# Answers: Refraction through Prisms

## Quick checks

**1 b** $r_1 + r_2 = A$, so $r_2 = 60^\circ - 35^\circ = 25^\circ$.

**2 b** $\delta = i + e - A = 50^\circ + 40^\circ - 60^\circ = 30^\circ$.

**3 b** The ray passes symmetrically: $i = e$ and $r_1 = r_2 = A/2$.

**4 b** $\mu - 1 = \dfrac{\delta}{A} = \dfrac{3.6^\circ}{6^\circ} = 0.6$, so $\mu = 1.6$.

**5 a** At minimum deviation the ray passes symmetrically, so $e = i$.

**6 c** At minimum deviation $i = e$ and $r_1 = r_2$, but $i$ is not equal to $r_1$: the light is bent at the first face, so $r_1 < i$.

**7 c** Light can leave the second face only if $r_2 \le C$. The largest $r_1$ can be is $C$ (at grazing incidence), so $r_2 = A - r_1 \ge A - C$. If $A > 2C$, then $r_2 > C$ for every ray, and all of them are totally reflected at the second face. If $A < 2C$ (options a and d), a ray falling at a large enough angle (so that $r_1$ is close to $C$) has $r_2 = A - r_1 < C$ and gets out. At exactly $A = 2C$ only the grazing ray gets out, and it leaves grazing the second face.

**8 c** $\sin r = \sin 50^\circ / 1.5 = 0.5107$, $r = 30.71^\circ$, and $A = 2r = 61.42^\circ$ (Example 3).

**9 b** At minimum deviation $r = A/2 = 30^\circ$, so $\sin i = \sqrt{2}\sin 30^\circ = \dfrac{\sqrt{2}}{2} = 0.707$ and $i = 45^\circ$.

**10 d** $\dfrac{A + \delta_m}{2} = \dfrac{180^\circ - A}{2} = 90^\circ - \dfrac{A}{2}$, so $\mu = \dfrac{\sin(90^\circ - A/2)}{\sin(A/2)} = \dfrac{\cos(A/2)}{\sin(A/2)} = \cot\dfrac{A}{2}$.

**11 b** $\sin\dfrac{A + \delta_m}{2} = \cot\dfrac{A}{2}\sin\dfrac{A}{2} = \cos\dfrac{A}{2} = \sin\left(90^\circ - \dfrac{A}{2}\right)$, so $\dfrac{A + \delta_m}{2} = 90^\circ - \dfrac{A}{2}$ and $\delta_m = 180^\circ - 2A$. (Question 6 the other way round.)

**8.**

a. Not through a glass prism in air: the deviation is never smaller than $\delta_m$, which is more than zero (37.2° for a 60° prism of glass). A ray can go through undeviated only if the prism is in a medium of the same refractive index (then nothing is bent, and the prism can't be seen), or if the prism's angle is zero, a slab, where the ray is only shifted.

b. To come back along its own path, the ray must meet the silvered face along its normal, so that it is reflected straight back. Then the angle of incidence on the silvered face is 0, and the angle of refraction at the first face equals the angle of the prism: $r_1 = A = 30^\circ$ (from $r_1 + r_2 = A$ with $r_2 = 0$).

$$\sin i = \mu \sin r_1 = \sqrt{2} \sin 30^\circ = \frac{\sqrt{2}}{2} = 0.707 \;\Rightarrow\; i = 45^\circ$$

The ray must fall on the unsilvered face at **45°**.

**9.**

a. The angle of incidence; the angle of the prism (a larger angle deviates more); the refractive index of the prism relative to the medium around it, which also depends on the color (wavelength) of the light, violet being deviated more than red.

b. $\sin r_1 = \dfrac{\sin 45^\circ}{1.5} = 0.4714$, $r_1 = 28.1^\circ$; $r_2 = 60^\circ - 28.1^\circ = 31.9^\circ$;

$$\sin e = 1.5 \sin 31.9^\circ = 1.5 \times 0.528 = 0.792 \;\Rightarrow\; e = 52.4^\circ$$

$$\delta = i + e - A = 45^\circ + 52.4^\circ - 60^\circ = 37.4^\circ$$

$e = 52.4^\circ$, $\delta = 37.4^\circ$ (this is the ray of Figure 16.4).

**10.**

a. A lateral shift is the gap between a ray and a parallel ray. The ray leaving a prism isn't parallel to the ray that went in: it has been turned through the deviation, and that angle describes it.

b. Normal incidence gives $r_1 = 0$, so $r_2 = A$. Leaving at 90° means $r_2$ is the critical angle. So $A = C$.

c. The ray makes 50° with the face, so with the normal it makes $i = 90^\circ - 50^\circ = 40^\circ$. From $\delta = i + e - A$:

$$e = \delta + A - i = 42^\circ + 60^\circ - 40^\circ = 62^\circ$$

This is the angle with the normal; with the second face itself the emergent ray makes $90^\circ - 62^\circ =$ **28°**.

**11.**

a. The angle of incidence, the angle of the prism, and the refractive index of the prism relative to its surroundings, which depends on the color of the light (question 9a).

b. $\delta = i + e - A$ gives $e = \delta + A - i = 30^\circ + 30^\circ - 60^\circ = 0^\circ$. The angle of emergence is 0°: the ray leaves along the normal of the second face, so it makes **90°** with the face. (Then $r_2 = 0$, $r_1 = 30^\circ$, and the prism's refractive index is $\sin 60^\circ / \sin 30^\circ = \sqrt{3} = 1.73$.)

**12.**

a. It becomes smaller, because the refractive index of glass relative to water, about 1.13, is much less than its index relative to air (Try it in section 16.3).

b. The light goes straight in and meets the long face at 45°. For total internal reflection the critical angle must be 45° or less: $\sin C = 1/\mu \leq \sin 45^\circ = 1/\sqrt{2}$, so $\mu \geq \sqrt{2}$. The least refractive index is $\sqrt{2} \approx 1.41$. Ordinary glass (1.5) is enough, which is why such prisms work in binoculars.

**13.**

a. Yes. $\delta = i + e - A$, and $e$ itself depends on $A$ through $r_2 = A - r_1$. For a thin prism, $\delta = (\mu - 1)A$: twice the angle gives twice the deviation. For larger prisms the deviation grows faster than the angle, until at $A > 2C$ no light gets out at all.

b. $i = e = \frac{3}{4} \times 60^\circ = 45^\circ$, so $\delta = 45^\circ + 45^\circ - 60^\circ = \mathbf{30°}$. (Since $i = e$, this is the minimum deviation; the prism's refractive index is $\sin 45^\circ / \sin 30^\circ = \sqrt{2}$.)

**14.**

a. Yes: for the same angle of incidence, a prism of larger angle deviates the light more (question 13a).

b. The angle of incidence equals the angle of emergence, $i = e$; the angles inside the glass are equal, $r_1 = r_2 = A/2$; the ray passes symmetrically through the prism, parallel to its base if the prism's two faces are equal.

c. As in Example 1: $r_1 = C = 41.8^\circ$, $r_2 = 18.2^\circ$, $\sin e = 1.5 \sin 18.2^\circ = 0.469$, so $e = 27.9^\circ$ and $\delta = 90^\circ + 27.9^\circ - 60^\circ = 57.9^\circ$.

**15.**

$$\sin \frac{A + \delta_m}{2} = \mu \sin \frac{A}{2} = 1.5 \sin 30^\circ = 0.75 \;\Rightarrow\; \frac{A + \delta_m}{2} = 48.6^\circ$$

The angle of incidence is $i = \dfrac{A + \delta_m}{2} = \mathbf{48.6°}$, and $\delta_m = 2 \times 48.6^\circ - 60^\circ = \mathbf{37.2°}$.

**16.**

$$i = \frac{A + \delta_m}{2} = \frac{60^\circ + 34^\circ}{2} = 47^\circ$$

$$\mu = \frac{\sin 47^\circ}{\sin 30^\circ} = \frac{0.731}{0.5} = 1.46$$

$i = 47^\circ$, $\mu = 1.46$.

**17.**

Normal incidence gives $r_1 = 0$ and $r_2 = A$. The ray just fails to emerge when $r_2$ equals the critical angle:

$$\sin C = \frac{1}{1.5} = 0.667 \;\Rightarrow\; C = 41.8^\circ$$

So $A = 41.8^\circ$.

**18.**

$\sin r = \dfrac{\sin 50^\circ}{1.5} = 0.5107$, so $r = 30.7^\circ$, and $A = 2r = \mathbf{61.4°}$ (Example 3).