Answers: Capacitors

Quick checks

1 b A capacitor works by keeping the charges on its two plates apart. The insulator (the dielectric), or simply a gap of air or vacuum, stops charge crossing between the plates. If the plates touched, or were joined by a wire, any charge would simply flow from one to the other.

2 c One plate carries +20 μC and the other −20 μC, so the total is zero. We still say "it stores 20 μC", meaning the charge on each plate. 180 μC (20 × 9) is not a charge at all: it is Q × V, which has the unit of energy (μJ).

3 c $C = Q/V = 60 \div 12 = 5$ μF, fixed by how the capacitor is built. At 18 V it holds $Q = CV = 5 \times 18 = 90$ μC. The charge follows the voltage; the capacitance doesn't change.

4 c $C = \dfrac{\varepsilon_0 A}{d}$: doubling A doubles C, doubling d halves it, and the two cancel.

5 d Filling the gap multiplies C by ε_r: 50 × 6 = 300 pF. (8.3 pF divides instead of multiplying; 56 pF adds.)

6 c Disconnected, the charge can't change. The glass is polarized, its surface charges weaken the field, so the p.d. $V = Ed$ falls, and $C = Q/V$ rises (by ε_r of the glass).

7 c $\dfrac{1}{C} = \dfrac{1}{6} + \dfrac{1}{6} + \dfrac{1}{6} = \dfrac{3}{6}$, so C = 2 μF. (n equal capacitors in series give C/n; in parallel, nC = 18 μF.)

8 c $C = \dfrac{2 \times 4}{2 + 4} = \dfrac{4}{3}$ μF, so $Q = CV = \dfrac{4}{3} \times 12 = 16$ μC on each. Then $V_1 = \dfrac{Q}{C_1} = \dfrac{16}{2} = 8$ V and $V_2 = \dfrac{16}{4} = 4$ V. In series the **smaller** capacitor takes the **larger** share of the p.d.

9 c $W = \frac{1}{2}CV^2$: three times the p.d. gives $3^2 = 9$ times the energy. (Its charge is 3 times as large; 27 is 3^3.)

10 b $Q = CV = 220 \times 10^{-6} \times 6 = 1.32 \times 10^{-3}$ C = 1.32 mC. (36.6 is 220 ÷ 6, a division where a product is needed.)

11 c At the same V, $Q = CV$ and $W = \frac{1}{2}CV^2$ are both proportional to C. P's capacitance is 10 times Q's, so both ratios are 10.

12 b At 12 V the charge is 30 mC: $C = \dfrac{Q}{V} = \dfrac{30}{12} = 2.5$ mF.
Energy = area under the graph = $\frac{1}{2}QV = \frac{1}{2} \times 30 \times 10^{-3} \times 12 = 0.18$ J = 180 mJ. (0.4 mF is V/Q, the slope of this graph, taken for C; 90 mJ is half the right energy.)

4. a. The **capacitance** of a capacitor is the charge stored on either plate per unit p.d. across it, $C = \dfrac{Q}{V}$. It is measured in farads (1 F = 1 C/V).

b. The line passes through 660 μC at 3 V (and 220 μC at 1 V, 440 μC at 2 V):

$$C = \frac{Q}{V} = \frac{660 \times 10^{-6}}{3} = 2.2 \times 10^{-4} \; =$$

(The slope of the Q–V line is the capacitance.)

5. a. In parallel, each capacitor has the full 12 V across it.
22 μF: $Q_1 = C_1 V = 22 \times 10^{-6} \times 12 = 2.64 \times 10^{-4}$ C = **264 μC**.
47 μF: $Q_2 = C_2 V = 47 \times 10^{-6} \times 12 = 5.64 \times 10^{-4}$ C = **564 μC**.
b. $C = C_1 + C_2 = 22 + 47 =$ **69 μF**. Check: it would hold 69 × 12 = 828 μC = 264 + 564 μC. The equivalent capacitance of a parallel combination is **larger than either** capacitor: joining capacitors in parallel is like making the plate area bigger, so together they store more charge at the same p.d.

6. a. In series, both capacitors carry the **same charge** Q, so the p.d. across each is $V = Q/C$: the smaller capacitor has the larger p.d. Find Q first:

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{20 \times 10}{20 + 10} = 6.67\ \mu, \qquad Q = CV = 6.67 \times 10^{-6} \times 12 = 80\ \mu$$

20 μF: $V_1 = \dfrac{80\ \mu}{20\ \mu} =$ **4 V**; 10 μF: $V_2 = \dfrac{80\ \mu}{10\ \mu} =$ **8 V**. Check: 4 + 8 = 12 V.
b. $\dfrac{1}{C} = \dfrac{1}{20} + \dfrac{1}{10} = \dfrac{3}{20}$, so C = **6.7 μF** (20/3 μF). The equivalent capacitance of capacitors in series is **smaller than the smallest** of them: it is like widening the gap between the plates, so the combination stores less charge at the same p.d. (80 μC, against 120 μC on the 10 μF capacitor alone at 12 V).

7. a. The line passes through 10 V at 470 μC:

$$C = \frac{Q}{V} = \frac{470 \times 10^{-6}}{10} = 4.7 \times 10^{-5} \ =$$

b. $W = \dfrac{Q^2}{2C}$, with $2C = 9.4 \times 10^{-5}$ F:

- 100 μC: $W = \dfrac{(1.00 \times 10^{-4})^2}{9.4 \times 10^{-5}} = 1.06 \times 10^{-4}$ J ≈ **0.11 mJ**;
- 200 μC: $W = \dfrac{(2.00 \times 10^{-4})^2}{9.4 \times 10^{-5}} = 4.26 \times 10^{-4}$ J ≈ **0.43 mJ** (4 times as much for twice the charge);
- 500 μC: $W = \dfrac{(5.00 \times 10^{-4})^2}{9.4 \times 10^{-5}} = 2.66 \times 10^{-3}$ J ≈ **2.7 mJ** (25 times the energy at 100 μC).

(The same answers come from the area under the graph, $\frac{1}{2}QV$: at 100 μC, V = 100 ÷ 47 = 2.13 V, and ½ × 100 μC × 2.13 V = 106 μJ.)
c. W ∝ Q², so the graph is a **parabola** through the origin, curving upward: flat at first, getting steeper.

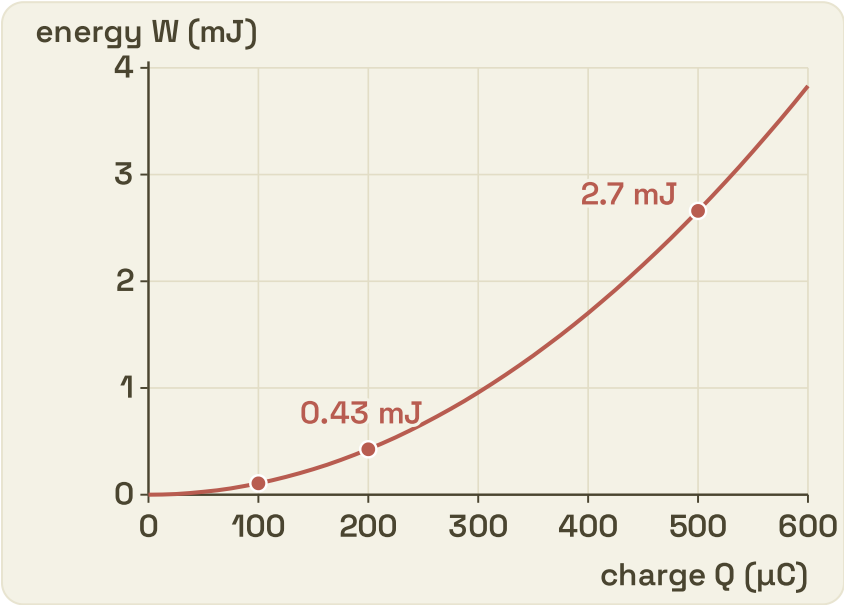


Figure 22.20 *For question 7c: the energy stored against the charge, $W = Q^2/2C$ for $C = 47$ μF, a parabola through the origin.*

8. **Before:** $V_1 = \dfrac{Q}{C_1} = \dfrac{2.47 \times 10^{-9}}{137 \times 10^{-12}} = 18.0$ V.

After: with a vacuum, $C_2 = \dfrac{C_1}{\varepsilon_r} = \dfrac{137}{3.1} = 44.2$ pF. The charge is still 2.47 nC, so

$$V_2 = \frac{Q}{C_2} = \frac{2.47 \times 10^{-9}}{44.2 \times 10^{-12}} = 55.9$$

(or simply $V_2 = 3.1 \times V_1$).

Change: the p.d. **rises by 55.9 − 18.0 ≈ 38 V**. Without the dielectric's surface charges, the field between the plates is 3.1 times stronger, and so is the p.d. (Pulling the dielectric out takes work, which shows up as extra stored energy.)

9. $$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2} \times 8.0 \times 10^{-6} \times (12)^2 = 5.76 \times 10^{-4} \approx$$