Unit 14

Answers: Reflection at Curved Mirrors

Quick checks

1 b Only a real image, where the light actually arrives, can light up a screen. A virtual image is where the light only seems to come from; for a real object, a real image is inverted and in front of the mirror.

2 c A convex mirror always spreads the light out, so the image in it is always virtual (for a real object). The other three are real images: light actually lands on the screen, the retina and the paper.

3 c $f = R/2 = 40/2 = 20$ cm. The focus is halfway between the pole and the center of curvature.

4 b $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{20} - \frac{1}{30} = \frac{1}{60}$, so $v = +60$ cm: positive, a real image 60 cm in front of the mirror (the object is between F and C, so the image is beyond C, enlarged).

5 b The minus sign means virtual (and, for a real object, upright); the size 0.5 means half as tall. This is what a convex mirror does.

6 c A convex mirror spreads the reflected light, so rays from a wide range of directions all reach the eye from one small mirror. That is why it is used as a rear-view mirror.

7 c A convex mirror diverges the reflected light, so it shows the widest area for its size, at the cost of diminished images.

8 a The object is between F and the pole of a concave mirror. The reflected rays spread out, and only their backward extensions meet, behind the mirror: the image is virtual, upright, and larger than the object.

9 a The object stands at the center of curvature C, and the image forms at C too, inverted and exactly the same size: $u = v = 2f$, so $m = v/u = 1$.

10 d For a real object (a.) the image is virtual, behind the mirror. For a virtual object (b.), light converging towards a point behind the mirror, the reflected rays really meet in front: a real image.

11 d Take $f = -10$ cm and the virtual object 5 cm behind: $u = -5$ cm. $\frac{1}{v} = -\frac{1}{10} + \frac{1}{5} = \frac{1}{10}$, $v = +10$ cm: real, in front of the mirror, and $m = v/u = -2$, upright and enlarged. (A virtual object farther behind than F gives a virtual image instead: with $u = -20$ cm, $v = -20$ cm.)

12 a A face between the pole and the focus of a concave mirror gives an upright, enlarged image.

13 **b** For every position of a real object, a convex mirror forms a virtual, upright and diminished image between P and F.

14 **a** A mirror's focal length is $R/2$, fixed by its shape. The law of reflection is the same in water, so the focal length doesn't change.

15 **d** Reflection turns every color through the same angle, so a mirror has one focus for all colors. (A lens's focal length does depend on color, since glass bends violet more than red.)

16 **c** $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{10} - \frac{1}{20} = -\frac{3}{20}$, so $v = -\frac{20}{3}$ cm ≈ -6.7 cm: virtual, 6.7 cm behind the mirror.

17 **d** Rays from a very distant point arrive parallel and meet at the focus: $v = f$, and $m = v/u = f/\infty = 0$. The image is a point at F (in practice a very small image, like the Sun's bright dot).

18 **b** Real image: $v = mu$. Then $\frac{1}{f} = \frac{1}{u} + \frac{1}{mu} = \frac{m+1}{mu}$, so $u = \frac{(m+1)f}{m}$. (Check: $m = 1$ gives $u = 2f$, the center of curvature.)

19 **c** An image of the same size, inverted, at the very place of the (virtual) object: the rays must be reflected straight back along themselves, so they were all heading for the center of curvature. So $R = 12$ cm and $f = R/2 = 6$ cm (-6 cm with the sign of a convex mirror).

20 **c** $\frac{1}{f} = \frac{1}{20} + \frac{1}{30} = \frac{5}{60}$, so $f = +12$ cm = 120 mm. Positive: a concave mirror (a convex mirror can't form a real image of a real object).

15. a.

Real image	Virtual image
formed where reflected (or refracted) rays really meet	formed where the rays only appear to meet, produced backwards
can be caught on a screen	cannot be caught on a screen
inverted, for a real object	upright, for a real object
e.g. the image on a cinema screen or the retina	e.g. the image in a plane or convex mirror

b. Yes, but only for a **virtual object**: light already converging towards a point behind the mirror.

c. A converging lens sends light towards a point O behind the mirror (Figure 14.15, b.). Before the rays get there, the plane mirror reflects them; by the law of reflection they then converge to the point I in front of the mirror, as far in front as O is behind. The rays really meet at I, so the image is real and can be caught on a screen. (The mirror formula agrees: $f = \infty$ gives $v = -u$; a virtual object, u negative, gives v positive, a real image.)

d. Your image is always as far behind the mirror as you are in front. If the mirror moves 10 m towards you in 1 s, your distance to it shrinks by 10 m, and so does the image's distance behind it: the image comes 20 m nearer. So the image approaches you at **20 m/s**, twice the mirror's speed.

16. a. A **concave** mirror with a fairly long focal length (say 30–60 cm). Held so that the face is between its pole and its focus, it gives an upright, enlarged, virtual image, which shows fine details for a close shave or careful make-up, while you still see yourself the right way up.

b. $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, where u is the object distance, v the image distance and f the focal length, with real distances positive and virtual ones negative (f positive for a concave mirror, negative for a convex one).

c. $f = -25$ cm, $u = +100$ cm, $O = 2$ cm.

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{25} - \frac{1}{100} = -\frac{5}{100}$, so $v = -20$ cm.
$m = \frac{v}{u} = \frac{-20}{100} = -0.2$, so $I = 0.2 \times 2 = 0.4$ cm.
The image is **20 cm behind the mirror, 0.4 cm tall, virtual, upright and diminished**.

17.

a. A spherical mirror of very large radius of curvature is almost flat. As $R \to \infty$, $f = R/2 \to \infty$, and the mirror formula gives $\frac{1}{u} + \frac{1}{v} = 0$, $v = -u$: the image is as far behind the mirror as the object is in front, upright and of the same size, just as in a plane mirror. A plane mirror is a spherical mirror of infinite radius.

b. No. A mirror's focal length is $R/2$, set by its shape alone.

c. It stays f. Reflection follows the same law in water as in air: the angle of reflection equals the angle of incidence, whatever the medium. So rays parallel to the axis are reflected along the same lines, and meet at the same focus. (Only a lens, which works by refraction, changes its focal length in water.)

18.

a. Look at your own image in each, first from close up and then from a few meters away. In the **convex** mirror you are always upright and smaller than life, however near or far you stand. In the **concave** mirror you look upright and enlarged from close up (inside its focus), and from farther away you see yourself upside down. (A plane mirror would always show you upright and life-size.)

b. Falling normally, the ray makes an angle of incidence of 0° with the normal. By the law of reflection the angle of reflection is also 0°: the reflected ray lies along the normal, back along the incident ray. For a spherical mirror, a ray along a radius (towards or through C) falls normally, which is why such a ray comes straight back through C.

19. Let the mirror be u m from the lamp. The image is real, on the screen; the screen is 8 m from the lamp along the axis, so $v = u + 8$ (screen farther from the mirror) or $v = u - 8$ (screen nearer).

Screen farther: $\frac{1}{3} = \frac{1}{u} + \frac{1}{u+8} = \frac{2u+8}{u(u+8)} \Rightarrow u^2 + 8u = 6u + 24 \Rightarrow u^2 + 2u - 24 = 0 \Rightarrow (u+6)(u-4) = 0$, so $u =$ **4 m** (the screen 12 m from the mirror; the image three times as large).

Screen nearer: $\frac{1}{3} = \frac{1}{u} + \frac{1}{u-8} \Rightarrow u^2 - 14u + 24 = 0 \Rightarrow u = 12$ m or 2 m. $u = 2$ m gives $v = -6$ m, a virtual image, which can't be on a screen, so $u =$ **12 m** (the screen 4 m from the mirror; the image a third of the size).

So the mirror can be **4 m** from the lamp (the usual answer, with an enlarged image) or 12 m from it.

20. $f = R/2 = -10$ cm (convex); the image is virtual, $v = -8$ cm.

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v} = -\frac{1}{10} + \frac{1}{8} = \frac{-4+5}{40} = \frac{1}{40}$$

$u =$ **40 cm** in front of the mirror.

21. An image on the object's side, in front of the mirror, is real: $v = +30$ cm, $u = +10$ cm.

$$\frac{1}{f} = \frac{1}{10} + \frac{1}{30} = \frac{4}{30} \Rightarrow f = +7.5$$

Positive: the mirror is **concave**, with focal length **7.5 cm**. (A convex mirror never forms a real image of a real object.)

22. The image is on a screen, so it is real: $m = +2$, $v = 2u$. The object and the screen are both in front of the mirror, the screen twice as far: $v - u = 1.5$ m, so $2u - u = 1.5$, $u = 1.5$ m and $v = 3.0$ m.
The mirror is **1.5 m from the object** (on the line from the screen through the object, 3 m from the screen).

$$\frac{1}{f} = \frac{1}{1.5} + \frac{1}{3.0} = \frac{2+1}{3} = 1 \Rightarrow f =$$

23. First find f. A real object 60 cm away beyond the mirror's focus with $m = 0.5$ gives a real image: $v = 0.5 \times 60 = 30$ cm.

$$\frac{1}{f} = \frac{1}{60} + \frac{1}{30} = \frac{3}{60} \Rightarrow f = 20$$

Now a virtual image twice the size: $m = -2$, $v = -2u$.

$$\frac{1}{20} = \frac{1}{u} - \frac{1}{2u} = \frac{1}{2u} \Rightarrow u =$$

in front of the mirror, between its focus and its pole.

24. $f = -1$ m. Find the image of each end.
Near end B, $u = 2$ m: $\frac{1}{v} = -1 - \frac{1}{2} = -\frac{3}{2}$, $v = -\frac{2}{3}$ m (0.667 m behind the mirror).
Far end A, $u = 6$ m: $\frac{1}{v} = -1 - \frac{1}{6} = -\frac{7}{6}$, $v = -\frac{6}{7}$ m (0.857 m behind).
Length of the image $= \frac{6}{7} - \frac{2}{3} = \frac{18-14}{21} = \frac{4}{21}$ m $= 0.19$ m $\approx$ **19 cm**.
A 4 m pole has an image only 19 cm long: a convex mirror squeezes the whole space in front of it, out to infinity, into the gap between P and F behind it.

25. The plane mirror is 32 cm from the object, so its image is 32 cm behind the plane mirror. The plane mirror is $40 - 32 = 8$ cm in front of the convex mirror, so that image is $32 - 8 = 24$ cm behind the convex mirror.
The convex mirror's image coincides with it, so for the convex mirror $u = +40$ cm and $v = -24$ cm (virtual).

$$\frac{1}{f} = \frac{1}{40} - \frac{1}{24} = \frac{3-5}{120} = -\frac{1}{60} \Rightarrow f =$$

The convex mirror's focal length is 60 cm (negative, as for every convex mirror). (The plane mirror covers only part of the convex mirror, so the object is seen in both at once.)