Unit 21

Answers: Potential, Potential Difference and Potential Energy

Quick checks

1 d $V = \dfrac{W}{Q} = \dfrac{8.0 \times 10^{-4}}{2.0 \times 10^{-6}} = 400$ V. Multiplying instead of dividing gives the first option.

2 b The two potentials, $+kQ/r$ and $-kQ/r$, add to zero. But both fields at the midpoint point the same way, from the + charge towards the − charge, so they add: the field is not zero.

3 b $|E| = \dfrac{\Delta V}{\Delta x} = \dfrac{50 - 20}{0.15} = 200$ V/m, and the minus sign in $E = -\Delta V/\Delta x$ means it points the way V falls, towards the 20 V end. 4.5 is ΔV × Δx; 333 is 50/0.15.

4 b Kinetic energy gained = qV: for the proton $e \times 500$ V = 500 eV, for the alpha particle $2e \times 500$ V = 1000 eV. The mass doesn't matter for the energy, only for the speed.

5 b The field is uniform: $E = \dfrac{V}{d} = \dfrac{240}{0.020} = 12{,}000$ V/m everywhere between the plates. The potential rises steadily from the − plate: a quarter of the way across, it is a quarter of 240 V, 60 V. 180 V is the potential a quarter of the way from the + plate.

6 c The field pulls the electron towards the higher potential. Its potential energy changes by $(-e)(+12\ ) = -12$ eV, and that energy becomes kinetic energy: +12 eV, which is $12 \times 1.60 \times 10^{-19} = 1.9 \times 10^{-18}$ J, not 12 J.

7 b For a point charge $V = kQ/r$ and $E = kQ/r^2$, so $\dfrac{V}{E} = r = \dfrac{120}{600} = 0.20$ m. 5.0 m is E/V.

8 c Gravity only attracts, so the potential near any mass, $-GM/r$, is negative. Near a positive charge the electric potential is positive. The other three are true of both.

9 b In a uniform field $V = Ed = 650 \times 0.12 = 78$ V. 7800 V comes from leaving the distance in centimeters (650 × 12), and 54 V from dividing by 12 instead of multiplying.

10 d The lines are straight, parallel and equally spaced: a **uniform** field. Field lines are drawn the same way for any field: the Earth's gravitational field over a small region, the magnetic field between the poles of a wide magnet, or the electric field between charged plates. Nothing in the picture says which.

11 b The potential belongs to the point, set by the charges that make the field, not by the charge placed there: it stays the same. The potential energy is qV: doubling q doubles it.

12 **d** $E = -\Delta V/\Delta x$: a potential that doesn't change from point to point has zero gradient, so zero field (as inside a charged metal sphere). The first two are false (Exercise 13); a constant field means a steadily changing potential, as between parallel plates, not a zero one.

13 **a** The two fields are equal and opposite, so they cancel; the two potentials, both positive, add: $2kQ/r$ (Exercise 11).

14 **e** The force is $F = qE$. The field is the same everywhere between the plates, and the two charges are the same size, e, so the forces are the same size, eE. The proton is pushed along the field, towards the negative plate; the electron against it, towards the positive plate. (The electron, about 1800 times lighter, gets a much larger **acceleration**.)

3. A test charge exerts forces on the charges that produce the field. A large test charge would **push or pull those charges out of place** (on a conductor, the charges move freely), so it would change the very field it is measuring. A small test charge disturbs them too little to matter, and the field is then $E = F/q$ in the limit of a very small q.

4. A negative one would do as well, if we remember its sign. The field is defined as $\vec{E} = \vec{F}/q$. The force on a negative charge points **opposite** to the field, but dividing by the negative q turns it round again, so $\vec{F}/q$ gives the same field. By convention the field's direction is the direction of the force on a **positive** charge, which is why a positive test charge is usually imagined: it points the right way without any thinking.

5. The field line through a point shows the direction of the field there: the direction of the force on a positive charge. If two lines crossed, the field at the crossing would have **two different directions** at once, and a charge placed there would be pushed two ways at once. But the field at a point is a single vector (the sum of all the charges' fields), so it has only one direction, and only one line can pass through any point.

6. The fields cancel where they are **equal in size and opposite in direction**. The field of $2Q$ is stronger at equal distances, so the point must be **nearer Q**.
a. **Opposite signs:** between the charges both fields point the same way (from + to −), so they can't cancel there. Outside, beyond the smaller charge Q, they point opposite ways. At a distance x beyond Q:
$$\frac{kQ}{x^2} = \frac{k(2Q)}{(x+\ell)^2} \;\Rightarrow\; x+\ell = \sqrt{2}\,x \;\Rightarrow\; x = \frac{\ell}{\sqrt{2}-1} \approx 2.4\,\ell$$
Yes: about **2.4ℓ beyond Q**, on the side away from 2Q. (Beyond 2Q there is no such point: there 2Q is both bigger and nearer.)
b. **Same signs:** between the charges the two fields point opposite ways. At a distance x from Q:
$$\frac{kQ}{x^2} = \frac{k(2Q)}{(\ell-x)^2} \;\Rightarrow\; \ell - x = \sqrt{2}\,x \;\Rightarrow\; x = \frac{\ell}{1+\sqrt{2}} \approx 0.41\,\ell$$
Yes: **between them, about 0.41ℓ from Q** (a little less than halfway). Outside, the two fields point the same way and can't cancel.

7. **Yes, no net work is done**: the work done is $q\,\Delta V$, and $\Delta V = 0$, whatever the path. **No, it doesn't mean there is no force.** The charge may go by a path where the field pushes it, doing positive work on one part and an equal negative work on another, so the total is zero. For example, carry a charge from a point 10 cm from a positive charge out to 20 cm and back round to another point 10 cm away: force acts all along, but the net work is zero. Only if the charge moves **along an equipotential** is the force at right angles to the motion at every step; even then the force isn't zero, it just does no work.

8. The field points the way the potential falls. A **positive charge** is pushed along the field, so it moves towards **lower potential**; a **negative charge** is pushed against the field, towards **higher potential**.
In each case the charge's **potential energy decreases**, turning into kinetic energy: for the positive charge $q > 0$ and $\Delta V < 0$; for the negative charge $q < 0$ and $\Delta V > 0$; either way $\Delta E_p = q\,\Delta V < 0$. Like a ball released on a slope, a charge left alone always moves to lower potential **energy**.

9. a. The **electric field** at a point is the **force** per unit positive charge there, $E = F/q$: a **vector**, in N/C. The **electric potential** is the **work (energy)** per unit positive charge to bring a charge there from infinity, $V = W/q$: a **scalar**, in volts. The field is minus the potential's gradient, $E = -\Delta V/\Delta x$: it depends on how the potential **changes**, not on its value, so either can be zero where the other is not.

   b. The **potential** belongs to a **point** in the field: it is the energy per unit charge, in volts, and doesn't depend on whether any charge is there. The **potential energy** belongs to a **charge** at that point: $E_p = qV$, in joules, and it is proportional to the charge (doubling the charge doubles it, as in Exercise 15).

10. The kinetic energy gained is $\frac{1}{2}mv^2 = eV$, so $v = \sqrt{\dfrac{2eV}{m}} \propto \sqrt{V}$. Four times the voltage gives $\sqrt{4} = 2$ times the speed: the final speed is **twice as great** (100% greater).

For 0.10 V, $v = \sqrt{\dfrac{2 \times 1.60 \times 10^{-19} \times 0.10}{9.11 \times 10^{-31}}} = 1.9 \times 10^5$ m/s; for 0.40 V, 3.7×10^5 m/s.

11. **Field: yes, at the midpoint.** There the two fields are equal in size (same charge, same distance) and opposite in direction, so they cancel. Anywhere else on the line one charge is nearer, and its field wins.
Potential: no. Each charge's potential, kQ/r, is positive everywhere, so their sum is positive everywhere and is never zero (except infinitely far away). At the midpoint it is $2kQ/r$, the field being zero there notwithstanding.

12. **Yes, if the particle is negatively charged.** Its change of potential energy is $\Delta E_p = q\,\Delta V$. Going to a higher potential, ΔV is positive; with q negative, ΔE_p is negative: the potential energy decreases. An electron released between charged plates does just this: it moves towards the positive plate, the higher potential, losing potential energy and gaining kinetic energy (21.6).

13. **Neither.** The field is minus the potential gradient, so it depends on how fast V **changes**, not on its value.

- **V = 0 but E ≠ 0:** halfway between equal and opposite charges, $+Q$ and $-Q$. The potentials $+kQ/r$ and $-kQ/r$ cancel, but both fields point from + to −, so they add. (Also the points 9 cm and 45 cm from O in the worked example of 21.3.)
- **E = 0 but V ≠ 0:** halfway between two equal positive charges (Exercise 11): the fields cancel, but the potential is $2kQ/r$. Also anywhere inside a charged hollow metal sphere: no field, but the potential is the sphere's, which may be thousands of volts.

14. $E_p = \dfrac{kQq}{r}$, taking it as zero when the charges are infinitely far apart.

- **Unlike charges:** Qq is negative, so E_p is **negative**. The pair has less energy than when far apart: work must be **done on them** to separate them. They are bound together, like an electron and a proton in an atom (or a planet and the Sun).
- **Like charges:** Qq is positive, so E_p is **positive**. The pair has more energy than when far apart: work was done to push them together, and if they are released the energy comes out as they fly apart.

19. $\Delta V = +65$ V and $q = -7.7 \times 10^{-6}$ C. The work done by the field is

$$W = -q\,\Delta V = -(-7.7 \times 10^{-6})(65) =$$

It is positive: a negative charge is pulled towards higher potential, so the field does the work (and the charge's potential energy falls by 5.0×10^{-4} J).

20.

$$E = \frac{V}{d} = \frac{220}{6.8 \times 10^{-3}} = 3.2 \times 10^{4}$$

The field is **3.2×10^{4} V/m** (N/C), pointing from the positive plate to the negative one.

21. The electron's kinetic energy must all turn into potential energy: $q\,\Delta V = -\Delta E_k = \frac{1}{2}mv^2$.

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 9.11 \times 10^{-31} \times (6.0 \times 10^{5})^2 = 1.64 \times 10^{-19}$$

With $q = -e$: $|\Delta V| = \dfrac{1.64 \times 10^{-19}}{1.60 \times 10^{-19}} =$ **1.0 V**.

The electron must be sent towards a point 1.0 V **lower** in potential ($\Delta V = -1.0$ V): a negative charge slows down as it goes to lower potential.

22.

$$Q = \frac{Vr}{k} = \frac{165 \times 0.15}{8.99 \times 10^{9}} =$$

(2.75 nC), positive, since the potential is positive.

23. a.

$$V = \frac{ke}{r} = \frac{8.99 \times 10^{9} \times 1.60 \times 10^{-19}}{0.53 \times 10^{-10}} =$$

b. The proton's pull keeps the electron in its circle (Unit 6): $\dfrac{ke^2}{r^2} = \dfrac{mv^2}{r}$, so

$$E_k = \tfrac{1}{2}mv^2 = \frac{ke^2}{2r} = \frac{8.99 \times 10^{9} \times (1.60 \times 10^{-19})^2}{2 \times 0.53 \times 10^{-10}} = = \frac{2.17 \times 10^{-18}}{1.60 \times 10^{-19}} = 13.6$$

c. The potential energy is $E_p = qV = (-e)(27.1\ ) = -27.1$ eV $= -4.34 \times 10^{-18}$ J (twice the kinetic energy, and negative). The total is

$$E = E_k + E_p = 13.6 - 27.1 = = -2.2 \times 10^{-18}$$

d. To free the electron and leave it at rest far away (total energy zero), **13.6 eV** (2.2×10^{-18} J) must be supplied: the ionization energy of hydrogen, just as measured.

24. When they just touch, their centers are $r = 1.2\times10^{-15} + 3.6\times10^{-15} = 4.8\times10^{-15}$ m apart. The potential of the silicon nucleus there is

$$V = \frac{k(14e)}{r} = \frac{8.99\times10^{9}\times 14\times 1.60\times10^{-19}}{4.8\times10^{-15}} = 4.2\times10^{6}$$

A proton (charge e) accelerated through a potential difference V gains energy eV, exactly what it needs to climb to that potential. So the voltage needed is about **4.2×10^{6} V** (4.2 MV): the proton needs 4.2 MeV.

(Some books print the silicon radius as 2.6×10^{-15} m. A nucleus of mass number A has a radius of about $1.2\times10^{-15}A^{1/3}$ m, which for silicon-28 is 3.6×10^{-15} m. With 2.6×10^{-15} m the same working gives 5.3 MV.)

25.

$$\tfrac{1}{2}mv^2 = eV \quad\Rightarrow\quad v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.60\times10^{-19}\times1200}{9.1\times10^{-31}}} = \sqrt{4.22\times10^{14}} =$$

about 7% of the speed of light (so ignoring relativity changes the answer by well under 1%).

26. a.

$$E = \frac{V}{d} = \frac{9.0}{4.5\times10^{-3}} =$$

from the positive plate to the negative plate.
b. The electron goes from the negative plate to the positive plate, 9.0 V higher, and gains $eV = 9.0$ eV $= 9.0\times1.60\times10^{-19} = 1.44\times10^{-18}$ J of kinetic energy:

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.44\times10^{-18}}{9.11\times10^{-31}}} = \sqrt{3.16\times10^{12}} =$$

(The same answer comes from the force: $a = eE/m$ over 4.5 mm, $v^2 = 2ad = 2eEd/m = 2eV/m$.)