Unit 12

Answers: Rate of Heat Flow

Quick checks

1 c In a metal, free electrons carry energy through the metal much faster than vibrations passing from atom to atom alone. Atoms in a solid don't travel; mass and density are not what decides it (lead is far denser than aluminium, yet conducts heat only about a seventh as well).

2 b In a lagged rod the temperature falls uniformly: by $(90 - 10) \div 40 = 2$ °C per cm. At 20 cm from the hot end it is $90 - 2 \times 20 = 50$ °C, halfway between the two ends. 45 °C is half of 90, forgetting that the cold end is at 10 °C, not 0 °C.

3 b $Q/t = kA(\theta_1 - \theta_2)/x$: doubling A doubles it, doubling x halves it, so it stays **40 W**.

4 b Natural convection needs gravity: warm fluid rises only because the denser fluid around it is pulled down more strongly and pushes it up. In orbit everything falls together, so nothing rises. Conduction and radiation don't depend on gravity, and fans still drive forced convection, which is why space stations are full of them.

5 b $a = 1 - r - t = 1 - 0.30 - 0.10 = 0.60$. 0.70 forgets the transmitted part; 0.90 forgets the reflected one.

6 c Soot reflects almost nothing and absorbs about 97% of what falls on it; by Kirchhoff's law it is also the best emitter. Silver and mirrors reflect nearly everything; white paint reflects most visible light (though, surprisingly, white paint absorbs and emits infrared well).

7 d $E \propto T^4$, and $(900/300)^4 = 3^4 = 81$.

8 a Natural convection happens because the denser, cooler fluid is pulled down more strongly than the warmer fluid and pushes it up: it needs gravity. Conduction and radiation work just as well without it.

9 a The power a surface gives out is $P = eA\sigma T^4$: it depends on its area A, its nature (emissivity e) and its own temperature T. The surroundings' temperature changes only what the surface **receives**, and so its **net** loss, not what it radiates. (Some books give "Stefan's constant" as the answer. σ is the same for every surface, but it is part of the law itself, so it can't be said to have no effect.)

10 d Stefan's law: $E = \sigma T^4$.

11 c k is a property of the material alone, like its density. The area and the temperature difference change the rate of flow of heat, $Q/t = kA(\theta_1 - \theta_2)/x$, not k.

12 d By Kirchhoff's law, a surface's emissivity equals its absorptance: a good absorber is a good emitter. (It is also a poor reflector and, if it absorbs well, a poor transmitter.)

13 d $E \propto T^4$: $2^4 = 16$.

14 b Every body above absolute zero radiates. Only at 0 K would $E = \sigma T^4$ be zero. (0 °C and 273 K are the same temperature, well above absolute zero.)

15 **a** From $Q/t = kA\,\dfrac{\theta_1 - \theta_2}{x}$, with $A = 1$ m² and a gradient of 1 °C/m (1 K/m), $Q/t = k$: this is the definition of thermal conductivity.

16 **c** Above the fire, the hot gases rising by convection carry heat to you as well as the radiation; beside it you get only the radiation. Radiation goes out equally in every direction.

17 **a** Both have the same surface, at the same temperature, so at first they lose heat at the same rate. But the hollow sphere has less mass, so it holds less heat, and the same loss lowers its temperature more: it cools faster.

18 **a** $Q/t = kA\Delta\theta/l \propto d^2/l$ (the area goes as the diameter squared). Ratio $= \dfrac{1^2/2}{2^2/1} = \dfrac{1/2}{4} = \dfrac{1}{8}$: **1 : 8**.

19 **a** In the steady state the temperature falls uniformly: $(100 - 25) \div 150 = 0.5$ °C per cm. P is 50 cm from B, so it is $0.5 \times 50 = 25$ °C warmer than B: $25 + 25 = 50$ °C.

20 **c** The same heat flows through both layers: $k_A A\,\dfrac{\Delta\theta_A}{x} = k_B A\,\dfrac{\Delta\theta_B}{x}$. With $k_A = k_B/3$: $\Delta\theta_A = 3\Delta\theta_B = 3 \times 15 = 45$ °C. The poorer conductor takes the larger drop.

21 **c** A black surface is the best emitter (a good absorber is a good emitter), so the black object radiates its heat fastest and cools first. The white one cools slowest.

15.

a. At night the ground and our bodies lose heat by radiating infrared into the sky. On a clear night, much of it escapes straight out to cold space. Clouds absorb that infrared and radiate part of it back down, and they also keep the warmer air below them from mixing upward. So under clouds less heat is lost, and the night is warmer; clear winter nights bring the hardest frosts.

b. i. In sunshine, the **blackened** bulb absorbs most of the radiation falling on it and the silvered one reflects most of it: the blackened thermometer reads **higher**.

ii. Facing a clear night sky, both lose heat by radiating to the cold sky. The blackened bulb is a good emitter and loses heat faster; the silvered one emits very little. So the blackened thermometer reads **lower**.

c. Beside the flame, heat reaches your fingers only by radiation, which is not much from a small flame. Above it, the hot gases from the flame rise by convection straight into your fingers, carrying much more heat.

16. a. The wooden chair. The metal chair, a good conductor, would carry heat away from the body fast and feel very cold. Wood is a poor conductor: its surface soon warms to your temperature and then takes little more heat.

b. A potato is a poor conductor, so in an oven or a fire its outside cooks long before the heat reaches its middle. A metal nail conducts heat straight into the center of the potato, so it cooks from the inside too.

c. Yes. Aluminium's thermal conductivity (about 237 W/(m K)) is about five times steel's (about 50 W/(m K)), so an aluminium nail carries heat into the potato faster.

d. The cotton of a new quilt is loose and fluffy, with many small spaces full of still air, a very poor conductor. In an old quilt the cotton is pressed down and holds much less air, so it conducts heat out of the bed faster.

e. Heat needed per second to evaporate the water:

$$\frac{Q}{t} = \frac{mL}{t} = \frac{0.390 \times 2.256 \times 10^6}{3 \times 60} = 4888$$

This heat flows through the steel bottom: $\dfrac{Q}{t} = kA\,\dfrac{\theta - 100}{x}$, so

$$\theta - 100 = \frac{(Q/t)\,x}{kA} = \frac{4888 \times 8.5 \times 10^{-3}}{50.2 \times 0.15} = 5.5$$

The bottom surface is at **105.5 °C**. (A metal pot bottom needs only a few degrees across it to carry nearly 5 kW.)

18. a. A thermos flask blocks all three roads of heat:

- the **vacuum** between its double glass walls stops conduction and convection across the gap;
- the walls are **silvered** on the sides facing the vacuum: the inner silver surface is a poor emitter, so little heat leaves the tea by radiation, and the outer one reflects back what does;
- the **stopper** is cork or plastic, a poor conductor, and closes the top against convection; the bottle rests on soft, insulating supports.

Heat escapes only slowly, mostly through the stopper and the glass neck, so the tea stays hot for hours.

b. The layer of air trapped between the two walls is a very poor conductor and can't circulate much, so it slows the loss of heat from inside to outside, and the snow walls themselves are full of trapped air.

c. $A = 2.0\ \text{m}^2$, $x = 3.0 \times 10^{-3}\ \text{m}$, $\theta_1 - \theta_2 = 20 - 5 = 15\ ^\circ\text{C}$.

$$\frac{Q}{t} = kA\,\frac{\theta_1 - \theta_2}{x} = 1.2 \times 2.0 \times \frac{15}{3.0 \times 10^{-3}} = 1.2 \times 10^4$$

12 kW. (A real window loses far less, because the thin layers of still air on its two faces insulate much better than the glass; see the example in Section 12.4.)

d. The thick, bushy tail traps a layer of still air round the squirrel's body, an insulating blanket, and it also covers part of the body's surface, so less heat is lost while the squirrel's body is still and cannot make much heat.

19.

a. The Earth receives only the tiny part of the Sun's radiation that falls on its small disc, 150 million km away, and it radiates its own heat into the cold sky in all directions. It warms until it radiates away as much as it absorbs, which happens at about 15 °C on average, not 5800 K. (The Sun also makes its own heat; the Earth only receives.)

b. Yes. Every body above absolute zero radiates, at a rate $e\sigma A T^4$ set by its own temperature. But it also absorbs radiation from the walls at 35 °C, which is more than it gives out, since $T_0^4 > T^4$. Its net exchange $e\sigma A(T^4 - T_0^4)$ is negative: it **gains** heat, and its temperature **rises** towards 35 °C instead of falling.

c. No: the air in the sun and in the shade are at almost the same temperature, since sunlight passes through air without warming it much. In the sun your body and clothes absorb the Sun's radiation **directly**; in the shade they don't. That radiation, not warmer air, is what you feel.

d. All the electrical power is radiated: $P = A\sigma T^4$.

$$T^4 = \frac{P}{A\sigma} = \frac{1000}{0.006 \times 5.7 \times 10^{-8}} = 2.92 \times 10^{124}$$

$$T = (2.92 \times 10^{12})^{1/4} =$$

about 1035 °C, which is why the element glows bright orange.

20.

a. The Earth receives less than one billionth of the Sun's radiation, and it loses heat by radiating into the cold sky (nearly absolute zero) in every direction. It settles at the temperature where it radiates as much as it absorbs, about 15 °C on average, far below the Sun's 5800 K. The Sun, meanwhile, keeps making new heat in its core.

b. Sunlight passes through the glass and is absorbed by the soil and plants. They give out their heat as long-wavelength infrared, which glass hardly transmits; and, more importantly, the glass stops the warmed air from rising and being carried away by convection. So the inside stays warmer than the outside.

c. $A = 4\pi r^2 = 4\pi(2.00 \times 10^{-2})^2 = 5.03 \times 10^{-3}$ m²; $T = -73 + 273 = 200$ K; $T_0 = 27 + 273 = 300$ K.

$T_0^4 - T^4 = 300^4 - 200^4 = 8.1 \times 10^9 - 1.6 \times 10^9 = 6.5 \times 10^9$ K⁴.

For a black surface ($e = 1$): $\dfrac{Q}{t} = A\sigma(T_0^4 - T^4)$, so

$$\sigma = \frac{Q/t}{A(T_0^4 - T^4)} = \frac{1.85}{5.03 \times 10^{-3} \times 6.5 \times 10^9} = \times^{-824})$$

d. Shiny aluminium paint reflects most of the Sun's radiation and absorbs little, so the petrol stays cooler: less of it evaporates, and the pressure of its vapor in the tank stays low and safe.

21. a. Stainless steel is a poor conductor for a metal (about 16 W/(m K)); copper is about 25 times better (about 400 W/(m K)). A copper bottom takes heat quickly from the flame and spreads it evenly over the whole base, so the food cooks evenly and doesn't burn in spots over the flame. The stainless steel inside doesn't react with food and is easy to clean.

b. A good cooking pot should have:

- a bottom of high thermal conductivity, to pass heat quickly and evenly to the food;
- a low heat capacity, so that it heats up quickly and wastes little fuel;
- a dull, dark bottom that absorbs the flame's radiation well, and polished sides that lose little heat by radiation;
- handles of a poor conductor (wood, Bakelite);
- a material that doesn't react with food and stays strong at cooking temperatures.

c. Heat reaching the ice per second:

$$\frac{Q}{t} = \frac{mL}{t} = \frac{0.025 \times 3.36 \times 10^5}{5 \times 60} = 28$$

This flows along the lagged rod: $\dfrac{Q}{t} = kA\,\dfrac{\theta_1 - \theta_2}{x}$, with $A = 3.14 \times 10^{-4}$ m², $x = 0.20$ m, $\theta_1 - \theta_2 = 100$ °C.

$$k = \frac{(Q/t)\,x}{A(\theta_1 - \theta_2)} = \frac{28 \times 0.20}{3.14 \times 10^{-4} \times 100} =$$

(a value between aluminium's and brass's).

22. Net loss: $P' = A\sigma(T^4 - T_0^4)$, so

$$T^4 = \frac{P'}{A\sigma} + T_0^4 = \frac{0.73}{10^{-4} \times 6 \times 10^{-8}} + 300^4 = 1.217 \times 10^{11} + 0.081 \times 10^{11} = 1.298 \times 10^{114}$$

$$T = (1.298 \times 10^{11})^{1/4} =$$

23.

$$T = \left[\frac{S}{\sigma}\left(\frac{r}{R}\right)^2\right]^{1/4}, \qquad \frac{r}{R} = \frac{1.5 \times 10^{11}}{7 \times 10^8} = 214.3$$

$$T^4 = \frac{1400}{5.67 \times 10^{-8}} \times 214.3^2 = 2.469 \times 10^{10} \times 4.592 \times 10^4 = 1.134 \times 10^{154}$$

$$T = (1.134 \times 10^{15})^{1/4} = 5.8 \times 10^3$$

The Sun's surface is at about **5800 K**.