ε_0:

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{F/m}$$

The **relative permittivity** ε_r of a material, also called its **dielectric constant** K, is how many times greater its permittivity is than that of a vacuum. Its **absolute permittivity** is then

$$\varepsilon = \varepsilon_r \varepsilon_0$$

Material	Relative permittivity ε_r
vacuum	1 (exactly)
air	1.0006 (taken as 1)
Teflon (PTFE)	2.1
polystyrene	about 2.6
paper	about 2–4
mica	about 5–7
glass	about 5–10
pure water	about 80
barium titanate (a ceramic)	1000 and more

Dielectrics used in capacitors include paper, mica (a mineral that splits into thin sheets), Teflon and other plastics, ceramics, and the oxides of metals such as aluminium and tantalum.

Deriving the capacitance

The charges +Q and −Q spread evenly over the inner faces of the plates. Between the plates, apart from near the edges, the field is **uniform**: straight, parallel field lines from the + plate to the − plate.

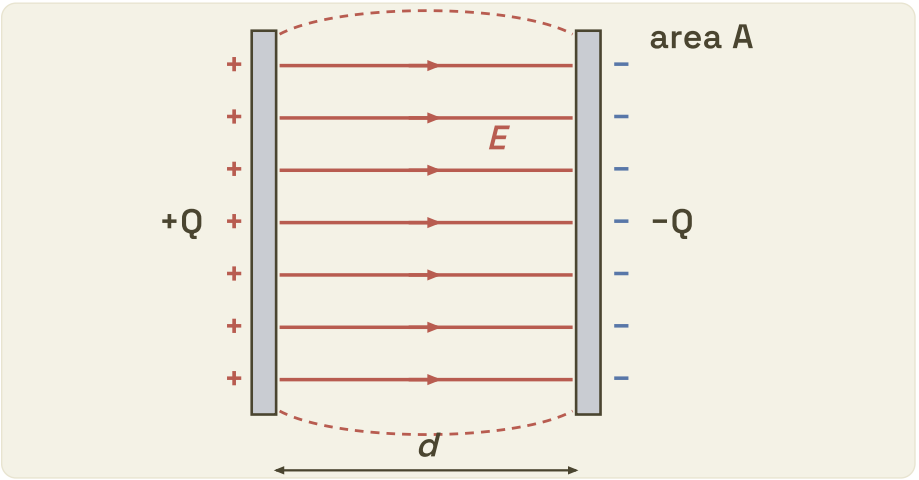


Figure 22.6 *A charged parallel-plate capacitor: plates of area A, a gap d. Between the plates the field E is uniform (evenly spaced, parallel lines) except near the edges, where it bulges out a little.*

The field between the plates. By Gauss's law (Unit 20), the field between two large, oppositely charged parallel plates, each with a charge σ per unit area, is $E = \frac{\sigma}{\varepsilon_0}$. Here $\sigma = \frac{Q}{A}$, so

$$E = \frac{Q}{\varepsilon_0 A}$$

The p.d. across a uniform field. In a uniform field, the p.d. between two points a distance d apart along the field is $V = Ed$ (Unit 21), so

$$V = \left(\frac{Q}{\varepsilon_0 A}\right) d$$

The capacitance. By definition,

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Q}{\varepsilon_0 A}\right) d}$$

and Q cancels:

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

With a dielectric filling the gap, replace ε_0 by the material's permittivity $\varepsilon = \varepsilon_r \varepsilon_0$:

$$\boxed{C = \frac{\varepsilon_r \varepsilon_0 A}{d}}$$

So C depends only on the **geometry** (A and d) and the **material** (ε_r), never on Q or V. A dielectric multiplies the capacitance by ε_r; with air the change is negligible.

This also gives the unit of ε_0: $\varepsilon_0 = \dfrac{Cd}{A}$ has the unit F m/m² = F/m, the farad per meter.

WORKED EXAMPLE

A pair of plates in air

Two parallel plates each have an area of 0.015 m² and are placed 2.0×10^{-3} m apart in air. Calculate the capacitance.

Answer

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12}\ \text{F/m} \times 0.015\ \text{m}^2}{2.0 \times 10^{-3}\ \text{m}} = 6.6 \times 10^{-11}\ \text{F} = \mathbf{66\ pF}$$

(With ε_0 rounded to 8.9×10^{-12} F/m, 6.7×10^{-11} F.) A tiny capacitance for plates as big as a sheet of exercise paper: that is why real capacitors use thin dielectrics, rolled foils and high permittivities.

Change the plates and the insulator below and watch the capacitance.

Check 4

The plates of a parallel-plate capacitor are moved twice as far apart, and their area is doubled. Its capacitance:

a doubles
b halves
c stays the same
d becomes four times as large

Check 5

A capacitor with air between its plates has a capacitance of 50 pF. The gap is filled with mica of relative permittivity 6. Its capacitance becomes:

a 50 pF
b 8.3 pF
c 56 pF
d 300 pF

22.5 Dielectric strength of dielectrics

Raise the p.d. across a capacitor and the field between its plates, $E = V/d$, grows. At some field the insulator can't hold out: electrons are torn loose from its atoms, the dielectric suddenly conducts, and a **spark** jumps between the plates. Usually that ruins a solid dielectric for good.

The **dielectric strength** of an insulator is the greatest electric field it can withstand without breaking down.

Dielectric	Dielectric strength
dry air at sea level	about 3 kV/mm (3×10^6 V/m)
paper	about 15 kV/mm
Teflon	about 60 kV/mm
mica	about 100 kV/mm and more

For air the exact value depends on the temperature, the pressure and how dry the air is. Lightning is air breaking down on a huge scale: the strong fields between charged clouds, or between a cloud and the ground, tear electrons off air molecules and open a conducting path. (A lightning stroke starts at fields lower than 3 kV/mm, because the field is far stronger close to raindrops and ice crystals, and the breakdown grows from there.)

So every capacitor has a **voltage rating**, printed beside its capacitance ("470 µF 25 V"): the highest p.d. it can safely take. A thin dielectric gives a large capacitance but breaks down at a lower voltage.

WORKED EXAMPLE

How thin can the dielectric be?

A capacitor is to work at up to 500 V with a Teflon film as its dielectric. What is the thinnest film it could use, and what thickness would you choose?

Answer

The field must stay below 60 kV/mm: $d = \dfrac{V}{E_{\max}} = \dfrac{500\ \text{V}}{60\,000\ \text{V/mm}} = 0.0083\ \text{mm}$, about 8 μm.

A designer would leave a safety margin, using a film several times thicker (say 25–40 μm), since a film has thin spots and the voltage may spike.

What a dielectric does

An insulator between the plates does two jobs for a capacitor's designer:

- it **keeps the plates apart** at a fixed, small distance, which a gap of air can't do for flexible foils;
- it **increases the capacitance**, by its relative permittivity.

Every material has a permittivity greater than a vacuum's. To include it, replace ε_0 by $\varepsilon = \varepsilon_r \varepsilon_0$ wherever it appears.

Why a dielectric increases the capacitance

A **dielectric** is an insulator that is **polarized** in an electric field. In many dielectrics each molecule is already slightly positive at one end and negative at the other (a polar molecule, like water); in others the field itself pulls each molecule's electrons slightly one way and its nuclei the other. Either way, in the capacitor's field the molecules line up, or are stretched, with their positive ends pointing along the field.

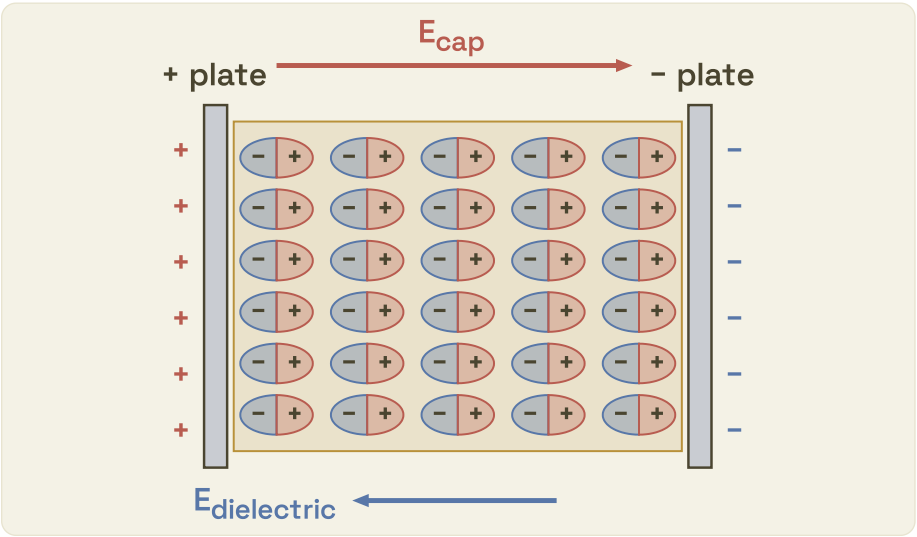


Figure 22.7 *A dielectric filling the gap of a charged capacitor. The capacitor's field, E_{cap}, points from the + plate to the − plate. The molecules line up with their + ends along the field. Inside, each + end sits next to a − end, but the faces are left with charge: negative next to the + plate, positive next to the − plate. These surface charges make a field $E_{dielectric}$ the other way, so the net field is smaller.*

Look at the dielectric's faces (Figure 22.7). Inside it, the + and − ends of neighboring molecules cancel. But the face next to the **positive plate** is left with a layer of **negative** ends, and the face next to the **negative plate** with **positive** ends. These surface charges make a field of their own, $E_{\text{dielectric}}$, pointing **against** the capacitor's field:

$$E_{\text{net}} = E_{\text{cap}} - E_{\text{dielectric}}$$

Now follow it through for a capacitor charged and then disconnected, so that Q can't change:

1. The net field between the plates is **smaller** (it is $E_{\text{cap}}/\varepsilon_r$).
2. The p.d. across the plates, $V = E_{\text{net}} d$, is smaller, since d is fixed.

3. The charge Q is unchanged, so the capacitance $C = \frac{Q}{V}$ is **larger**, by the factor ε_r.

Another way to say it: the dielectric's negative face partly cancels the positive plate's charge, lowering its potential, and its positive face raises the potential of the negative plate. The p.d. between the plates falls.

If the capacitor stays connected to a battery instead, the p.d. stays at the battery's voltage, and the battery pushes **more charge** onto the plates until the net field is back to V/d: again Q/V goes up by ε_r.

What happens to the stored energy of the disconnected capacitor? It **falls**, because the p.d. falls while Q stays the same (Section 22.7). The field pulls the dielectric into the gap, doing work on it: a slab held at the edge of a charged capacitor is sucked in. This simple picture of surface charges explains most of what a dielectric does; a full treatment also counts how the field inside the material varies from molecule to molecule.

Check 6

A capacitor is charged and disconnected from the battery. A slab of glass is then slid in to fill the gap. Which of these goes **up**?

a the charge on the plates
b the p.d. between the plates
c the capacitance
d the field between the plates

Practical capacitors

To get a large capacitance into a small space, designers make the plate area A large and the gap d tiny, and choose a dielectric with a high permittivity.

- **Ceramic** capacitors: small discs or chips with a ceramic dielectric of high ε_r; picofarads to microfarads.
- **Film** capacitors: metal foil or metallized plastic film, rolled up with a plastic film between; nanofarads to microfarads. A ceiling fan's capacitor is one.
- **Electrolytic** capacitors: the largest capacitance for their size, microfarads to farads.

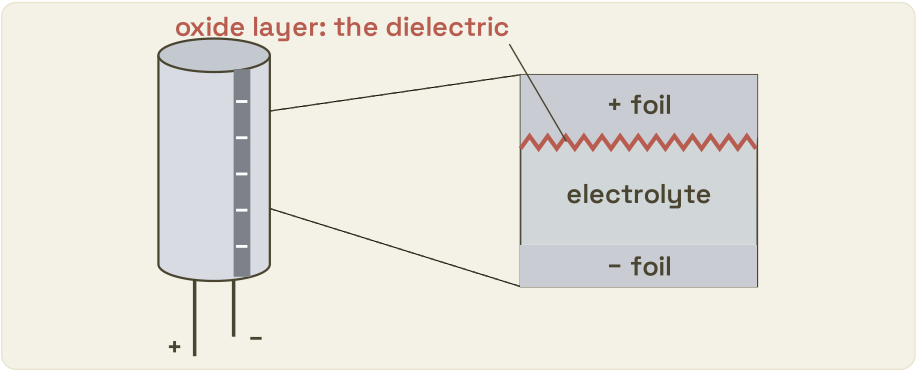


Figure 22.8 *Inside an aluminium electrolytic capacitor. Two strips of aluminium foil, with paper soaked in a conducting liquid (the electrolyte) between them, are rolled up into a can. The + foil is coated with a layer of aluminium oxide far thinner than a micrometer: that oxide is the dielectric, and the electrolyte, touching it, acts as the other plate.*

In an **aluminium electrolytic capacitor** (Figure 22.8), the layers are rolled up together, rather like rolling up three carpets laid one on another. The dielectric is a layer of **aluminium oxide** grown on the positive foil, so thin (well under a micrometer) that d is tiny, and the foil is etched rough so that its area is many times its flat size: both make C large. The conducting electrolyte in the paper fills every pit of the rough surface and carries the charge to the other foil.

An electrolytic capacitor must be connected the **right way round**: its oxide layer is kept in place only while the + foil is at the higher potential. Connected backwards, the oxide is broken down, a current flows, the electrolyte boils, and the can may burst. That is why such capacitors have a stripe marking the − lead, and a + on the circuit symbol.

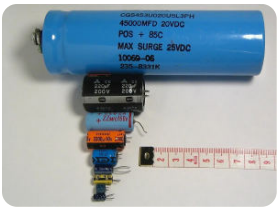


Figure 22.9 *Aluminium electrolytic capacitors, from 45 000 µF (the large can, for up to 20 V) down to a few microfarads. Each is marked with its capacitance and its highest voltage, and a + sign or a stripe on the sleeve shows which lead is which.*
Photo: Omegatron, CC BY-SA 3.0

WORKED EXAMPLE

Air and polystyrene

In a laboratory experiment, two parallel plates, each of area 100 cm², are 1.5 cm apart. Calculate the capacitance if the gap between the plates is filled with a. air, b. polystyrene (take its relative permittivity as 3).

Answer

a. Convert to SI: $A = 100 \times 10^{-4} = 1.0 \times 10^{-2}$ m²; $d = 0.015$ m.

$$C_1 = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 1.0 \times 10^{-2}}{0.015} = 5.9 \times 10^{-12}\ \text{F} = \mathbf{5.9\ pF}$$

b. Filling the gap multiplies the permittivity, and so the capacitance, by ε_r:

$$\frac{C_2}{C_1} = \frac{\varepsilon_r \varepsilon_0 A/d}{\varepsilon_0 A/d} = \varepsilon_r \quad \Rightarrow \quad C_2 = C_1 \times \varepsilon_r = 5.9 \times 10^{-12} \times 3 = 1.8 \times 10^{-11}\ \text{F} = \mathbf{18\ pF}$$

(Measured, polystyrene's relative permittivity is about 2.6, which would give 15 pF.)

22.6 Combination of capacitors

Capacitors are made in a set of standard values (1.0, 2.2, 4.7, 10 µF…). To get a value that isn't made, or a larger capacitance, or one that can take a higher voltage, we connect capacitors together. Like resistors, they can be joined in **parallel**, in **series**, or in a mixture of both.

DID YOU KNOW?

Supercapacitors

Supercapacitors (ultracapacitors) are compact capacitors with capacitances of hundreds or even thousands of farads, at a few volts each. They are used instead of batteries, or beside them: to back up a memory when the power fails, in emergency lighting, and to catch the energy of a braking bus or tram and give it back when it pulls away. Unlike a rechargeable battery, which wears out after a few hundred or thousand cycles, a supercapacitor can be charged and discharged hundreds of thousands of times. It stores far less energy than a battery of the same size, but takes it in and gives it out much faster.

22.6.1 Capacitors in parallel

In **parallel**, each capacitor is connected directly across the same two points (Figure 22.10). For two or more capacitors in parallel:

The p.d. is the same across each capacitor, the supply's p.d. V (each plate is joined by wire to one terminal).

The charges add. Charge is conserved, so the total charge the supply delivers is shared among the capacitors:

$$Q = Q_1 + Q_2 + \dots$$

Each charge is $Q_1 = C_1 V$, $Q_2 = C_2 V$.

One equivalent capacitor C, with the same V, would hold the same total Q: $Q = CV$. So

$$CV = C_1 V + C_2 V + \dots$$

Dividing by V:

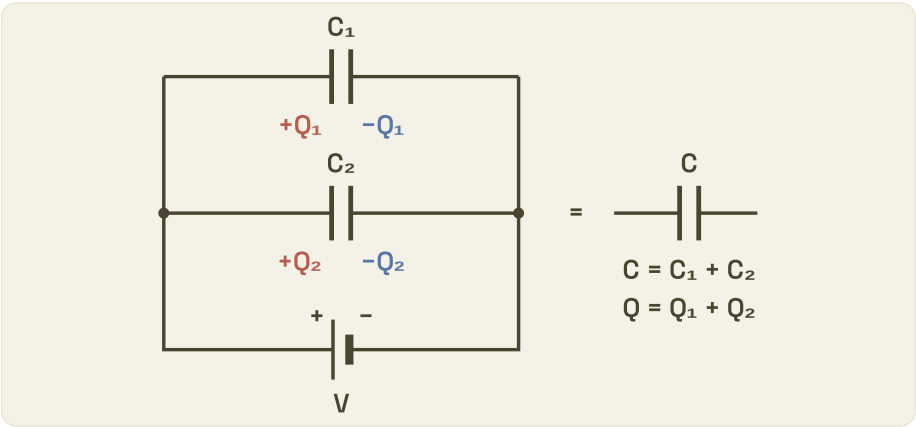


Figure 22.10 *Two capacitors in parallel across a supply of p.d. V. Each has the whole p.d. V across it; their charges Q_1 and Q_2 add up to the charge Q that the supply delivered. They can be replaced by one capacitor of $C = C_1 + C_2$.*

The total capacitance of capacitors in parallel is the **sum** of their capacitances, larger than any one of them. It is as if their plate areas were added: the combination stores more charge at the same p.d.

22.6.2 Capacitors in series

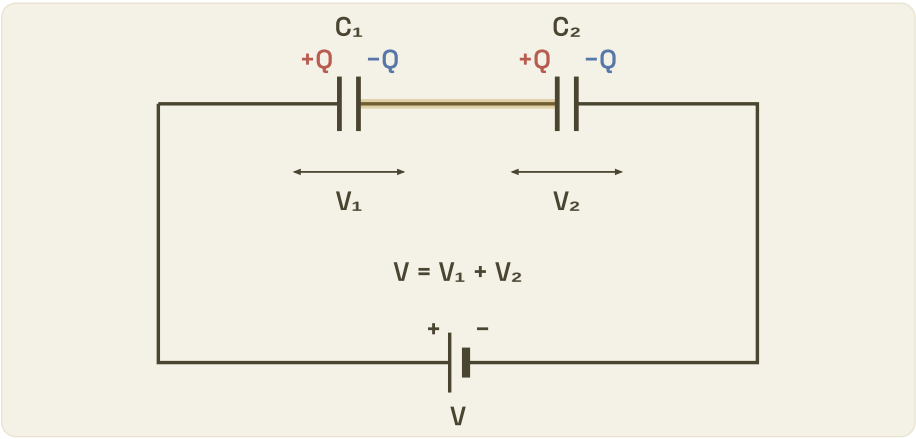


Figure 22.11 *Two capacitors in series. The supply moves charge Q onto the outer plates; the inner two plates, joined to each other but to nothing else, take −Q and +Q by induction. Every capacitor holds the same Q, and the p.d.s add: $V = V_1 + V_2$.*

In **series**, the capacitors are joined one after the other in a single line (Figure 22.11).

Why do they all store the same charge, even when their capacitances differ? The supply is connected only to the outer plates: the left plate of C_1 and the right plate of C_2. These take +Q and −Q as electrons flow off one and onto the other. The two inner plates are joined to each other by a wire, but cut off from the supply by the dielectrics on either side. The +Q on C_1's left plate attracts electrons to C_1's right plate, which becomes −Q; those electrons come from C_2's left plate, which is left with +Q. The two inner plates together stay neutral. So each capacitor has +Q on one plate and −Q on the other: the **same charge Q**, whatever its capacitance. (Each capacitor's total charge is zero, as always.)

For two or more capacitors in series:

The charge is the same on each capacitor: $Q = Q_1 = Q_2$.

The p.d.s add to the supply's p.d. (energy is conserved: each coulomb gives up its energy in the capacitors one after the other):

$$V = V_1 + V_2 + \dots$$

Each p.d. is $V_1 = \dfrac{Q}{C_1}, V_2 = \dfrac{Q}{C_2}$.

One equivalent capacitor C would hold the same Q at the same total V: $V = \dfrac{Q}{C}$. So

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \dots$$

Dividing by Q:

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots}$$

For **two** capacitors this gives a handy form: $C = \dfrac{C_1 C_2}{C_1 + C_2}$ (product over sum).

The total capacitance of capacitors in series is **less than the smallest** of them: the combination stores less charge at a given p.d. It is as if their gaps were added. But the supply's p.d. is shared among them, so a series string can take a higher voltage than one capacitor alone.

CAREFUL

The opposite of resistors

The two rules are the reverse of the rules for resistors (Unit 23): capacitances **add in parallel** and add as reciprocals **in series**. Resistances add in series and as reciprocals in parallel. Keep them apart by the picture: parallel capacitors are like bigger plates (more C); series capacitors are like a wider gap (less C).

Check 7

Three capacitors of 6 μF each are joined in series. Their combined capacitance is:

a 18 μF
b 6 μF
c 2 μF
d 0.5 μF

Check 8

A 2 μF and a 4 μF capacitor are in series across 12 V. The p.d. across the 2 μF capacitor is:

a 4 V
b 6 V
c 8 V
d 12 V

Mixed capacitor combinations

To find the capacitance of a network, and the charge and p.d. of each capacitor in it:

1. Combine capacitors that are clearly in series or in parallel, using the formulas above.
2. Redraw the circuit after every combination.
3. Repeat steps 1 and 2 until only a single equivalent capacitor is left.
4. Find the charge on that single capacitor, using $Q = CV$.
5. Work backwards through the diagrams to the original one, finding the charge and p.d. of each capacitor on the way, using these facts:
   - A. the capacitor equation: $C = \dfrac{Q}{V}$;
   - B. capacitors in parallel: $C = C_1 + C_2$;
   - C. capacitors in parallel all have the same p.d. V, as does their equivalent capacitor;
   - D. capacitors in series: $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$;
   - E. capacitors in series all have the same charge Q, as does their equivalent capacitor.

WORKED EXAMPLE

A small network

Calculate the capacitance of the network below.

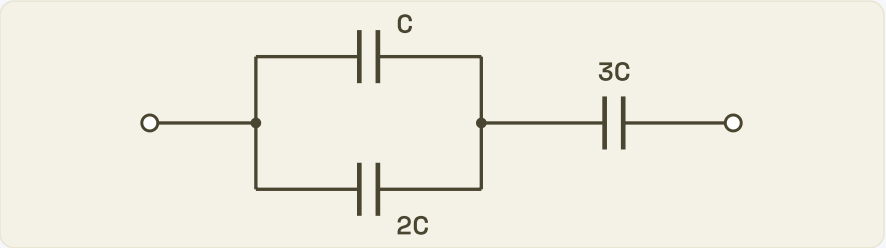


Figure 22.12 *Capacitors of C and 2C in parallel, joined in series with a capacitor of 3C.*

Answer

The capacitors C and 2C are in parallel: $C + 2C = 3C$.

This 3C is in series with the other 3C:

$$\frac{1}{C_{\text{total}}} = \frac{1}{3C} + \frac{1}{3C} = \frac{2}{3C} \quad \Rightarrow \quad C_{\text{total}} = \frac{3C}{2} = \mathbf{1.5C}$$

YOUR TURN

Working backwards

In the network above, C = 2 μF, and 12 V is applied across the whole network. Find the charge on each capacitor and the p.d. across it.

Answer

The whole network: $C_{\text{total}} = 1.5 \times 2 = 3$ μF, so the supply delivers $Q = 3 \times 12 = 36$ μC.

Series (fact E): the 3C capacitor (6 μF) and the parallel pair (also 6 μF together) each hold 36 μC. So the 6 μF capacitor has $V = 36 \div 6 = \mathbf{6\ V}$, and the parallel pair has the other **6 V**.

Parallel (fact C): C and 2C each have 6 V across them: C (2 μF) holds $2 \times 6 = \mathbf{12\ \mu C}$; 2C (4 μF) holds $4 \times 6 = \mathbf{24\ \mu C}$. Check: $12 + 24 = 36$ μC, the charge of the pair.

22.7 Energy stored in a capacitor

Charge a 1000 μF capacitor from a 9 V battery, take it out of the circuit, and touch its leads to a small LED (with a 100 Ω resistor in series). The LED flashes brightly for a moment, with no battery in sight. The flash is the evidence: **a charged capacitor stores energy**. A camera's flash and the defibrillator that restarts a heart both work this way, storing energy slowly and giving it out all at once.

How much energy is stored depends on the capacitance and on the p.d. the capacitor was charged to. (A tiny 1000 pF capacitor charged to 3 V holds only about 4.5 billionths of a joule: far too little to light anything you could see.)

Where the energy comes from

An electron arriving at the negative plate is **repelled** by all the electrons already on it, so work must be done to push it on. An electron leaving the positive plate is **attracted** back by the positive charge there, so work must be done to pull it off (Figure 22.13). The battery or power supply does this work. The more charge already on the plates, the harder each new bit is to move.

The work is not lost: it is stored as **electric potential energy** of the separated charges, in the field between the plates. It comes from the energy of the battery, and it comes back out when the capacitor discharges.

Deriving the energy stored

By definition of p.d., moving a charge Q through a p.d. V takes work W = QV. But while a capacitor charges, its p.d. doesn't stay the same: it grows in step with the charge, Q = CV.

A small step. At a moment when the p.d. is V, adding a small charge ΔQ (so small that V hardly changes) takes work

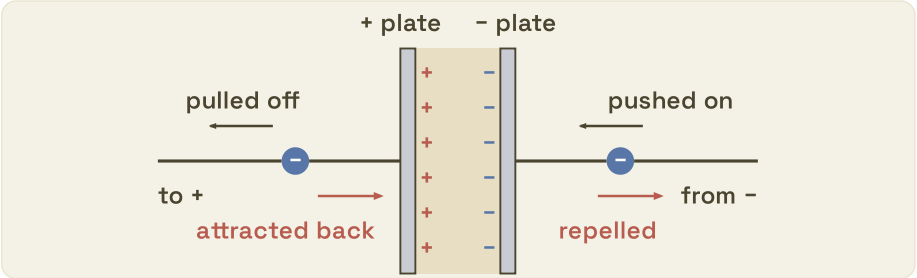


Figure 22.13 *Charging a capacitor. An electron arriving at the negative plate is repelled by the electrons already there: it has to be pushed on. An electron leaving the positive plate is attracted back by the positive charge there: it has to be pulled away. The battery does both.*

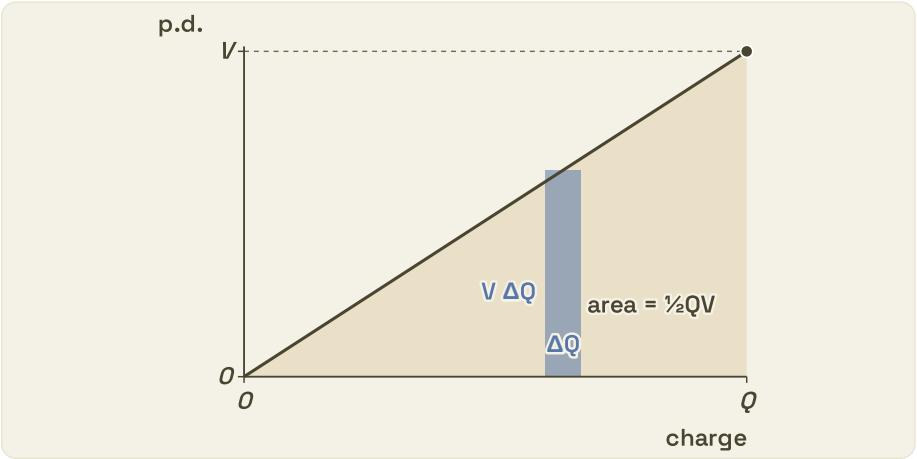


Figure 22.14 *The p.d. across a capacitor against its charge: a straight line through the origin. Adding a small charge ΔQ when the p.d. is V takes work V × ΔQ, the area of a thin strip. All the strips together fill the triangle under the line, whose area, ½QV, is the energy stored.*

On the graph of V against Q (Figure 22.14), this is the area of the thin strip of height V and width ΔQ.

All the steps. Adding all the strips from zero to the final charge Q gives the area under the graph. So the **area under a p.d.–charge graph equals the work done**, which is the energy stored.

The triangle. The graph is a straight line from the origin to (Q, V), so the area is a triangle:

$$W = \tfrac{1}{2} \times \text{base} \times \text{height} = \tfrac{1}{2}QV$$

where V is the p.d. when the charge is Q.

In terms of C and V. Put Q = CV:

$$W = \tfrac{1}{2}(CV)V = \tfrac{1}{2}CV^2$$

In terms of Q and C. Put $V = \dfrac{Q}{C}$:

$$W = \tfrac{1}{2}Q\left(\frac{Q}{C}\right) = \frac{Q^2}{2C}$$

So there are three equations for the energy stored, all the same thing:

$$\boxed{W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}}$$

By calculus: the work to add dQ at p.d. $V = Q/C$ is $dW = \dfrac{Q}{C}\,dQ$, and $W = \displaystyle\int_0^Q \frac{Q}{C}\,dQ = \frac{Q^2}{2C}$.

Why the half? Each bit of charge is carried across the p.d. of its moment, which rises from 0 to V: on average across V/2.

For a given capacitor, W ∝ V²: **doubling the p.d. quadruples the energy stored.**

CAREFUL

Q² over 2C

The third form is $\frac{Q^2}{2C}$, with C **under** the line; some books misprint it as ½Q²C. Check with units: Q²/C has the unit C²/F = C²/(C/V) = C V = J, an energy. Q²C would be C³/V, which is not.

WORKED EXAMPLE

A large capacitor

Calculate the energy stored in a 5000 μF capacitor charged to a p.d. of 25 V.

Answer

$$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2} \times 5000 \times 10^{-6}\ \text{F} \times (25\ \text{V})^2 = \tfrac{1}{2} \times 5 \times 10^{-3} \times 625 = 1.56\ \text{J} \approx \mathbf{1.6\ J}$$

YOUR TURN

The flash and its power

A camera's flash capacitor of 200 μF is charged to 300 V. a. How much energy does it store? b. It empties through the flash tube in about 1 ms. What is the average power of the flash?

Answer

a. $W = \tfrac{1}{2}CV^2 = \tfrac{1}{2} \times 200 \times 10^{-6} \times 300^2 = \mathbf{9.0\ J}$.

b. $P = \dfrac{W}{t} = \dfrac{9.0}{1 \times 10^{-3}} = \mathbf{9000\ W}$, for a thousandth of a second. The camera's battery charges the capacitor over a few seconds, at a few watts; the capacitor lets the energy out about a thousand times faster.

DID YOU KNOW?

Restarting a heart

A **defibrillator** stores energy in a capacitor charged to well over a thousand volts and delivers about 120–360 J through two pads on a patient's chest in a few milliseconds. The pulse stops the chaotic quivering of a heart in cardiac arrest, so that its own pacemaker can start a normal beat again. A battery alone couldn't give that much energy that quickly.

Check 9

The p.d. across a capacitor is increased from 3 V to 9 V. The energy it stores becomes:

a 3 times as large
b 6 times as large
c 9 times as large
d 27 times as large

IN SHORT

Key ideas of Unit 22

- A **capacitor** is two conductors separated by an insulator (the dielectric). Charged, its plates hold +Q and −Q; its total charge is zero.
- **Capacitance** $C = \dfrac{Q}{V}$, the charge stored per volt; unit the **farad**, 1 F = 1 C/V = 1 A² s⁴ kg⁻¹ m⁻²; usually μF, nF, pF.
- **Parallel-plate capacitor:** $C = \dfrac{\varepsilon_0 A}{d}$ in vacuum or air; $C = \dfrac{\varepsilon_r \varepsilon_0 A}{d}$ with a dielectric; $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m.
- A **dielectric** is polarized; its surface charges weaken the field, so it raises C by ε_r. Its **dielectric strength** is the largest field it can take before it breaks down (air about 3 kV/mm).
- **Parallel:** same V, charges add, $C = C_1 + C_2 + \ldots$. **Series:** same Q, p.d.s add, $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots$
- **Energy stored** = area under the V–Q graph: $W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \dfrac{Q^2}{2C}$.

Exercise

Choose the correct option

Check 10

1. What is the charge stored on a 220 μF capacitor with a 6 V p.d. across it?

a 36.6 mC
b 1.32 mC
c 2.70 mC
d 3.96 mC

Check 11

2. Two capacitors, P and Q, are both charged by the same 12 V battery. The capacitance of P is 4200 μF and that of Q is 420 μF. Which gives the correct ratios of the energy and the charge stored by each capacitor?

a energy in P ÷ energy in Q = 1; charge on P ÷ charge on Q = 1
b energy in P ÷ energy in Q = 10; charge on P ÷ charge on Q = 1
c energy in P ÷ energy in Q = 10; charge on P ÷ charge on Q = 10
d energy in P ÷ energy in Q = 1; charge on P ÷ charge on Q = 10

Check 12

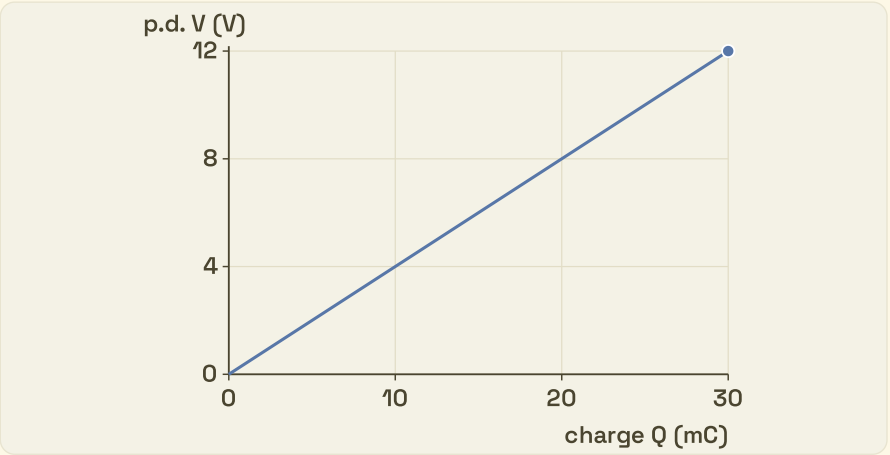


Figure 22.15 *For question 3: the p.d. across a capacitor against the charge stored on it.*

3. The graph above shows how the p.d. across a capacitor varies with the charge stored on it. Which gives the correct capacitance and the energy stored when the p.d. across it is 12 V?

a capacitance 2.5 mF; energy 90 mJ
b capacitance 2.5 mF; energy 180 mJ
c capacitance 0.4 mF; energy 90 mJ
d capacitance 0.4 mF; energy 180 mJ

Answer these questions

4. The p.d. of a test circuit was measured as a capacitor was charged, and the graph below was plotted from the data.

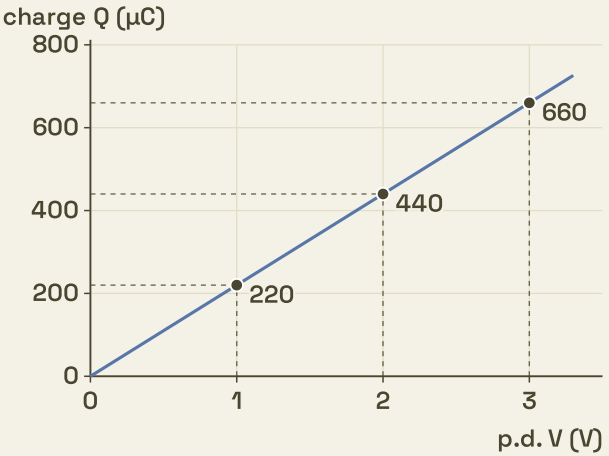


Figure 22.16 *For question 4: the charge on the capacitor against the p.d. across it.*

a. Explain what is meant by the term “capacitance”.
b. Calculate the capacitance of the capacitor.

5. In the figure below, a p.d. of 12 V is applied between P and Q.

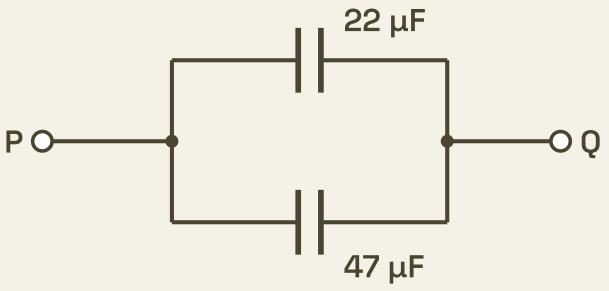


Figure 22.17 *For question 5: capacitors of 22 μF and 47 μF in parallel between P and Q.*

a. Calculate the charge on each capacitor.
b. What single capacitor placed between P and Q would be equivalent to the two shown? Comment on your answer.

6. In the figure below, a p.d. of 12 V is applied between P and Q.

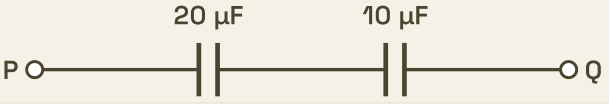


Figure 22.18 *For question 6: capacitors of 20 μF and 10 μF in series between P and Q.*

a. What is the p.d. across each capacitor? Explain your answer.
b. What single capacitor placed between P and Q would be equivalent to the two shown? Comment on your answer.

7. The figure below shows how the p.d. across a capacitor changes as it is charged.

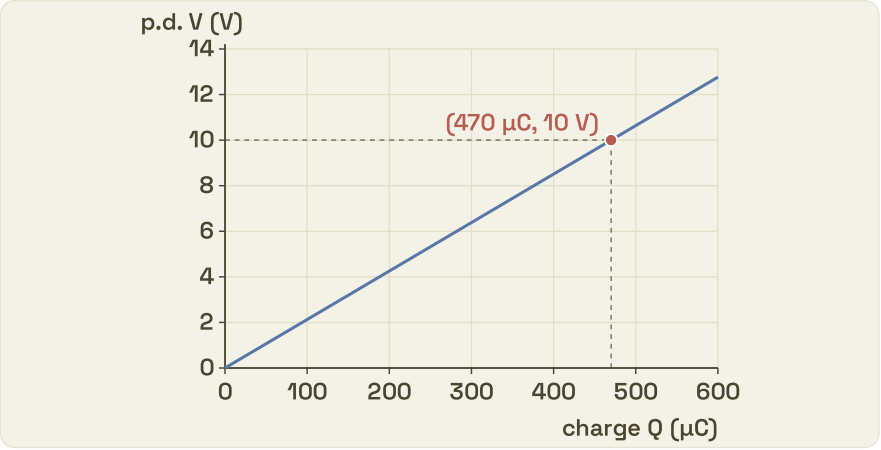


Figure 22.19 *For question 7: p.d. against charge for a capacitor.*

a. What is the capacitance of the capacitor?

b. Calculate the energy stored in the capacitor when it has a charge of 100 μC, 200 μC and 500 μC.

c. Sketch a graph of the energy stored against the charge.

8. A capacitor with a capacitance of 137 pF is charged until it stores a charge of 2.47 nC, and is then disconnected. It has a dielectric with a relative permittivity of 3.1. The dielectric is removed, so there is now a vacuum between the plates, but nothing else is changed. Calculate the change in the p.d. across the capacitor.

9. An 8.0 μF capacitor is fully charged from a 12 V supply. Calculate the energy stored by the capacitor.

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.