Unit 19

Electric Charges

This unit online, with its interactives:
science-playground.com/s/11f19

On a dry winter morning in the Kathmandu Valley, you walk across a carpet, reach for a metal door handle, and a small spark jumps to your finger with a sharp little sting. Brush your freshly washed hair hard and the hairs fly apart. A plastic comb run through your hair picks up bits of paper, but a metal comb doesn't. In every case two things were rubbed together. Why should rubbing make forces and sparks? What kind of force is it? And why do metals behave differently from plastics?

Those are the questions this unit begins with. Their answers lead to one of the great laws of physics, **Coulomb's law**, which tells you exactly how hard two charges push or pull on each other. The same law holds an atom together, sticks a balloon to a wall and makes lightning possible. The next two units build on it: Unit 20 describes the space round a charge as an electric field, and Unit 21 the energy stored in it.

A STORY FROM SCIENCE

The engineer who weighed a spark's force

Charles-Augustin de Coulomb (1736–1806) was a French army engineer before he was a physicist. He spent years building forts in the Caribbean, and his first scientific work was on very practical things: how strong beams and walls are, how friction acts in machines, and how much work people and animals can do in a day without wearing themselves out. That last study made him one of the founders of what is now called **ergonomics**.

In 1785 he turned to electricity with an instrument he had perfected himself, the **torsion balance**: a light rod hanging from a fine silver wire, a small charged ball at one end. When a second charged ball was brought near, the push turned the rod and twisted the wire, and the twist measured the force, so delicately that it could detect a force of less than a millionth of a newton. Bringing the balls to half the distance needed about four times the twist. From measurements like these he concluded that the electric force falls off as the square of the distance. The unit of charge, the coulomb, is named after him.



Figure 19.1 *Charles-Augustin de Coulomb (1736–1806) in his engineer's uniform. His hand rests on the glass case of a torsion balance.*
Photo: Louis Hierle, Public domain

19.1 Introduction

Everything round you is made of atoms, and every atom is made of charged particles: **protons** (positive) in the tiny nucleus at its center, and **electrons** (negative) round it, with neutrons (no charge) in the nucleus too. An everyday object contains an enormous number of them, but usually exactly as many protons as electrons. The positive and negative charges then cancel: the object has no overall charge, and is said to be **neutral**.

The protons are locked in the nuclei and can't be moved about in ordinary processes. But the outermost electrons of an atom are held less tightly, and some of them can be moved from one object to another. That is the whole secret of charging:

- An object given **extra electrons** becomes **negatively charged**.
- An object that **loses electrons** becomes **positively charged** (it now has more protons than electrons).

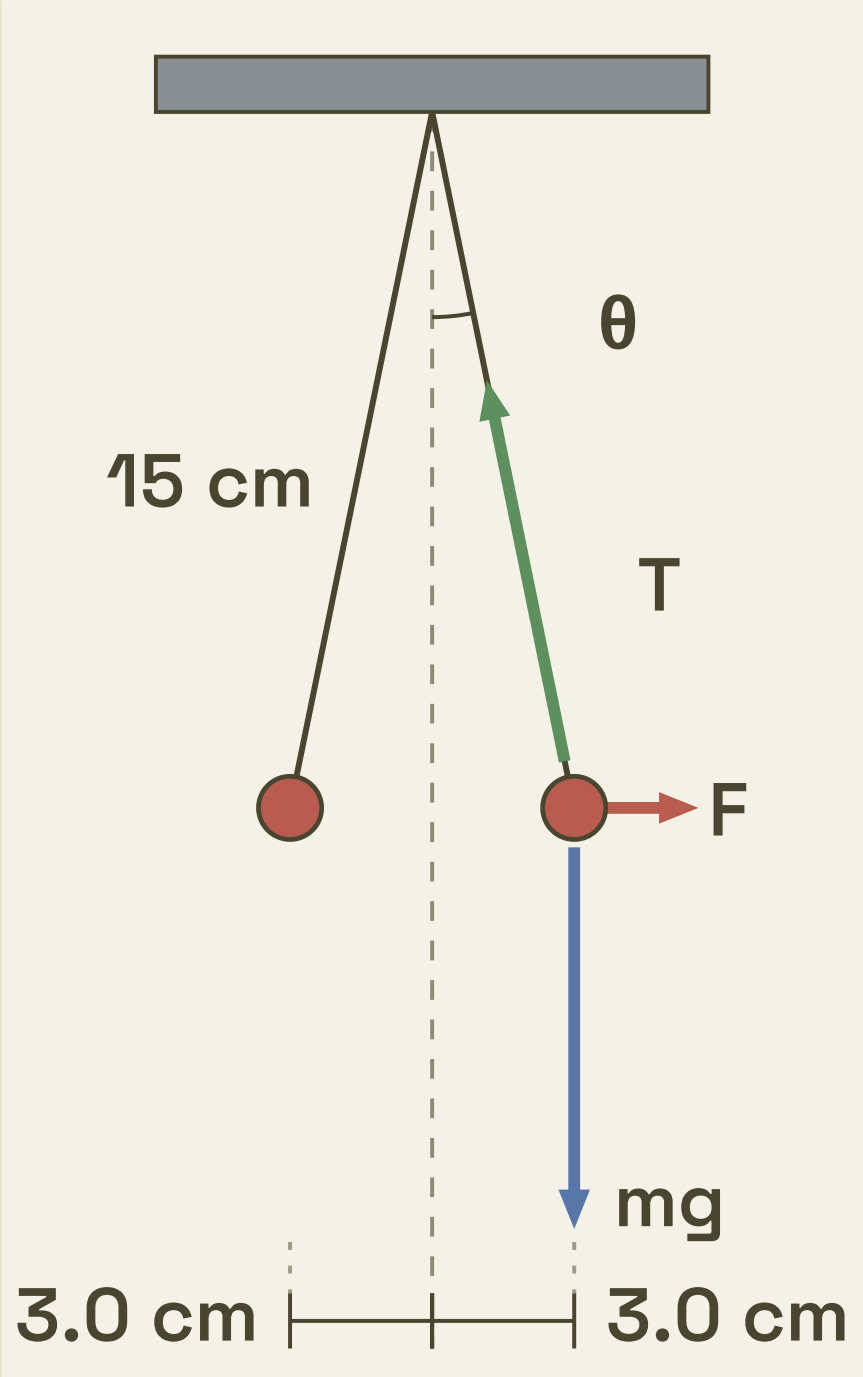
So "charging" never creates charge. It only moves electrons from one place to another: whatever charge one object gains, another loses. This is the law of **conservation of charge**: the total charge of an isolated system never changes.

Charge comes in lumps

Charge is measured in **coulombs** (C). The charge of one electron is −e and of one proton +e, where

Figure 19.14 *The two spheres, to scale, and the three forces on one of them: its weight mg, the electric force F and the tension T in the thread.*

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.



$$e = 1.6 \times 10^{-19}\ \text{C}$$

is the **elementary charge**. Since charge moves only as whole electrons (or whole ions), every charge you can find on an object is a whole number of these lumps:

$$Q = ne, \quad n = 0, \pm 1, \pm 2, \ldots$$

Charge is said to be **quantized**. One coulomb is a lot: it is the charge of $1/(1.6 \times 10^{-19}) = 6.25 \times 10^{18}$ electrons. The charge on a rubbed comb is only a few nanocoulombs (1 nC = 10^{-9} C), and on a charged sphere in the laboratory a few microcoulombs (1 μC = 10^{-6} C) at most.

Charge carriers

The charged particles that are free to move and carry charge from place to place are called **charge carriers**. What they are depends on the material:

- in **metals**, free electrons, which wander between the atoms;
- in **solutions** such as salt water and in molten salts, **ions**, positive and negative, moving in opposite directions;
- in **gases**, ions and electrons, made when a spark or flame knocks electrons off molecules;
- in **semiconductors** such as silicon, electrons and **holes** (places where an electron is missing, which behave like positive carriers).

A material with many charge carriers is a **conductor**; one with very few is an **insulator** (more about these in 19.3).

> **Check 1**
> A plastic rod is given a charge of -4.8×10^{-9} C by rubbing. How many electrons has it gained?
>
> a 7.7×10^{-28} b 3.3×10^{-11}
> c 3.0×10^{10} d 4.8×10^{10}

19.2 Charging by friction

When two different materials rub together, their surfaces come into very close contact at many tiny points, and electrons cross from one to the other. Which way they go depends on how tightly each material holds its electrons.

Brush dry hair with a plastic brush, for example, and electrons move from the hair to the brush. The **brush gains electrons** and becomes **negatively charged**; the **hair**

loses electrons and is left **positively charged**. Now two things happen that you can see:

- the brush and the hair, with opposite charges, **attract**: the hair is pulled towards the brush;
- the hairs, all with the same (positive) charge, **repel each other**, and stand apart.

In school experiments, **insulating rods are rubbed with cloth**: one of the two becomes positive and the other negative, as electrons cross between them. A plastic (polythene) rod rubbed with wool takes electrons from the wool and becomes **negative**; a glass rod rubbed with silk gives electrons to the silk and becomes **positive**. Insulators are used because they have very few free charge carriers: the charge stays where the rubbing put it. A metal rod held in the hand can be charged by rubbing too, but the charge doesn't stay: it flows away at once through the metal and through your body.



Figure 19.2 *A balloon rubbed on hair takes electrons from it. The positively charged hairs are pulled towards the negative balloon, and push each other apart.*

Photo: MikeRun, CC BY-SA 4.0

So: **when two materials are rubbed together, electrons are transferred between their surfaces**, and the two are left with equal and opposite charges.

CAREFUL

Rubbing moves charge; it doesn't make it

After the rubbing, the rod's charge and the cloth's charge are equal and opposite: added together, they are zero, just as before. And only electrons move. A rod becomes positive by *losing* electrons, never by gaining protons.

DID YOU KNOW?

Why winter shocks are worse

In the dry months the air holds little water. Damp air (and the thin film of water it leaves on surfaces) lets charge leak away slowly, so in the monsoon you hardly notice it building up. In dry winter air it stays on you as you walk across a carpet or slide off a car seat, until you touch metal and it all goes at once, in a spark.

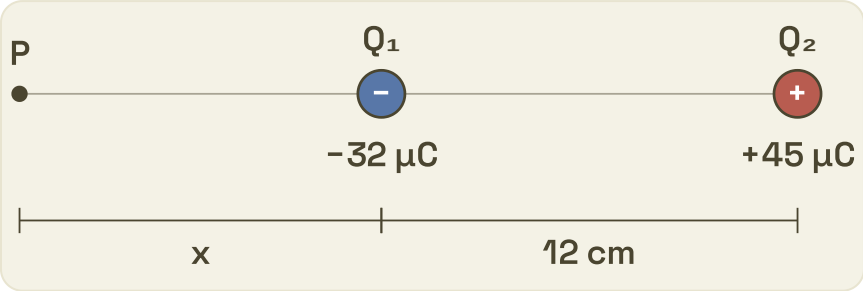


Figure 19.11 *P, on the line through the two charges, beyond Q_1. Not to scale.*

3. You are given two unknown point charges, Q_1 and Q_2. At a point on the line joining them, one third of the way from Q_1 to Q_2, the electric field is zero. What is the ratio Q_1/Q_2?

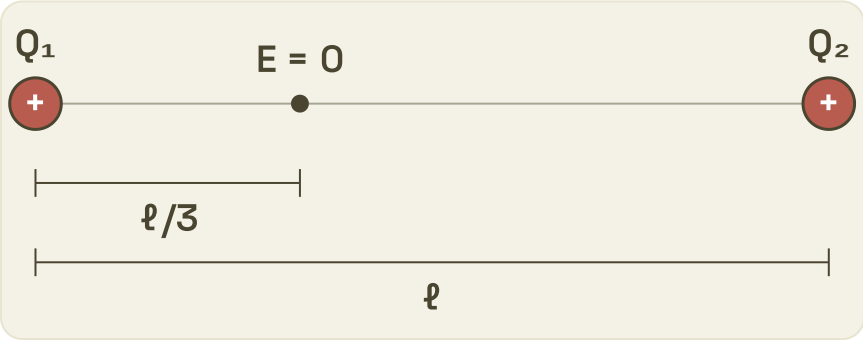


Figure 19.12 *The field is zero at a point $\ell/3$ from Q_1, between the charges.*

4. Two point charges of +30 nC and −30 nC form an electric dipole of length 0.25 m.
   a. Calculate the force between the charges.
   b. Calculate the electric field strength at the midpoint between the charges.
   c. Sketch the shape of the electric field in the region of the dipole.

5. Two light polystyrene spheres, coated in a conducting metallic paint, have each been given a positive charge by a high-voltage supply. They hang from one point on cotton threads 15 cm long, and their repulsion pushes them apart until each is 3.0 cm from the vertical, as shown. The mass of each sphere is 0.08 g. Calculate the charge on each sphere ($g = 9.8\ m/s^2$), assuming the two charges are equal.

Check 2

A glass rod rubbed with silk becomes positively charged. This is because:

a protons move from the silk to the glass
b electrons move from the glass to the silk
c electrons move from the silk to the glass
d positive charge is created in the glass by friction

19.3 Charging by induction

A charged object can also cause charge to move in a conductor **without touching it**. Bring a charged rod near a metal object, and the free electrons in the metal are pushed away or pulled closer, even though nothing passes between the rod and the metal.

Follow Figure 19.3, two metal spheres A and B on insulating stands:

1. **(a)** The spheres touch, so together they form a single conductor, uncharged.
2. **(b)** A negatively charged rod is brought near A. The free electrons in the metal are repelled by the rod and move as far from it as they can, to the far side of B. They stop when their own repulsion on each other balances the rod's push. A's side near the rod is left with too few electrons: positive. The charge has been **redistributed**, but the two spheres together are still neutral.
3. **(c)** B is moved away from A **while the rod is still in place**, so the electrons can't flow back.
4. **(d)** The rod is taken away. A is left **positively charged** and B **negatively charged**, by equal amounts; each sphere's charge now spreads evenly over its surface.

This is **charging by induction**. The rod never touched the spheres and it keeps all of its own charge, so it can charge any number of pairs of spheres, again and again.

You can do the same with **one** sphere. With the charged rod near it, touch the far side of the sphere with your finger: electrons pushed away by the rod flow out through you to the ground. Take your finger away first, then the rod, and the sphere is left positive, opposite to the rod. Connecting a conductor to the Earth like this is called **earthing** (or **grounding**).

These experiments show that **charge can be passed from one object to another only when they touch** (or when a spark jumps between them): contact is needed for charge to be *transferred*. Induction doesn't contradict this: in induction the rod's

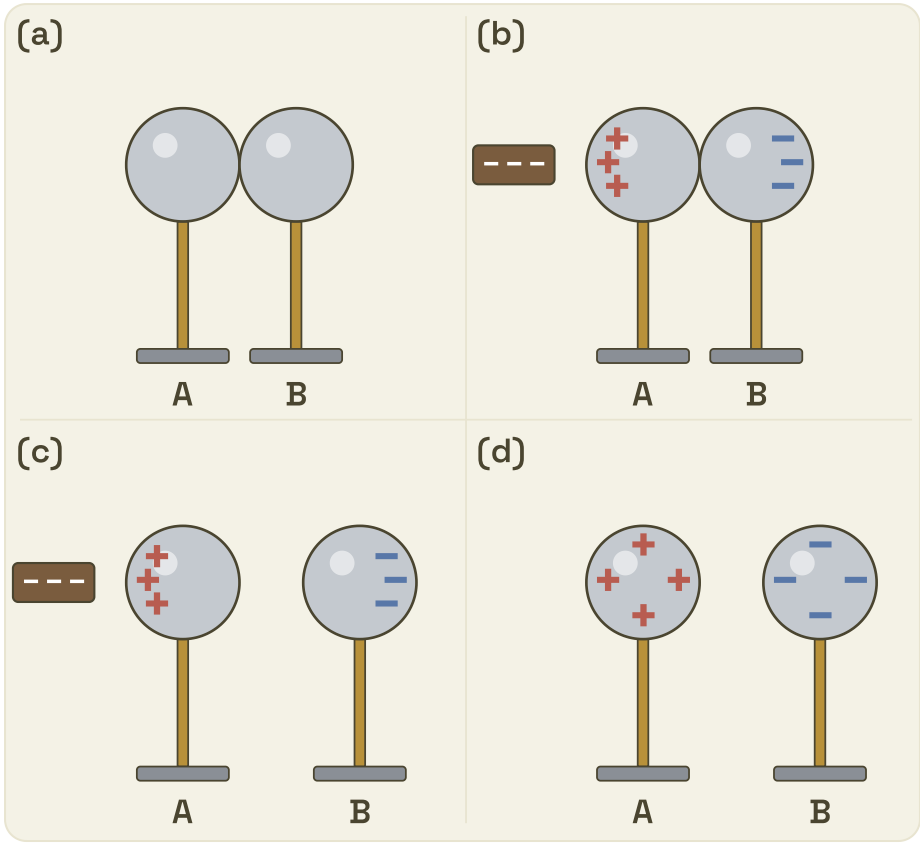


Figure 19.3 *Charging two metal spheres by induction. (a) A and B touch: one uncharged conductor. (b) A negatively charged rod is brought near A: free electrons are pushed to the far side of B, leaving A's near side positive. (c) With the rod still there, B is moved away. (d) The rod is taken away: A is left positive and B equally negative, each charge spread evenly over its sphere.*

charge stays on the rod, and it only *pushes the charges already in the conductor about*. The two spheres of Figure 19.3 were charged by touching each other, not by touching the rod.

And they show there are two kinds of material with very different electric properties:

- **Conductors**, such as metals, salt water, the human body and the Earth, contain many charge carriers. Charge moves through them freely, so charge put on one place spreads over the whole object at once.
- **Insulators**, such as plastic, glass, rubber, dry wood and dry air, contain very few. Charge put on them stays where it was put.

IN SHORT

Key ideas of Unit 19

- Matter is made of protons (+) and electrons (−); an object is **charged** when electrons are added (negative) or removed (positive). Charge is **conserved** and **quantized**: $Q = ne$, $e = 1.6 \times 10^{-19}$ C.
- **Charging by friction**: rubbing transfers electrons between two surfaces, leaving equal and opposite charges.
- **Charging by induction**: a charged object near a conductor pushes its free charges about without touching it; separating the parts (or earthing it) before the charged object is removed leaves a charge of the opposite sign.
- Charge is transferred only by contact; **conductors** let charge move freely, **insulators** don't. Removing charge by touching is **discharging**.
- **Coulomb's law**: $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1 Q_2}{r^2}$, along the line joining the charges, repulsive for like and attractive for unlike charges; $k = 9.0 \times 10^9$ N m²/C², $\varepsilon_0 = 8.85 \times 10^{-12}$ C²/(N m²). In a medium, ε replaces ε_0.
- Uniformly charged spheres act as point charges at their centers. Forces from several charges add as vectors (**superposition**).
- Electric field strength $E = F/q$ (N/C); for a point charge $E = kQ/r^2$, away from + and towards −.

Exercise

Choose the correct option

Check 5

1. Two charged particles, −2Q and +Q, a distance d apart, attract each other with a force F. The charge of each particle is changed by +Q, and the distance between them is increased to 2d. What is the force between them now?

a an attractive force of F/4
b a repulsive force of F/4
c an attractive force of F/2
d a repulsive force of F/2

Numerical problems

2. Two point charges, $Q_1 = -32$ μC and $Q_2 = +45$ μC, are 12 cm apart. The electric field at the point P shown is zero. How far is P from Q_1?

WORKED EXAMPLE

Example 2: the field between two charges

Two point charges are 10.0 cm apart: $Q_1 = -25\ \mu\text{C}$ and $Q_2 = +50\ \mu\text{C}$. (a) Find the size and direction of the electric field at a point P between them, 2.0 cm from the negative charge. (b) An electron (mass 9.11×10^{-31} kg) is placed at rest at P and let go. What is its acceleration at that instant?

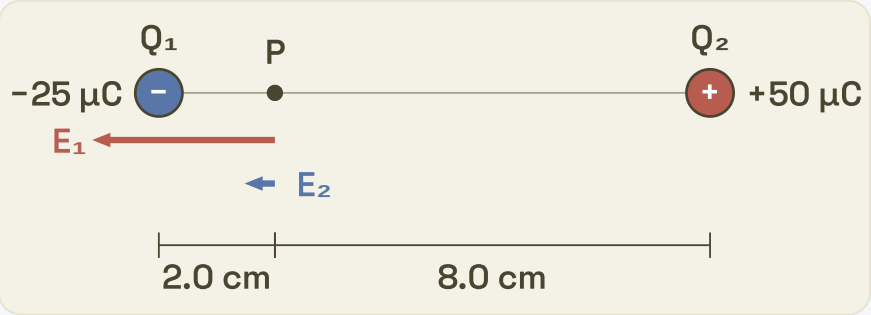


Figure 19.10 *The fields of the two charges at P both point to the left, towards Q_1. To scale: E_1 is eight times E_2.*

Answer

(a) The field at P is the vector sum of the fields of Q_1 and Q_2 separately. The field of the **negative** Q_1 points **towards** Q_1, to the left; the field of the **positive** Q_2 points **away from** Q_2, also to the left. Both point the same way, so their sizes add:

$$E = E_1 + E_2 = k\left(\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2}\right) = 9.0 \times 10^9 \left(\frac{25 \times 10^{-6}}{(2.0 \times 10^{-2})^2} + \frac{50 \times 10^{-6}}{(8.0 \times 10^{-2})^2}\right)$$

$$E = 9.0 \times 10^9\,(0.0625 + 0.0078) = 5.63 \times 10^8 + 0.70 \times 10^8 = 6.3 \times 10^8\ \text{N/C}$$

(Here Q_1 and Q_2 stand for the sizes of the charges; the directions were worked out first.)

(b) The field points to the left, so the force on the **negative** electron points to the **right**: so does its acceleration.

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{(1.6 \times 10^{-19}) \times (6.3 \times 10^8)}{9.11 \times 10^{-31}} = 1.1 \times 10^{20}\ \text{m/s}^2\text{, to the right}$$

DID YOU KNOW?

A machine that prints with static

A photocopier or laser printer in any office in Nepal works by charges. A drum coated with a special material is given an even static charge in the dark. Light reflected from the page (or a laser) falls on it, and where light falls the material becomes a conductor and the charge leaks away. The charge left behind is a picture of the page, and fine black powder (toner), given the opposite charge, sticks only there. The powder is then pressed onto paper and melted in with heat.

Check 3

A positively charged rod is held near (but not touching) an uncharged metal sphere on an insulating stand. The sphere is touched with a finger, the finger is removed, and then the rod is taken away. The sphere is now:

a uncharged
b positively charged
c negatively charged
d positive on one side and negative on the other

19.4 Coulomb's law for the electric force

A charged object pushes or pulls on other charged objects across empty space, without touching them. The region round a charged object in which another charge feels this force is its **electric field** (or electrostatic field): Unit 20 is about fields. First, how big is the force?

In 1785, using his torsion balance, Coulomb measured the force between two small charged spheres and found that the force between two **point charges**:

- is **directly proportional to the product of the two charges**, $F \propto Q_1 Q_2$;
- is **inversely proportional to the square of the distance between them**, $F \propto \frac{1}{r^2}$;
- acts **along the line joining them**: **repulsive** between charges of the same sign, **attractive** between charges of opposite sign.

Combining the two proportions,

$$F \propto \frac{Q_1 Q_2}{r^2}$$

so for two point charges Q_1 and Q_2 a distance r apart in a vacuum,

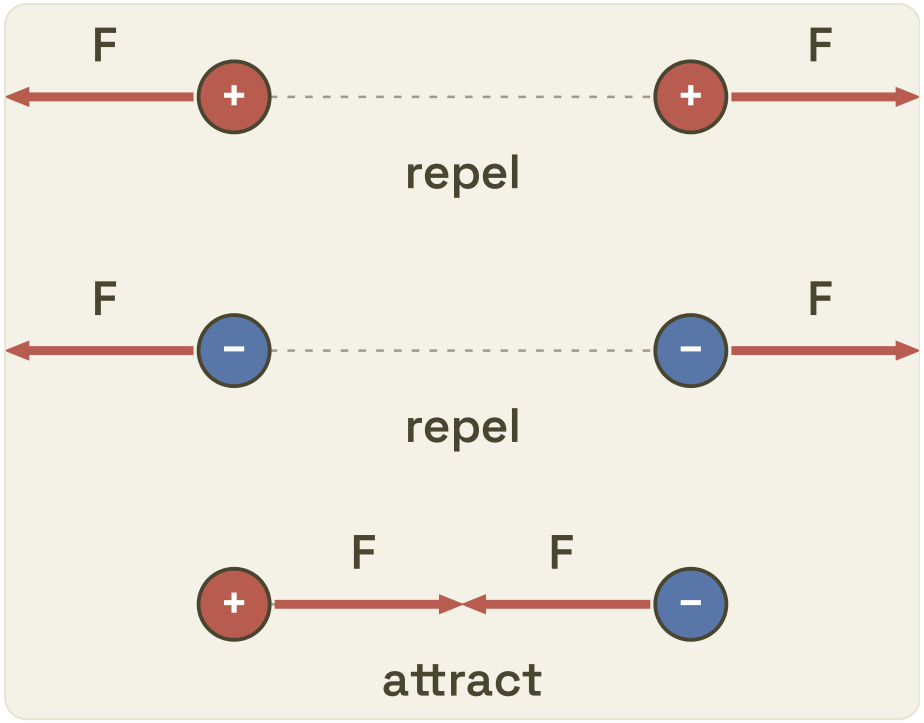


Figure 19.6 *The forces between two charges a distance r apart: like charges repel, unlike charges attract. In each pair the two forces are equal and opposite.*

where k is a constant, **Coulomb's constant**. It is usually written as $\dfrac{1}{4\pi\varepsilon_0}$, so **Coulomb's law** is

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}}$$

The constant ε_0 is called the **permittivity of free space** (of a vacuum):

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/(N\,m^2)}, \qquad k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 \approx 9.0 \times 10^9\ \mathrm{N\,m^2/C^2}$$

(The 4π is there because a charge's influence spreads out over a sphere, whose area is $4\pi r^2$; Unit 20's Gauss's law shows why this way of writing it is the natural one.)

The force on each charge is directed **along the line joining the two charges**. If the charges have the same sign, the force on each points away from the other; if opposite signs, towards the other (Figure 19.6). The force that Q_1 exerts on Q_2 is equal and opposite to the force Q_2 exerts on Q_1, as **Newton's third law** requires: a small charge pulls a big one exactly as hard as the big one pulls it.

WORKED EXAMPLE

Example 1: the field of a single point charge

Calculate the size and direction of the electric field at a point P, 30 cm to the right of a point charge $Q = -3.0 \times 10^{-6}$ C.

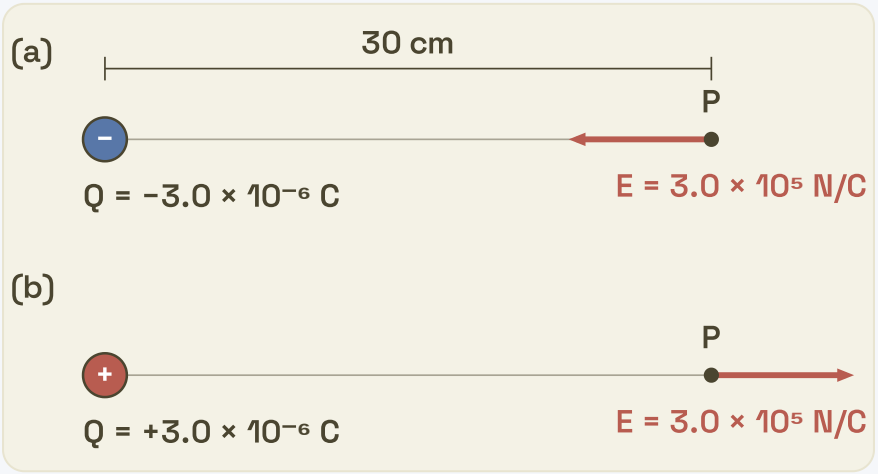


Figure 19.9 *The field at P, 30 cm from Q: towards a negative charge (a), away from a positive one (b).*

Answer

The size of the field depends only on the size of Q:

$$E = \frac{kQ}{r^2} = \frac{(9.0 \times 10^9) \times (3.0 \times 10^{-6})}{(0.30)^2} = \frac{2.7 \times 10^4}{0.090} = 3.0 \times 10^5\ \mathrm{N/C}$$

Its direction is **towards the charge Q, to the left**, as in Figure 19.9(a): the direction of the field is that of the force on a positive test charge, which a negative Q would attract. If Q were positive, the field at P would point **away** from it, to the right, as in (b).

There is no charge at P. But there is an electric field there: the only real charge is Q.

Check 4

Two point charges repel each other with a force F. If the distance between them is made three times as large, the force becomes:

a 3F
b F/3
c 9F
d F/9

WORKED EXAMPLE

Three charges in a line

Three point charges lie on a straight line: A = +4.0 μC, then B = −2.0 μC 10 cm to its right, then C = +1.0 μC a further 10 cm to the right. Find the net force on C.

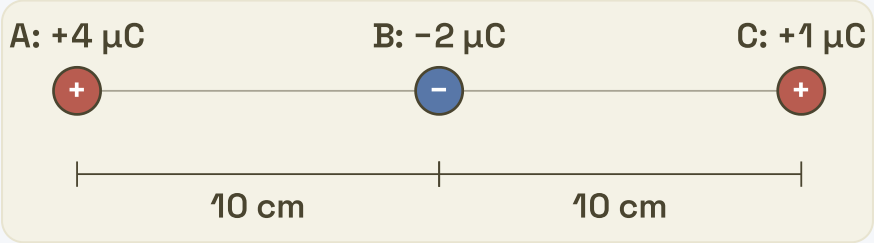


Figure 19.8 *The three charges, 10 cm apart in a line.*

Answer

Take the direction to the right as positive.

Force from A (like charges, 0.20 m apart, a push to the right):

$$F_A = \frac{9.0 \times 10^9 \times 4.0 \times 10^{-6} \times 1.0 \times 10^{-6}}{(0.20)^2} = \frac{3.6 \times 10^{-2}}{0.040} = 0.90 \text{ N to the right}$$

Force from B (unlike charges, 0.10 m apart, a pull to the left):

$$F_B = \frac{9.0 \times 10^9 \times 2.0 \times 10^{-6} \times 1.0 \times 10^{-6}}{(0.10)^2} = \frac{1.8 \times 10^{-2}}{0.010} = 1.8 \text{ N to the left}$$

Net force on C: $0.90 - 1.8 = -0.90$ N, that is **0.90 N to the left**, towards B. B is smaller than A, but it is twice as near, and $1/r^2$ makes that count four times over.

The sign of the force

If you put the charges into the formula with their signs, the sign of F tells you which way the force acts:

- F **positive** (Q_1 and Q_2 both positive or both negative): **repulsion**;
- F **negative** (one positive, one negative): **attraction**.

The direction of the force is always along the line joining the charges; the sign only says whether it points towards the other charge or away from it.

Charges in a material

The law above is for charges in a vacuum. If the charges are in another medium, such as water or oil, the force between them is smaller, and the permittivity of the medium, ε, takes the place of ε_0:

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{r^2}$$

(the "0" is kept for the vacuum only). The ratio $\varepsilon_r = \varepsilon/\varepsilon_0$ is the medium's **relative permittivity** (or **dielectric constant**): it says how many times weaker the force is in the medium.

Medium	Relative permittivity ε_r	Permittivity ε (C²/(N m²))
vacuum	1 (exactly)	8.85×10^{-12}
air	1.0006	8.86×10^{-12}
paper	about 3	about 3×10^{-11}
glass	5 to 10	about 4×10^{-11} to 9×10^{-11}
water (room temperature)	about 80	about 7.1×10^{-10}

Air is so close to a vacuum that ε_0 is used for both. Water is the striking one: two charges in water attract only about an eightieth as strongly as in air. That is why salt dissolves in water: the sodium and chloride ions of a salt crystal, held together by Coulomb's attraction, are pulled apart far more easily once water surrounds them.

Charged spheres act as point charges

Coulomb's law is for **point charges**, charges so small compared with the distance between them that their size doesn't matter: protons, electrons, ions, or small charged balls far apart. But it also works exactly for **uniformly charged spheres**, however close they are, as long as r is measured from **center to center**: outside it, a uniformly

charged sphere acts as if all its charge were concentrated at its center. (Unit 20 proves this with Gauss's law; it is the same result as for a planet's gravity in Unit 7.)

CAREFUL

Center to center

For two charged spheres of radius 1 cm whose surfaces are 3 cm apart, $r = 1 + 3 + 1 = 5$ cm, not 3 cm. And "uniformly charged" matters: two *metal* spheres very close together push their charges about by induction, to the far sides (like charges) or the near sides (unlike), and then the law gives only an approximate answer.

The force from several charges

When several charges act on one charge, each pushes or pulls on it **exactly as if the others weren't there**, by Coulomb's law. The total (net) force is the **vector sum** of these separate forces:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$$

This is the **principle of superposition**. It sounds obvious, but it is a fact of nature, found by experiment: one charge doesn't change the force another exerts. Since the forces are vectors, they are added as vectors (Unit 2): along one line, with signs for their directions; at an angle, by the parallelogram law or by components.

A first look at the electric field

The two worked examples below use an idea that Unit 20 develops fully. Instead of asking about the force between two charges, ask: what does a charge Q do to the space round it? Place a small positive **test charge** q at a point. The **electric field strength** E there is the **force on each coulomb** of test charge:

$$E = \frac{F}{q}$$

in newtons per coulomb (N/C). Its direction is the direction of the force on a *positive* test charge. For a point charge Q, Coulomb's law gives $F = kQq/r^2$, so

$$E = \frac{F}{q} = \frac{kQ}{r^2}$$

pointing **away from** a positive Q and **towards** a negative one. The field is there whether or not a test charge is placed to feel it. Once you know E at a point, the force on any charge q put there is $F = qE$.

WORKED EXAMPLE

The force inside a hydrogen atom

Find the size and direction of the electric force on the electron of a hydrogen atom, exerted by the single proton that is its nucleus. Take the average distance between them to be 0.53×10^{-10} m.

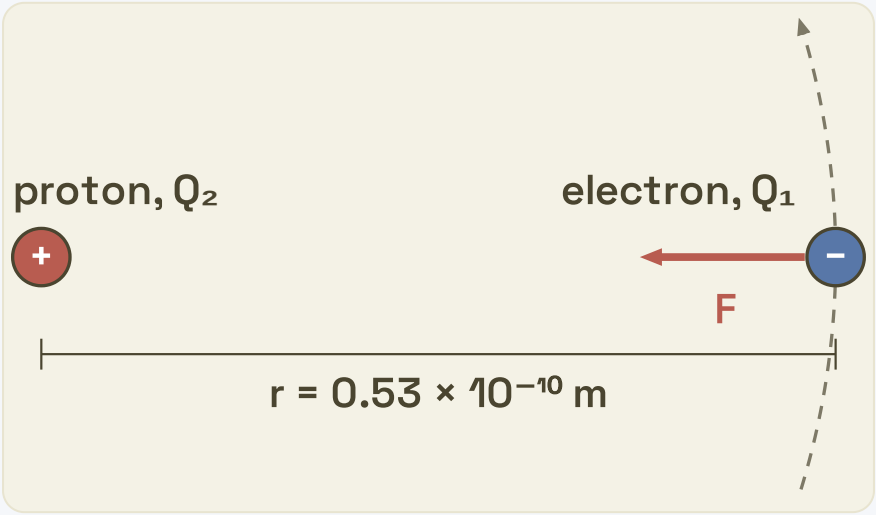


Figure 19.7 *The electron, a distance r from the proton, is pulled towards it.*

Answer

Both charges have size $e = 1.6 \times 10^{-19}$ C.

$$F = \frac{kQ_1Q_2}{r^2} = \frac{(9.0 \times 10^9) \times (1.6 \times 10^{-19}) \times (1.6 \times 10^{-19})}{(0.53 \times 10^{-10})^2} = \frac{2.30 \times 10^{-28}}{2.81 \times 10^{-21}} =$$

The charges have opposite signs, so the force is **attractive**: the force on the electron points **towards the proton**. It is this pull that keeps the electron round the nucleus.

Tiny as 8.2×10^{-8} N sounds, it acts on an electron of mass 9.1×10^{-31} kg: it would give it an acceleration of about 9×10^{22} m/s².