Answers: Lenses

Quick checks

1 b One face convex and one concave makes it a meniscus. Thinner in the middle means the concave face is curved more: a **convexo-concave** lens, a diverging lens. (A concavo-convex lens is thicker in the middle and converges.)

2 c $\dfrac{1}{f} = (1.5-1)\left(\dfrac{1}{10}+\dfrac{1}{\infty}\right) = \dfrac{0.5}{10} = \dfrac{1}{20}$, so $f = 20$ cm. The flat face contributes nothing ($1/\infty = 0$), so a plano-convex lens is half as strong as an equiconvex lens with the same curve.

3 c $\dfrac{1}{F} = \dfrac{1}{20}+\dfrac{1}{30} = \dfrac{3+2}{60} = \dfrac{1}{12}$, so $F = 12$ cm: shorter than either, because the pair converges more strongly than either alone. Adding the focal lengths (50 cm) is the usual mistake.

4 b $P = -4 + 2.5 = -1.5$ D, negative, so the pair diverges. $F = \dfrac{1}{-1.5}$ m = –0.667 m = **−66.7 cm**.

5 b $P = 12 + (-2) = 10$ D, so $F = \dfrac{1}{10}$ m = 10 cm.

6 a $u = f$ gives $\dfrac{1}{v} = \dfrac{1}{f} - \dfrac{1}{f} = 0$: the image is at infinity, so the rays leave **parallel**. (This is how a torch or a slide projector's lamp sends out a parallel beam.)

7 d A real image of the same size needs $m = \dfrac{v}{u} = 1$, so $u = v = 2f$ and the distance is $d = u + v = 4f$. This is also the least distance there can be between an object and its real image, of any size. (Proof: $\frac{1}{u}+\frac{1}{v} = \frac{1}{f}$ gives $uv = f(u+v) = fd$; and $(u-v)^2 = d^2 - 4uv = d^2 - 4fd \geq 0$, so $d \geq 4f$.) An object nearer to its screen than 4f can never be focused on it.

8 a Virtual and upright: $m = \dfrac{v}{u} = -2$, so $v = -2u$. $\dfrac{1}{16} = \dfrac{1}{u} - \dfrac{1}{2u} = \dfrac{1}{2u}$, so $u = 8$ cm (inside the focus, as a virtual image needs).

9 **a** Take the lens as equiconvex ($\mu = 1.5$ gives $f = R$), so R = 20 cm.
The lens is surrounded by the liquid, but the light starts and ends in air beyond the cell's flat walls. The liquid on each side of the lens fills the space between a flat wall and a convex glass face: it forms **two plano-concave liquid lenses**, each with $\frac{1}{f} = (1.6 - 1)\left(-\frac{1}{20}\right) = -0.03$ cm^{-1}.
The glass lens in air: $\frac{1}{f_g} = (1.5 - 1)\left(\frac{1}{20} + \frac{1}{20}\right) = +0.05$ cm^{-1}.
All three are in contact: $\frac{1}{F} = 0.05 - 0.03 - 0.03 = -0.01$ cm^{-1}, so $F = \mathbf{-100\ cm}$, a diverging system.
(The lens alone in an endless liquid would have $\frac{1}{f} = \left(\frac{1.5}{1.6} - 1\right) \times \frac{2}{20} = -0.00625$, $f = -160$ cm; the cell's flat walls add the liquid's own power.)

10 **c** Every length of the image is m times the object's; an area is a length times a length, so it is $m \times m = m^2$ times as large.

11 **d** $P = \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{f_2 + f_1}{f_1 f_2}$. (The second option is the focal length F, not the power.)

12 **b** Light passes through the lens, is reflected by the silvered face and passes through the lens again: the lens converges it twice and the silvered (inwardly curved, seen from inside) face converges it as well. The whole acts as a **concave mirror**, of power $2P + P$.

13 **c** The water fills the space between the lens's convex lower face and the flat mirror, so it forms a **plano-concave water lens**, a diverging lens of negative power. The light passes through the glass lens and the water lens twice: the system's power is $2(P + P)$ with $P < 0$, so it **decreases** (its focal length grows, as in the pin-and-mirror sample question of 17.4).

14 **d** Relative to the liquid, the lens's refractive index is 1, so $\frac{1}{f} = (1 - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = 0$: f is **infinite**. Light passes straight through without bending, and the lens can't be seen.

15 **a** $P = \frac{1}{f_1} - \frac{1}{f_2}$ is positive (converging) when $\frac{1}{f_1} > \frac{1}{f_2}$, that is when $f_1 < f_2$: the convex lens must be the stronger. ($f_1 = f_2$ gives zero power, a glass plate.)

12. a. **In air.** Its power is $(\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ with μ relative to the medium. Glass relative to water ($1.5/1.33 = 1.13$) is smaller than relative to air (1.5), so $\mu - 1$ and the power are smaller in water (about a quarter).

b. Power measures how strongly a lens bends light. A lens of short focal length bends parallel rays steeply, to a nearby focus, and one of long focal length gently: the shorter f, the stronger the lens, so the strength is measured by $1/f$. This also makes powers of lenses in contact simply add, $P = P_1 + P_2$, which is how opticians combine and prescribe lenses.

c. $P_1 = \dfrac{1}{0.10} = +10$ D; $P_2 = \dfrac{1}{-0.20} = -5$ D. $P = 10 - 5 = \mathbf{+5\ D}$ (a converging lens of focal length 20 cm).

13. a. A **water drop in air** is a **converging** lens: water is denser than the air around it, $\mu = 1.33 > 1$, and its faces bulge outwards (it is a strong ball lens; a drop on a leaf can focus sunlight). An **air bubble in water** is a **diverging** lens: relative to the water, air's refractive index is $1/1.33 = 0.75 < 1$, so $\mu - 1$ is negative though the bubble is convex (17.4).

b. When the **refractive index of the liquid equals that of the lens**. Then light is neither bent nor reflected at its surfaces, so there is nothing to show where the lens is; its focal length is infinite.

c. $P = +6 + (-4) = +2$ D, positive: the combination is **converging** (it acts as a convex lens). $F = \dfrac{1}{2}$ m $= \mathbf{50\ cm}$.

14. a. Yes. Relative to the liquid, the lens's refractive index $\mu = \mu/\mu$ is less than 1, so $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ is negative: the converging lens becomes a **diverging** lens, its power changing from positive to negative.

b. Yes: it **increases**. In water ${}_w\mu_g = 1.5/1.33 = 1.13$, so $\mu - 1$ falls from 0.5 to 0.13, and the focal length becomes about $0.5/0.128 = 3.9$ times as long (a lens of 24 cm in air has 93.9 cm in water, 17.4).

c. $\dfrac{1}{v} = \dfrac{1}{f} - \dfrac{1}{u} = \dfrac{1}{10} - \dfrac{1}{20} = \dfrac{1}{20}$, so $v = \mathbf{+20\ cm}$: a real image 20 cm on the other side of the lens (the object is at 2F, so the image is at 2F, inverted and the same size).

15.

a. Yes, if the **concave lens is the stronger** of the two (its focal length shorter in size). The power of the pair, $P = P + P$, is then negative: for example, convex +4 D with concave −6 D gives −2 D, a diverging pair of focal length −50 cm.

b. Its power **decreases**. $P = ({}_w\mu_g - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ in water, and ${}_w\mu_g = 1.5/1.33 = 1.13$ is less than the 1.5 it has relative to air, so $\mu - 1$ is smaller (0.13 instead of 0.5): the power falls to about a quarter. The glass and the water are optically more alike, so light bends less at their boundary.

c. A convex lens makes a smaller image only when it is real; a smaller **virtual** image comes only from a **diverging (concave) lens**, so $f = -12$ cm.

Virtual and upright, one third the size: $m = \frac{v}{u} = -\frac{1}{3}$, so $v = -\frac{u}{3}$.
$\frac{1}{-12} = \frac{1}{u} - \frac{3}{u} = -\frac{2}{u}$, so $u =$ **24 cm** and $v = -\frac{24}{3} =$ **−8 cm**: the object is 24 cm from the lens, and the virtual image 8 cm from the lens on the same side as the object.

16.

a. Their powers are equal and opposite, so the total power is zero and the focal length infinite: the pair behaves like a **plain glass plate**, sending parallel light on parallel (17.6).

b. At **2f** (2F): then $\frac{1}{v} = \frac{1}{f} - \frac{1}{2f} = \frac{1}{2f}$, so $v = 2f$ and $m = \frac{v}{u} = 1$: a real, inverted image of the same size at 2F on the other side. The ray diagram is Figure 17.5 in 17.3.

c. A lens's two faces are parts of two spheres. The **centers of curvature** C_1 and C_2 are the centers of those spheres, and the **radii of curvature** R_1 and R_2 their radii (Figure 17.3). The line through C_1 and C_2 is the principal axis. (The radius of a flat face is infinite.)

d. Convex face: $R_1 = +15$ cm (bulging out); concave face: $R_2 = -30$ cm (curving in).

$$\frac{1}{f} = (1.6 - 1)\left(\frac{1}{15} - \frac{1}{30}\right) = 0.6 \times \frac{2-1}{30} = \frac{0.6}{30} = \frac{1}{50}$$

$f =$ **+50 cm**: a converging lens, as a concavo-convex lens (thicker in the middle) must be.

17.

$\frac{P_w}{P_a} = \frac{{}_w\mu_g - 1}{{}_a\mu_g - 1}$, with ${}_w\mu_g = \frac{1.5}{1.33} = 1.128$.
$P_w = 5 \times \frac{1.128 - 1}{1.5 - 1} = 5 \times \frac{0.128}{0.5} =$ **+1.28 D** (with water's index taken as 4/3, ${}_w\mu_g = 1.125$ and $P_w = 1.25$ D).

18.

$f_1 = +5$ cm, $f_2 = -10$ cm.
$\frac{1}{F} = \frac{1}{5} - \frac{1}{10} = \frac{2-1}{10} = \frac{1}{10}$, so $F =$ **+10 cm**: positive, a **converging** combination (the convex lens is the stronger).

19. $R_1 = 12$ cm, $R_2 = \infty$ (flat).
$\dfrac{1}{f} = (1.5 - 1)\left(\dfrac{1}{12} + 0\right) = \dfrac{0.5}{12} = \dfrac{1}{24}$, so $f = \mathbf{24\ cm}$.

20. A virtual image smaller than the object can only be made by a **diverging (concave) lens**.

Virtual and upright: $m = \dfrac{v}{u} = -\dfrac{2}{3}$, so $v = -\dfrac{2}{3} \times 20 = \mathbf{-13.3\ cm}$: the image is 13.3 cm from the lens on the same side as the object.

$\dfrac{1}{f} = \dfrac{1}{20} + \dfrac{1}{-13.33} = 0.05 - 0.075 = -0.025$, so $f = \mathbf{-40\ cm}$: a diverging lens of focal length 40 cm.