Unit 13

Answers: Ideal Gas

Quick checks

1 b Each degree adds $\frac{1}{273}$ of 546 cm³, which is 2 cm³. Fifty degrees add 100 cm³: $546 + 100 = 646$ cm³. (Doubling the volume needs a rise of 273 degrees, to 273 °C, not a doubling of the Celsius temperature.)

2 c $PV/T = R$ for one mole of any ideal gas. The mass of a mole and $r = R/M$ depend on the gas's molar mass, and so does the molecules' speed (Section 13.10).

3 b Water's molar mass is 18 g/mol, so 9 g is $n = 9/18 = 0.5$ mol, and $N = 0.5 \times 6.02 \times 10^{23} = 3.01 \times 10^{23}$ molecules.

4 b $P = \frac{1}{3}\rho c^2$: doubling c multiplies P by $2^2 = 4$. Each molecule hits twice as often and twice as hard.

5 c $E' = \frac{3}{2}P = 1.5 \times 2.0 \times 10^5 = 3.0 \times 10^5$ J/m³.

6 c $c = \sqrt{3RT/M}$: proportional to $\sqrt{T}$, and to $1/\sqrt{M}$, not $\sqrt{M}$.

7 d $P = \frac{1}{3}\rho c^2$.

8 c $\dfrac{c_{O_2}}{c_{H_2}} = \sqrt{\dfrac{M_{H_2}}{M_{O_2}}} = \sqrt{\dfrac{2}{32}} = \dfrac{1}{4}$.

9 b $E' = \frac{1}{2}\rho c^2 = \frac{3}{2} \times \frac{1}{3}\rho c^2 = \frac{3}{2}P$.

10 c The kinetic energy is proportional to the absolute temperature, which goes from 273 K to 546 K: it doubles.

11 d $\frac{1}{2}mc^2 = \frac{3}{2}k_BT$ depends only on the temperature, which is the same. The oxygen molecule is 16 times heavier and moves 4 times slower.

12 d $\frac{1}{2}mc^2 = \frac{3}{2}k_BT$, the same for every gas at the same T. The rms speed and the density depend on the molar mass.

13 c $c \propto \sqrt{T}$, so doubling c needs $4T$: $4 \times 273 = 1092$ K $= 819$ °C.

14 b $T = 527 + 273 = 800$ K. $\frac{3}{2}k_BT = 1.5 \times 1.36 \times 10^{-23} \times 800 = 1.632 \times 10^{-20}$ J. (With the accurate $k_B = 1.38 \times 10^{-23}$ J/K it is 1.656×10^{-20} J.)

15 a Equal $\sqrt{T/M}$: $\dfrac{T_{N_2}}{28} = \dfrac{400}{32}$, so $T_{N_2} = 350$ K $= 77$ °C. (350 is the answer in kelvin, not °C.)

16 b From $P = n_V k_B T$, $n_V \propto P/T$: $n_V = 2.70 \times 10^{25} \times \dfrac{1.33 \times 10^{-4}}{1.01 \times 10^5} \times \dfrac{273}{223} = 4.35 \times 10^{16}$ m^{-3}.

17 **b** Charles's law: $V_2 = V_1 \dfrac{T_2}{T_1} = 150 \times \dfrac{273}{300} = 136.5$ cm³.

18 **c** For one mole of any ideal gas, $C_p - C_v = R$, the universal gas constant: heating at constant pressure takes an extra R joules per kelvin, the work $P\Delta V = R\Delta T$ the gas does as it expands (Section 13.4, "Physical meaning of R"; you will meet this as Mayer's relation in Grade 12). The others depend on the kind of gas.

19 **b** From $PV = mrT$, $\rho = \dfrac{m}{V} = \dfrac{P}{rT}$, so $\rho \propto P/T$: $\rho_2 = 0.09 \times \dfrac{4.0}{1.0} \times \dfrac{300}{400} = 0.27$ kg/m³.

20 **d** $PV = Nk_BT$: at the same volume and temperature, $P \propto N$. Twice as many molecules hit the walls twice as often.

16.

a. The molecules are always moving and keep hitting the walls. At each bounce a molecule's momentum perpendicular to the wall is reversed, so the wall pushes it back, and by Newton's third law it pushes on the wall. The countless kicks together make a steady force on each unit area of wall: the pressure.

b. No. The container is rigid, so the load doesn't change its volume; the number of molecules and their speeds (the temperature) are unchanged, so they hit the walls just as often and as hard as before. The load presses on the container, not on the gas.

c. $V = \dfrac{nRT}{P} = \dfrac{1 \times 8.3 \times 273}{1.013 \times 10^5} = 22.4 \times 10^{-3}$ **m³**, about 22.4 L, a cube about 28 cm on each side.

17.

a. Carbon dioxide molecules are heavy (M = 44 g/mol, 22 times hydrogen's 2 g/mol). At the same temperature, $c \propto 1/\sqrt{M}$, so they move about $\sqrt{22} \approx 4.7$ times slower than hydrogen molecules, and almost none of them ever reach the escape speed of 11.2 km/s. Hydrogen's fast molecules escape steadily.

b. The Moon's gravity is weak: its escape speed is only 2.4 km/s, against the Earth's 11.2 km/s. In the Moon's daytime the surface reaches about 120 °C, where a good share of the fastest gas molecules exceed 2.4 km/s and escape. Over its 4.5 billion years, the Moon has lost whatever gases it had, even heavy ones.

c. $P_1 = 2 \times \dfrac{9.8}{10^{-42}} = 1.96 \times 10^5$ Pa, $T_1 = 293$ K, $T_2 = 323$ K. At constant volume,

$$P_2 = P_1 \frac{T_2}{T_1} = 1.96 \times 10^5 \times \frac{323}{293} = \ \times \ \approx 2.2 \times 10^5$$

(This treats the reading as the whole pressure of the air in the tyre, as the question intends. A real tyre gauge reads the pressure **above** the atmosphere's; the gas law needs the whole pressure, $1.96 \times 10^5 + 1.01 \times 10^5 = 2.97 \times 10^5$ Pa. That becomes $2.97 \times 10^5 \times 323/293 = 3.27 \times 10^5$ Pa, and the gauge then reads $3.27 \times 10^5 - 1.01 \times 10^5 = 2.26 \times 10^5$ Pa, about 2.3 kgf/cm².)

18. a. A has **twice as many** molecules as B. From $PV = Nk_BT$, $N = \dfrac{PV}{k_BT}$: with the same P and T, $N \propto V$. (This is Avogadro's law.)

b. Boyle's law: $P_2 = P_1\dfrac{V_1}{V_2} = 3.4 \times \dfrac{0.110}{0.390} =$ **0.96 atm**.

19. a. No. Collisions keep sharing energy unevenly among the molecules, so their speeds are spread out from nearly zero to several times the rms speed (Maxwell's distribution), and each molecule's speed changes at every collision.

b. **Average kinetic energy:** $\frac{1}{2}mc^2 = \frac{3}{2}k_BT$, so it rises in proportion to the absolute temperature. **Speeds:** the whole distribution shifts to higher speeds and spreads out wider; c grows as $\sqrt{T}$, and many more molecules are very fast. **Pressure:** the container is sealed and rigid, so the density is unchanged, and $P = \frac{1}{3}\rho c^2 \propto T$: the molecules hit the walls more often and harder, and the pressure rises in proportion to the absolute temperature.

c. As in the worked example of Section 13.11: $P = 0.65 \times 13\,600 \times 9.8 = 8.66 \times 10^4$ Pa, and $n_V = \dfrac{P}{k_BT} = \dfrac{8.66 \times 10^4}{1.38 \times 10^{-23} \times 300} = 2.09 \times 10^{25}$ per m³, so 1 cm³ (10^{-6} m³) holds 2.09×10^{19} **molecules**.

20. a. The model gives $P = \frac{1}{3}\rho c^2$, so for one mole $PV = \frac{1}{3}Mc^2$. The gas equation, from experiment, gives $PV = RT$. Putting the two together, $\frac{1}{2}Mc^2 = \frac{3}{2}RT$, and per molecule $\frac{1}{2}mc^2 = \frac{3}{2}k_BT$: the average kinetic energy of the molecules is proportional to the absolute temperature. Temperature **is** a measure of that energy.

b. Not because fewer molecules are left: the temperature depends on the average kinetic energy of the molecules, not on how many there are. If the pumping is slow, so that the gas keeps exchanging heat with the walls, its temperature stays the same and only its pressure falls ($P \propto N$). (If the gas is pumped out quickly, the gas left behind expands and does work pushing gas out, and it does cool for a while, until the walls warm it again.)

c. At STP: $c_0 = \sqrt{\dfrac{3P}{\rho}} = \sqrt{\dfrac{3 \times 1.01 \times 10^5}{0.1785}} = 1303$ m/s at 273 K. At 30 °C, 303 K:

$$c = c_0\sqrt{\frac{303}{273}} = 1303 \times 1.0535 =$$

21. At constant volume, $P \propto T$. $T_1 = 47 + 273 = 320$ K, so doubling the pressure needs

$$T_2 = 2T_1 = 640 = 640 - 273 =$$

(The volume and the starting pressure aren't needed.)

22. $$n = \frac{PV}{RT} = \frac{1.01 \times 10^5 \times 1}{8.31 \times 273} =$$

(Or $1^3 \div 22.4 \times 10^{-33}/ = 44.6$ mol.)

$$N = nN_A = 44.5 \times 6.02 \times 10^{23} = \ \times$$

23. a. $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ with $P_2V_2 = 4P_1V_1$: $T_2 = 4T_1 = 4 \times (41.0 + 273) = 4 \times 314 =$ 1256 K $=$ **983 °C**.

b. $n = \frac{PV}{RT} = \frac{1.30 \times 1.01 \times 10^5 \times 2.60 \times 10^{-3}}{8.314 \times 314} = 0.1308$ mol, so the mass is $0.1308 \times 4.00 =$ **0.523 g** (5.23×10^{-4} kg).

24. $$P_2 = P_1 \frac{V_1}{V_2}\frac{T_2}{T_1} = 1 \times \frac{500}{900} \times \frac{270}{300} =$$

At about 5.5 km up, the air's pressure is about half its sea-level value, so a balloon that rises that far nearly doubles its volume: this is why weather balloons are launched only partly filled.

25. From $PV = \frac{m}{M}RT$, $\rho = \frac{m}{V} = \frac{PM}{RT}$:

$$\rho = \frac{5 \times 1.013 \times 10^5 \times 16 \times 10^{-3}}{8.314 \times 293} =$$

26. $n_V = \frac{N}{V} = \frac{1 \times 10^{25}}{1 \times 10^{-3}} = 1 \times 10^{28}$ m^{-3}.

$$P = \frac{1}{3}\, m\, n_V\, c^2 = \frac{1}{3} \times 2.7 \times 10^{-28} \times 1 \times 10^{28} \times 400^2 = \ \times$$

No real molecule is this light: 2.7×10^{-28} kg is less than a sixth of the mass of a hydrogen atom (1.67×10^{-27} kg), so this is an imagined gas. Real oxygen molecules (5.3×10^{-26} kg) with the same numbers would give 2.8×10^7 Pa, about 280 atmospheres.