Unit 18

Answers: Dispersion

Quick checks

1 a $v = c/\mu$, and μ is smallest for red, so **red light travels fastest** in glass (and is bent least). In a vacuum, and very nearly in air, all colors travel at the same speed; red has the *lower* frequency.

2 b Angular dispersion $(\mu_v - \mu_r)A$ doubles with A; the dispersive power $(\mu_v - \mu_r)/(\mu - 1)$ has no A in it and stays the same.

3 a $\frac{0.02}{20} + \frac{0.04}{-40} = 0.001 - 0.001 = 0$ ✓, and $\frac{1}{20} - \frac{1}{40} = \frac{1}{40}$: F = +40 cm ✓. The second pair has $\frac{0.04}{20} - \frac{0.02}{40} \neq 0$; two convex lenses can never cancel; the last pair would have no power at all.

4 a When light goes from air into glass or water its speed and wavelength both change, but its **frequency** stays the same, and so does its color: a red light under water still looks red. So frequency is what fixes the color. (Wavelengths are quoted for light in a vacuum or air, where they go with the frequency one to one.)

5 a Each color is focused at a different distance, so the image is fringed with **color**: where the screen is set for one color, the others form colored blurs round it.

6 d Achromatic means red and violet are focused at the same point: $F_r = F_v$, so $F_r - F_v = 0$.

7 b White light goes in and comes out spread into a band of colors: **dispersion**. (Refraction and deviation happen too, but they don't split light; chromatic aberration is a defect of lenses.)

8 b No dispersion: $\omega(\mu - 1)A = \omega'(\mu' - 1)A'$, so $A = \frac{0.045 \times 0.66 \times 5^\circ}{0.03 \times 0.53} = \frac{0.1485}{0.0159} = 9.34^\circ$.

9 b $f = \frac{25 + 26}{2} = 25.5$ cm, and $\omega = \frac{f_r - f_v}{f} = \frac{1}{25.5} = 0.039$. (Dispersive power is never negative.)

10 c $\frac{f_2}{f_1} = -\frac{\omega_2}{\omega_1} = -3$, so $f_2 = -3f_1$. Then $\frac{1}{40} = \frac{1}{f_1} - \frac{1}{3f_1} = \frac{2}{3f_1}$, so $f_1 = \frac{80}{3} = 26\frac{2}{3}$ cm and $f_2 = -80$ cm.

11 **a** $\dfrac{f_2}{f_1} = -\dfrac{0.03}{0.015} = -2$, so $f_2 = -2f_1$, and $\dfrac{1}{100} = \dfrac{1}{f_1} - \dfrac{1}{2f_1} = \dfrac{1}{2f_1}$: $f_1 = 50$ cm (crown, convex) and $f_2 = -100$ cm (flint, concave). A doublet needs two lenses, so the options give pairs.

9.

a. The refractive index of a medium depends on the wavelength of light: it is larger for shorter wavelengths (violet) than for longer ones (red). So at a slanting surface each color is refracted through a different angle, and white light, a mixture of colors, is split.

b. Monochromatic light has one wavelength, so the medium has a single refractive index for it and all of it is bent through the same angle. There are no different colors to separate.

c. A mirror forms images by **reflection**, and the law of reflection, angle of reflection = angle of incidence, is the same for every wavelength. No color is turned more than another, so a mirror focuses all colors at one point: it has no chromatic aberration. (This is why the largest telescopes use mirrors.)

10.

a. Yes. It decreases as the wavelength increases, roughly as Cauchy's relation $\mu = A + B/\lambda^2 + C/\lambda^4$ says, so it is larger for violet than for red.

b. Angular dispersion is the angle between the violet and red rays leaving a prism, $\delta_v - \delta_r = (\mu_v - \mu_r)A$ for a thin prism; it depends on the material and on the prism's angle. Dispersive power is the ratio of the angular dispersion to the mean deviation, $\omega = \dfrac{\mu_v - \mu_r}{\mu - 1}$; it depends only on the material.

c. $f_r - f_v = \omega f$, so $f = \dfrac{f_r - f_v}{\omega} = \dfrac{0.351}{0.026} = \mathbf{13.5\ cm}$.

11.

a. The colors are separated at the first face, but the second face is parallel to it, and refraction there turns each color back to the direction it came in. They all come out parallel to the incident beam and to one another, shifted sideways only a little (a fraction of a millimeter in an ordinary slab). Over the width of a beam these slightly shifted colors overlap and mix back to white, and only the extreme edges of the beam are faintly tinted.

b. **Blue.** Its wavelength is shorter, so glass has a larger refractive index for it, and the deviation $(\mu - 1)A$ is larger.

c. $f = \dfrac{26 + 25}{2} = 25.5$ cm; chromatic aberration $f_r - f_v = \mathbf{1\ cm}$; $\omega = \dfrac{1}{25.5} = \mathbf{0.039}$.

12. a. $\omega = \dfrac{\mu_v - \mu_r}{\mu - 1}$, where μ_v, μ_r and μ are the refractive indices for violet, red and the mean color.

b. A prism deviates light towards its base. With the prism's apex upwards, as usually drawn, the base is at the bottom; violet, with the largest refractive index, is deviated most, so it goes lowest, and red, deviated least, stays highest (Figure 18.2).

c. $\dfrac{f_2}{f_1} = -\dfrac{0.03}{0.015} = -2$, and $\dfrac{1}{100} = \dfrac{1}{f_1} - \dfrac{1}{2f_1} = \dfrac{1}{2f_1}$, so $f_1 = \mathbf{50\ cm}$ (crown, convex) and $f_2 = \mathbf{-100\ cm}$ (flint, concave).

13. a. The failure of a lens to bring all colors to a focus at one point on its axis: violet is focused nearer the lens than red, and the image has colored edges. It is measured by $f_r - f_v = \omega f$.

b. Achromatism is freedom from chromatic aberration: a combination of lenses (usually a convex crown lens and a concave flint lens, with $\omega/f + \omega'/f' = 0$) that brings red and violet to the same focus.

c. $\omega(\mu - 1)A = \omega'(\mu' - 1)A'$: $A = \dfrac{0.045 \times 0.66 \times 5^\circ}{0.03 \times 0.53} = \mathbf{9.34°}$.

14. a. Place two thin prisms of different glasses (angles A and A′) with their apexes in opposite directions. **Deviation without dispersion:** choose the angles so that their dispersions are equal and opposite, $(\mu_v - \mu_r)A = (\mu'_v - \mu'_r)A'$; the colors recombine, while the deviations, which differ because the glasses differ, leave a net deviation. **Dispersion without deviation:** choose the angles so that the deviations of the mean color cancel, $(\mu - 1)A = (\mu' - 1)A'$; the mean color goes straight through, while the dispersions, which differ, leave a spectrum (a direct-vision prism).

b. Angular dispersion depends on the material and on the angle of the prism; dispersive power depends only on the material (and, for both, on which colors are compared).

c. As in 12b: violet has the largest refractive index and is deviated most, towards the base, which is at the bottom when the apex points up; red is deviated least.

d. $\dfrac{f_2}{f_1} = -\dfrac{0.026}{0.013} = -2$, and $\dfrac{1}{100} = \dfrac{1}{2f_1}$, so $f_1 = \mathbf{50\ cm}$ (crown) and $f_2 = \mathbf{-100\ cm}$ (flint). (These are the same as in 12c: only the ratio of the dispersive powers matters, and it is 2 in both.)

15. a. Put a convex lens of the glass with the smaller dispersive power (crown) in contact with a concave lens of the glass with the larger dispersive power (flint), with focal lengths chosen so that $\frac{\omega}{f} + \frac{\omega'}{f'} = 0$. The convex lens must be the stronger, so the pair still converges.

b. As in the worked example of 18.8: $f_1 = -\frac{2}{3} f_2$ and $\frac{1}{30} = -\frac{1}{2f_2}$, so the flint lens has $f_2 = \mathbf{-15\ cm}$ and the crown lens $f_1 = \mathbf{+10\ cm}$.

16. For a concave lens, in the real-is-positive convention, $\frac{1}{f} = -(\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

with $R_1 = 25$ cm and $R_2 = 30$ cm: $\frac{1}{25} + \frac{1}{30} = 0.07333\ \text{cm}^{-1}$.

Violet: $\frac{1}{f_v} = -0.523 \times 0.07333 = -0.03835$, so $f_v = -26.07$ cm.

Red: $\frac{1}{f_r} = -0.5204 \times 0.07333 = -0.03816$, so $f_r = -26.20$ cm.

Chromatic aberration $f_r - f_v = -26.20 - (-26.07) = -0.13$ cm: its size is **0.13 cm**, and, as for any concave lens, the violet focus is nearer the lens.

17. $F = \frac{1}{P} = \frac{1}{5}\ \text{m} = 20$ cm.

$\frac{f_2}{f_1} = -\frac{0.030}{0.015} = -2$, so $f_2 = -2f_1$, and $\frac{1}{20} = \frac{1}{f_1} - \frac{1}{2f_1} = \frac{1}{2f_1}$.

So $f_1 = \mathbf{+10\ cm}$ (crown, convex) and $f_2 = \mathbf{-20\ cm}$ (flint, concave).

(Crown glass's dispersive power is about 0.015, half flint's. If a book prints a much larger value for crown, such as 0.115, the numbers can't work: a convex lens of the more dispersive glass can't make a converging achromat with a concave lens of the less dispersive one.)

18. $f_2 = -2f_1$ (as in 17), and $\frac{1}{80} = \frac{1}{2f_1}$, so $f_1 = \mathbf{40\ cm}$ (crown, convex) and $f_2 = \mathbf{-80\ cm}$ (flint, concave).

19. Take each mean index as the average of violet and red: crown $\mu = 1.520$, flint $\mu' = 1.657$.

$\omega_1 = \frac{1.523 - 1.517}{1.520 - 1} = \frac{0.006}{0.520} = 0.01154$, $\omega_2 = \frac{1.664 - 1.650}{1.657 - 1} = \frac{0.014}{0.657} = 0.02131$.

$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$ and $\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{50}$. From the first, $\frac{1}{f_2} = -\frac{\omega_1}{\omega_2} \cdot \frac{1}{f_1} = -\frac{0.5415}{f_1}$, so $\frac{1 - 0.5415}{f_1} = \frac{1}{50}$.

$f_1 = 50 \times 0.4585 = \mathbf{22.9\ cm}$ (crown), and $f_2 = -\frac{f_1}{0.5415} = \mathbf{-42.3\ cm}$ (flint).

(Rounding ω to three figures first, 0.0115 and 0.0213, gives about 23.0 cm and −42.6 cm.)

20. $\theta = (\mu_v - \mu_r)A = (1.66 - 1.54) \times 5^\circ = 0.12 \times 5^\circ = \mathbf{0.6°}$.

21. Mean indices: crown $\dfrac{1.523 + 1.513}{2} = 1.518$, flint $\dfrac{1.773 + 1.743}{2} = 1.758$.

Crown: $\omega = \dfrac{1.523 - 1.513}{1.518 - 1} = \dfrac{0.010}{0.518} = \mathbf{0.0193}$.

Flint: $\omega' = \dfrac{1.773 - 1.743}{1.758 - 1} = \dfrac{0.030}{0.758} = \mathbf{0.0396}$.