Answers: Refraction at Plane Surfaces

Quick checks

1 b Light slows down going into water, the optically denser medium, so it bends towards the normal. (By Snell's law, $\sin r = \sin 40^\circ / 1.33$, so $r = 28.9^\circ$.)

2 c $\mu = \dfrac{c}{v} = \dfrac{3 \times 10^8}{2 \times 10^8} = 1.5.$

3 b By the principle of reversibility, ${}_g\mu_w = \dfrac{1}{{}_w\mu_g} = \dfrac{1}{1.13} = 0.885.$

4 b $\mu_w \sin i = \mu_g \sin r$: $\sin r = \dfrac{1.33 \sin 60^\circ}{1.5} = 0.768$, $r = 50.2^\circ$. It bends towards the normal, but only a little, since glass is only a little denser than water.

5 b $a = \dfrac{R}{\mu} = \dfrac{40}{4/3} = 30$ cm. It seems raised by 10 cm.

6 b $\mu = \dfrac{1}{\sin C} = \dfrac{1}{\sin 30^\circ} = \dfrac{1}{0.5} = 2.0.$

7 c 55° is more than the critical angle, and the light is going from the denser medium towards the rarer, so it is totally reflected.

8 c The frequency is set by the source: every wave crest that arrives at the boundary leaves it, so the number per second is the same on both sides. The speed changes, and so the wavelength, $\lambda = v/f$, changes with it.

9 c The frequency doesn't change; the speed falls by a factor of 1.5 in glass, so the wavelength $\lambda = v/f$ falls by the same factor.

10 b $v = \dfrac{c}{\mu} = \dfrac{3 \times 10^8}{1.5} = 2 \times 10^8$ m/s.

11 b $v = \dfrac{3 \times 10^8}{1.5} = 2 \times 10^8$ m/s, so $t = \dfrac{6 \times 10^{-3}}{2 \times 10^8} = 3 \times 10^{-11}$ s = 0.03 ns.

12 c Real depths: $6 \times 1.5 = 9$ cm from one face and $4 \times 1.5 = 6$ cm from the other. The thickness is $9 + 6 = 15$ cm.

13 c $\sin C = \dfrac{\mu_w}{\mu_g} = \dfrac{4/3}{5/3} = \dfrac{4}{5}.$

14 a $\mu = \dfrac{1}{\sin 30^\circ} = 2$, so $v = \dfrac{3 \times 10^8}{2} = 1.5 \times 10^8$ m/s.

15 d Light inside the core meets its edge at more than the critical angle and is totally reflected, again and again.

16 **d** $\sin C = 1/\mu$, and μ is largest for violet light, so its critical angle is the smallest.

17 **a** Its speed doubles, so it goes from a denser into a rarer medium with relative refractive index 2: $\sin C = \dfrac{1}{2}$, $C = 30^\circ$. Up to 30° it gets out; beyond 30° it is totally reflected.

11.

a. Light travels at different speeds in different media. When it meets the boundary at a slant, the part of the wavefront that enters first changes speed first, so the wavefront swings round and the light changes direction. (Along the normal it changes speed but not direction.)

b. The lateral shift is the perpendicular distance between the incident ray, produced, and the emergent ray when light passes through a parallel-sided slab: $L = t\sin(i - r)/\cos r$. It is zero at normal incidence, grows with the angle of incidence (in proportion to it for small angles) and approaches the slab's thickness at grazing incidence.

c. **Red** light. Glass's refractive index is smallest for red light, and $v = c/\mu$, so red travels fastest in glass, and violet slowest. (In a vacuum all colors travel at the same speed.)

12.

a. No. Along the normal, $i = 0$ and $r = 0$, so $\dfrac{\sin i}{\sin r} = \dfrac{0}{0}$, which has no value. The light still changes speed, so the refractive index is still $\mu = c/v$; it just can't be measured from angles at normal incidence. Snell's law is used with oblique rays.

b. Light from a point on the bottom bends away from the normal as it leaves the water. To the eye, the rays seem to come from a point higher up, where they meet when produced back: the image of the bottom is raised. Looking straight down, the apparent depth is the real depth ÷ 1.33, about three quarters of it; looking across at a slant, the bottom seems even higher.

c. The apparent shift is $d = t(1 - 1/\mu)$. Glass has a larger refractive index for violet light than for any other color, so violet letters are raised the most.

d. $v = \dfrac{c}{\mu} = \dfrac{3 \times 10^8}{1.33} = 2.26 \times 10^8$ m/s, so $t = \dfrac{d}{v} = \dfrac{500}{2.26 \times 10^8} = 2.2 \times 10^{-6}$ **s**.

13. a. On the nature of the medium; on the color (wavelength) of the light, larger for violet than for red; on temperature (hot liquids and gases have smaller refractive indices); and, for gases, on pressure. A relative refractive index also depends on the other medium.

b. $\sin C = \dfrac{1}{\mu}$, so $C = \sin^{-1}\left(\dfrac{1}{\mu}\right)$, where μ is the refractive index of the denser medium with respect to the rarer one.

c. The **mirage**: on a hot day the air near the ground is hottest, with the smallest refractive index. Light from the sky or a distant object heading down is bent more and more away from the normal by the layers of hotter air until it turns upward into the eye, in effect totally reflected. The eye sees it as coming from below the ground: the sky seen there looks like water, with upside-down images of objects in it. (The sparkle of dew drops and of a diamond is another.)

d. Apparent depth $= 4 - 1.25 = 2.75$ cm, so $\mu = \dfrac{4}{2.75} = \mathbf{1.45}$.

14. a. The hot ground heats the air above it, which rises in uneven, moving pockets of different temperatures and so of slightly different refractive indices. Light from a distant object is bent by changing amounts as it passes through them, so the image keeps moving: the object seems to shimmer.

b. Light from the Sun bends downward as it comes into the denser air near the ground (the atmosphere is denser lower down), so we see the Sun higher than it really is. At the horizon, light from the Sun's lower edge passes through more and denser air than light from its upper edge, so it is lifted more, by about 0.1° more. The bottom is raised towards the top and the disc looks squashed from top to bottom, an oval. (The same bending lets us see the Sun for about two minutes after it has really set.)

c. $\sin C = \dfrac{\mu}{\mu} = \dfrac{1.50}{1.72} = 0.872$, so $C = \mathbf{60.7°}$.

15. a. No. Light going from air into glass goes from a rarer into a denser medium and bends towards the normal; for every angle of incidence there is an angle of refraction (always less than the critical angle). Total internal reflection needs light going from the denser towards the rarer medium.

b. A glass prism whose cross-section is a right-angled isosceles triangle (90°, 45°, 45°). Light entering a face along the normal meets another face from inside at 45°, more than glass's critical angle (about 42°), and is totally reflected, turned through 90° (or through 180° if it enters the long face). It is used in binoculars and periscopes in place of a mirror.

c. A crack is a thin film of air inside the glass. Light inside the glass that meets the crack's surface at more than the critical angle (about 42°) is totally reflected. From a direction in which this reflected light reaches the eye, the crack looks brilliantly bright, like silver.

16.

a. The black road absorbs sunlight and heats the air just above it, so the air is hottest, least dense and of lowest refractive index near the road. Light from the sky heading down at a low angle passes into rarer and rarer air, bending away from the normal more and more until it turns upward into the driver's eye. The eye sees this bright sky light as coming from the road far ahead: it looks like a shining pool of water reflecting the sky. This is a **mirage**.

b. Diamond's refractive index is very large, 2.42, so its critical angle is only 24.4°. The diamond is cut so that light entering it meets the faces at the back at more than 24.4°, and is totally reflected several times before coming out through the front in a few directions. So it looks very bright from those directions and flashes as it moves; its large dispersion also splits the light into flashes of color.

c. No. Light going from the rarer medium into the denser bends **towards** the normal (Figure 15.2, a.), and $\sin r = \sin i/\mu$ is always less than 1, so there is always a refracted ray, however large the angle of incidence. In Figure 15.8, total internal reflection happens only to light inside the glass, heading out.

d. Light from water into glass: $\mu_w \sin i = \mu_g \sin r$, so

$$\sin r = \frac{1.33 \times \sin 70^\circ}{1.5} = \frac{1.33 \times 0.940}{1.5} = 0.833 \;\Rightarrow\; r = 56.4^\circ$$

17. $\mu_w \sin i = \mu_g \sin r$:

$$\sin r = \frac{1.33 \times \sin 30^\circ}{1.5} = \frac{1.33 \times 0.5}{1.5} = 0.443 \;\Rightarrow\; r = 26.3^\circ$$

The angle of refraction is **26.3°**.

18.

$$\sin C = \frac{\mu_w}{\mu_g} = \frac{1.32}{1.62} = 0.815 \;\Rightarrow\; C = 54.6^\circ$$

19. Looking straight down, each layer's apparent thickness is its real thickness divided by its refractive index:

$$a = \frac{t_g}{\mu_g} + \frac{t_w}{\mu_w} = \frac{6}{1.5} + \frac{4}{1.33} = 4 + 3.0 = 7.0$$

The object appears **7.0 cm below the surface of the water**, though it is really 10 cm below it: raised by 3 cm.

20. For light inside the glass heading into the water,

$$\sin C = \frac{{}_a\mu_w}{{}_a\mu_g} = \frac{1.33}{1.5} = 0.887 \;\Rightarrow\; C = 62.5^\circ$$