Unit 11

Quantity of Heat

This unit online, with its interactives:
science-playground.com/s/11f11

Put a pot of water on the stove in Kathmandu and it boils at about 95 °C. Keep the flame high and it goes on boiling for twenty minutes, swallowing heat the whole time, and its temperature doesn't move by a degree. Where does all that heat go?

This unit is about measuring heat. You will meet the heat equation and the calorimeter, see why water is the best coolant and the best hot-water bag, follow a hot cup of tea as it cools by Newton's law, and measure the heat that melts ice and boils water without warming them, the **latent heat**. Then pressure: why rice cooks quicker in a pressure cooker, why a wire can pass through a block of ice and leave it whole, and the one temperature and pressure at which ice, water and steam all exist together.

11.1 Introduction

A STORY FROM SCIENCE

Henri Victor Regnault, the measurer

Henri Victor Regnault (1810–1878) was orphaned at eight and worked in a Paris drapery shop before he won a place at the École Polytechnique. He became the most careful measurer of heat of his century. Working for the French government on the steam engine, he spent years finding the specific heats of dozens of solids, liquids and gases, the pressure of steam at every temperature, and the expansion of mercury and of gases, with apparatus he designed himself. He showed that no two gases expand exactly alike and that Boyle's law is only nearly true for real gases. A young William Thomson (later Lord Kelvin) worked in his laboratory in 1845, and learned there the habit of precise measurement. Regnault's tables of the properties of steam were used by engineers for decades.

When two bodies at different temperatures are in thermal contact, the hotter one cools and the colder one warms until both reach the same temperature, **thermal equilibrium**. Energy passes from the hotter body to the colder one; this energy **in transit**, passed because of a temperature difference, is **heat**. It flows as long as there is a temperature difference. Heat is a form of energy, a scalar, measured in joules.

We can't measure the "heat in a body", only the heat passed from one body to another. That heat may change a body's temperature, and how much depends on the body's **mass** and on **what it is made of**. Knowing the mass, the material (its specific heat capacity) and the change of temperature, we can calculate the heat.

A temperature can also be raised **without** heat. Shake a closed thermos flask of water hard for a few minutes and the water gets warmer; pump up a bicycle tyre and the pump gets warm. The **work** done on the system raises its internal energy, just as heat would.



Figure 11.1 *Henri Victor Regnault in the 1860s.*
Photo: Unknown (Mondadori Publishers), Public domain

Calorimetry

The measurement of the heat passed from one body to another is **calorimetry**: the branch of physics that deals with the experimental methods for measuring heat exchanged. The vessel used for it is a **calorimeter**.

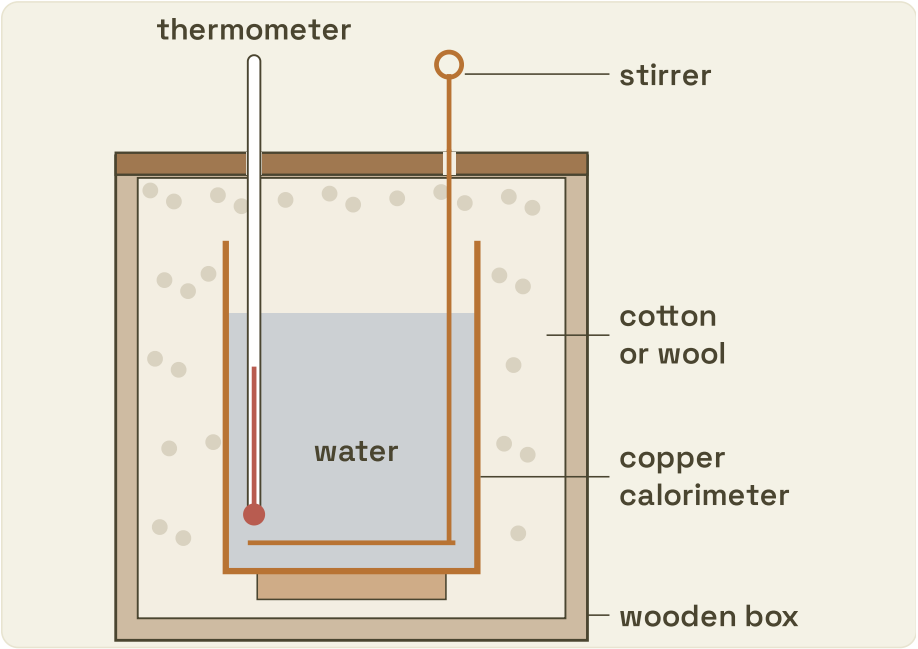


Figure 11.2 *A calorimeter. A thin, polished copper vessel with a copper stirrer stands on a cork, packed round with cotton or wool inside a wooden box; a thermometer passes through a hole in the box's lid.*

A calorimeter is a thin-walled, polished cylindrical vessel, usually of **copper**, with a stirrer of the same metal. It is insulated so that as little heat as possible passes between it and the surroundings: it stands on a poor conductor inside a wooden box, packed round with cotton, wool or another insulator. A thermometer passes through a hole in the lid to read the temperature of the calorimeter and its contents.

Why copper?

- Copper's **specific heat capacity is low** (about 385 J/(kg K)), so the calorimeter itself takes little of the heat being measured, and it nearly stays constant over the usual range of temperatures.
- Copper is a **good conductor**, so the calorimeter quickly comes to the same temperature as its contents.
- **Polished** walls lose and take in little heat by radiation.

11.2 Heat equation

When a body takes in heat and stays in the same state (solid, liquid or gas), its temperature rises; when it gives out heat, its temperature falls. Experiments show that the heat Q taken in or given out is

1. **proportional to the mass** m of the body: $Q \propto m$;
2. **proportional to the change in temperature** $\Delta\theta$: $Q \propto \Delta\theta$.

Combining these, $Q \propto m\,\Delta\theta$, and with a constant s,

$$\boxed{Q = m\,s\,\Delta\theta} \tag{11.1}$$

This is the **heat equation**. The constant s is the **specific heat capacity** of the material (Section 11.4).

An older unit of heat is the **calorie**: **1 calorie is the heat needed to raise the temperature of 1 g of pure water by 1 °C** (from 14.5 °C to 15.5 °C, since the heat needed changes very slightly with temperature).

$$1 \text{ cal} = 4.2 \text{ J (more exactly } 4.186 \text{ J)}$$

The Calorie printed on food packets, with a capital C, is a kilocalorie: 1000 calories, about 4.2 kJ.

11.3 Heat capacity (thermal capacity)

The heat capacity (thermal capacity) of a body is the heat needed to change its temperature by 1 °C (or 1 K).

From the heat equation, the heat to change the temperature of a body of mass m by $\Delta\theta = 1$ K is $Q = ms \times 1$. So

$$\text{heat capacity} = m\,s \tag{11.2}$$

Heat capacity depends on **both** the material and the mass of the body: a bucket of water has a far larger heat capacity than a cup of it. Its SI unit is **J/K** (J K^{-1}).

11.4 Specific heat capacity

Rearranging the heat equation,

$$s = \frac{Q}{m\,\Delta\theta} \tag{11.3}$$

Put $m = 1$ kg and $\Delta\theta = 1$ K, and $s = Q$. So:

The specific heat capacity of a substance is the heat needed to change the temperature of unit mass (1 kg) of it by 1 °C (or 1 K).

- It depends only on the **substance** (and its state: ice and water differ), not on how much of it there is: it is the heat capacity **per kilogram**.
- Its SI unit is **J/(kg K)**.

Water's specific heat capacity, 4200 J/(kg K), is one of the highest of all common solids and liquids: only a few substances, such as liquid ammonia, have higher values.

Substance	Specific heat capacity (J/(kg K))
Water	4200
Copper	380
Aluminium	910
Iron	470
Brass	372
Steel	447
Mercury	138
Sand (dry)	800
Glass	677
Kerosene	2090
Ice	2100
Alcohol (ethanol)	2440

CAREFUL

Heat capacity or specific heat capacity?

Heat capacity belongs to a body (this kettle, this brick): ms, in J/K. **Specific heat capacity** belongs to a substance (water, iron): s, in J/(kg K). A big iron pot and a small iron spoon have the same specific heat capacity but very different heat capacities.

Check 1

A 2 kg block of aluminium (910 J/(kg K)) has a heat capacity of:

a 455 J/K
b 910 J/K
c 1820 J/K
d 1820 J/(kg K)

11.5 Specific heat capacity and change in temperature

Give the **same heat** to two bodies of **equal mass** m but of different specific heat capacities s_1 and s_2, and let their temperatures rise by $\Delta\theta_1$ and $\Delta\theta_2$. Then

$$m\,s_1\,\Delta\theta_1 = m\,s_2\,\Delta\theta_2 \quad\Rightarrow\quad s_1\,\Delta\theta_1 = s_2\,\Delta\theta_2 \quad\Rightarrow\quad \frac{\Delta\theta_1}{\Delta\theta_2} = \frac{s_2}{s_1}$$

So for a given heat (and mass), $\Delta\theta \propto \frac{1}{s}$: **the change in temperature is inversely proportional to the specific heat capacity.** A substance with a high specific heat capacity changes its temperature little; one with a low specific heat capacity changes a lot.

11.6 Uses of high and low specific heat capacity

High specific heat capacity: water. A substance with a high specific heat capacity can **take in** a lot of heat from a hotter body while its own temperature rises only a little, so it cools that body a great deal. So water is used

- as the **coolant** in car, bus and motorcycle engines, and in many machines that get hot;
- to bring down a high **fever**: a cloth soaked in water on the forehead takes heat from the skin (and its evaporation takes more, Section 11.15).

It can also **give out** a lot of heat while its own temperature falls only a little, so it keeps something warm for a long time. So water is used

- in **hot-water room heaters** (radiators), where hot water carries heat round a building;
- in a **hot-water bag**, which stays warm for hours.

Low specific heat capacity. A substance with a low specific heat capacity takes in or gives out very little heat for a change of temperature, so it soon comes to the temperature of whatever it touches, and it warms up quickly on a little heat.

- **Mercury** in a thermometer: it takes very little heat from the body it measures, so it barely changes that body's temperature, and it quickly reaches the same temperature.
- **Copper** for a calorimeter: it takes little of the heat being measured.
- **Cooking oil** for frying: it gets very hot quickly on the same flame, and can be heated far above 100 °C without boiling, so food fries fast.

WORKED EXAMPLE

An iron ball takes in heat

An iron ball of mass 150 g is given 2100 J of heat. By how much does its temperature rise? (Specific heat capacity of iron = 470 J/(kg K).)

Answer

$m = 0.150$ kg, $Q = 2100$ J, $s = 470$ J/(kg K).

$$\Delta\theta = \frac{Q}{ms} = \frac{2100}{0.150 \times 470} = 29.8 \text{ K} \approx \mathbf{30\ K}$$

A rise of 30 K is a rise of 30 °C. The same heat would warm 150 g of water by only 3.3 °C.

WORKED EXAMPLE

A waterfall warms its water

Find the difference in temperature between the water at the top and the bottom of a waterfall 200 m high, if all the energy of the fall turns into heat in the water. (Specific heat capacity of water = 4200 J/(kg K), g = 10 m/s².)

Answer

Each kilogram loses potential energy mgh and gains heat $ms\Delta\theta$:

$$ms\,\Delta\theta = mgh \quad\Rightarrow\quad \Delta\theta = \frac{gh}{s} = \frac{10 \times 200}{4200} = \mathbf{0.48\ K}$$

The mass cancels: every waterfall 200 m high warms its water by about half a degree. James Prescott Joule, on his honeymoon in the Alps, is said to have tried to measure exactly this with a thermometer at a waterfall.

Sample questions

11.7 Newton's law of cooling

A bowl of hot soup is too hot to eat for the first few minutes; then it cools to a pleasant temperature and stays warm for a long while. It cools **fast at first and slowly later**. How fast a hot body cools depends on how much hotter it is than its surroundings, on its surface area, on what it is and what holds it; but when everything except the temperature difference is kept the same, Isaac Newton found the rule in 1701:

Newton's law of cooling: the rate at which a body loses heat is proportional to the difference between its temperature and the temperature of its surroundings.

$$-\frac{dQ}{dt} \propto (\theta - \theta_0)$$

where θ is the body's temperature and θ_0 the surroundings' (the minus sign because the body **loses** heat).

Its equation

Take a hot body (a liquid in a vessel, say) of mass m and specific heat capacity s at temperature θ, in surroundings at θ_0. In a short time dt it loses heat dQ.

By Newton's law, with a constant K:

$$-\frac{dQ}{dt} = K(\theta - \theta_0) \tag{11.4}$$

K depends on the body's surface area exposed to the surroundings and on the nature of its surface. Then, step by step:

1. Losing dQ, its temperature falls by $d\theta$ (a negative change): $dQ = ms\,d\theta$.
2. Put this into (11.4): $-ms\dfrac{d\theta}{dt} = K(\theta - \theta_0)$. So the **rate of cooling**, $\dfrac{d\theta}{dt} = -\dfrac{K}{ms}(\theta - \theta_0)$, is also proportional to the temperature difference.
3. Separate the variables: $\dfrac{d\theta}{\theta - \theta_0} = -\dfrac{K}{ms}\,dt$.
4. Integrate both sides: $\ln(\theta - \theta_0) = -\dfrac{K}{ms}\,t + C$, where C is the constant of integration.

$$\ln(\theta - \theta_0) = -\frac{K}{ms}\,t + C \tag{11.5}$$

This is of the form $y = mx + c$. A graph of $\ln(\theta - \theta_0)$ against t is a **straight line** with a negative slope $-K/ms$; plotting the measured temperatures this way and getting a straight line is how the law is tested. (Taking the exponential of both sides, $\theta - \theta_0 = (\theta_1 - \theta_0)\,e^{-Kt/ms}$: the excess temperature falls by the same **fraction** in every equal interval of time.)

When does it hold? Newton's law is a good description when

1. the **temperature difference is small** (a few tens of degrees);
2. the body loses heat in the **same way** throughout: the same surface, the same draught of air. For a small difference, both radiation and convection carry heat away at a rate nearly proportional to the difference; for a large one, radiation (which depends on the fourth power of the absolute temperature) and convection in still air grow faster than the difference;
3. the **temperature of the surroundings stays constant** while the body cools.

> CAREFUL
> **"Small difference"**
>
> Some books give the limit as a difference of less than about 35 °C, or say the heat must be lost by radiation only. Radiation follows Newton's law only while the difference is small, so "small" is the condition that matters. An answer right in every book: the temperature difference must be small, the heat must be lost in the same way throughout (as by radiation), and the surroundings must stay at a constant temperature.

Where we see it:

1. A cup of hot tea or coffee cools fast at first and then more and more slowly as its temperature nears the room's.
2. In forensic science, the body's temperature and the room's give an estimate of the time of death.
3. Tea cools faster in winter than in summer: the room is colder, so the difference, and with it the rate of cooling, is larger.
4. A hot liquid cools faster than a warm one in the same room: its temperature difference is larger.
5. Animals eat more in winter: they lose heat faster and must make more.

WORKED EXAMPLE

How cold is the room?

A body cools from 50 °C to 45 °C in 3 minutes and from 45 °C to 40 °C in 5 minutes. Find the temperature of its surroundings, and the time it will take to cool from 40 °C to 35 °C.

Answer

Over a short interval, use the **average** temperature in Newton's law: rate of fall of temperature $= k(\theta_{\text{avg}} - \theta_0)$, with $k = K/ms$.

First interval: fall 5 °C in 3 min, average $\frac{50+45}{2} = 47.5$ °C:

$$\frac{5}{3} = k(47.5 - \theta_0) \quad (1)$$

Second interval: fall 5 °C in 5 min, average 42.5 °C:

$$\frac{5}{5} = k(42.5 - \theta_0) \quad (2)$$

Divide (1) by (2): $\dfrac{5}{3} = \dfrac{47.5 - \theta_0}{42.5 - \theta_0}$, so $5(42.5 - \theta_0) = 3(47.5 - \theta_0)$, $212.5 - 5\theta_0 = 142.5 - 3\theta_0$, $2\theta_0 = 70$:

$\theta_0 = \mathbf{35\ °C}$.

From (2), $k = \dfrac{1}{42.5 - 35} = \dfrac{1}{7.5}$ per minute.

Third interval: fall 5 °C in t min, average 37.5 °C:

$$\frac{5}{t} = \frac{1}{7.5}(37.5 - 35) \quad \Rightarrow \quad t = \frac{5 \times 7.5}{2.5} = \mathbf{15\ min}$$

11.8 Principle of calorimetry

When two bodies at different temperatures are put in thermal contact, the hotter one loses heat and the colder one gains it until both reach the same temperature. If no heat is lost to the surroundings,

Principle of calorimetry: in a closed (isolated) system, the heat lost by the hotter bodies equals the heat gained by the colder bodies.

$$\text{heat lost} = \text{heat gained}$$

A closed, or **isolated**, system is one that exchanges no heat with its surroundings; a well-insulated calorimeter is close to one. The principle is the law of conservation of energy applied to heat, and it is the basis of every calorimeter measurement in the next sections.

Check 2

100 g of water at 80 °C is mixed with 300 g of water at 20 °C in an insulated flask. The final temperature is:

a 50 °C
b 35 °C
c 40 °C
d 27.5 °C

11.9 Specific heat capacity of a solid by the method of mixtures

The **method of mixtures** uses the principle of calorimetry: heat a solid to a known temperature, drop it into water in a calorimeter, and measure how much the water warms. The solid's heat is found from the water's. Regnault designed an apparatus that heats the solid in steam and drops it into the calorimeter quickly.

Method:

1. Weigh the clean, dry calorimeter with its stirrer: m_1.
2. Fill it about two-thirds with water and weigh it again: m_2.
3. Place it in its insulating box with a thermometer, and note the temperature of the water and calorimeter: θ_1.

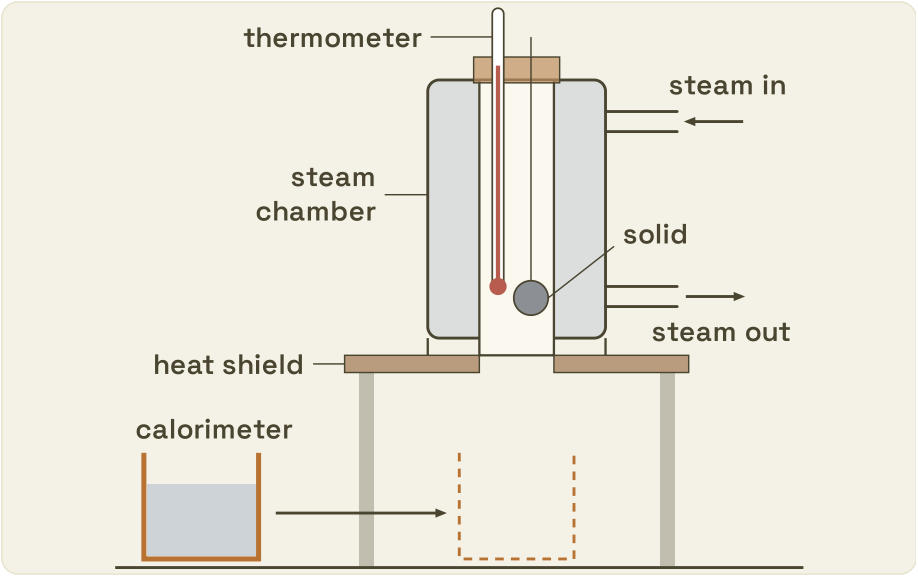


Figure 11.3 *Regnault's apparatus. The solid hangs by a thread inside a steam chamber, a thermometer beside it; steam enters at the top and leaves at the bottom. The calorimeter waits to one side behind a heat shield (so the steam chamber doesn't warm it); when the solid is ready, the calorimeter is slid under the chamber and the solid is lowered into it.*

4. Meanwhile, hang the solid in the steam chamber and pass steam until its thermometer shows a steady temperature: θ_2 (near 100 °C, less at altitude).
5. Slide the calorimeter under the chamber and drop the solid in quickly, so it loses no heat on the way.
6. Stir gently until the temperature stops rising, and note the highest steady temperature of the mixture: θ.
7. Weigh the calorimeter with its water and the solid: m_3.

Calculation. Mass of water $m_w = m_2 - m_1$; mass of the solid $m_s = m_3 - m_2$.

The solid's temperature falls by $\theta_2 - \theta$; the water's and calorimeter's rises by $\theta - \theta_1$.

- Heat lost by the solid: $Q_L = m_s\, s\,(\theta_2 - \theta)$.
- Heat gained by the water and the calorimeter: $Q_G = m_w s_w(\theta - \theta_1) + m_1 s_c(\theta - \theta_1) = (m_w s_w + m_1 s_c)(\theta - \theta_1)$,

where s_w and s_c are the specific heat capacities of water and of the calorimeter's metal, from tables.

By the principle of calorimetry, $Q_L = Q_G$:

$$s = \frac{(m_w s_w + m_1 s_c)(\theta - \theta_1)}{m_s(\theta_2 - \theta)} \tag{11.6}$$

WORKED EXAMPLE

The specific heat capacity of a metal

A 200 g block of metal at 150 °C is dropped into a copper calorimeter of mass 270 g holding 150 cm³ of water at 27 °C. The final temperature is 40 °C. Find the specific heat capacity of the metal. (Copper 390 J/(kg K), water 4200 J/(kg K).)

Answer

Mass of water = 150 cm³ × 1 g/cm³ = 150 g = 0.150 kg.
Heat lost by the block: $0.200 \times s \times (150 - 40) = 22s$.
Heat gained by the calorimeter and the water:

$$0.270 \times 390 \times (40 - 27) + 0.150 \times 4200 \times (40 - 27) = 1368.9 + 8190 = 9558.9 \text{ J}$$

Heat lost = heat gained: $22s = 9558.9 \Rightarrow s =$ **434.5 J/(kg K)**, about 4.3×10^2 J/(kg K): close to iron's or steel's.

YOUR TURN

A metal block in a copper calorimeter

A copper calorimeter of mass 300 g holds 500 g of water at 15 °C. A 560 g block of metal at 100 °C is dropped in, and the temperature rises to 22.5 °C. Find the specific heat capacity of the metal, ignoring heat lost to the surroundings. (Copper 390 J/(kg K), water 4200 J/(kg K).)

Answer

Heat gained: $(0.300 \times 390 + 0.500 \times 4200)(22.5 - 15) = (117 + 2100) \times 7.5 = 16\,627.5$ J.
Heat lost: $0.560 \times s \times (100 - 22.5) = 43.4s$.
$s = \dfrac{16\,627.5}{43.4} =$ **383 J/(kg K)**, close to copper or brass. (It can't be aluminium, whose specific heat capacity is 910 J/(kg K).)

11.10 Change of phase

Matter is found in three states: **solid**, **liquid** and **gas**. In a solid the particles are packed closest and held in place; in a liquid they are about as close but free to move past each other; in a gas they are far apart and move freely. Which state a substance is in depends on its temperature and the pressure on it. A change of state is a **physical** change.

The changes come in pairs, each the reverse of the other:

- **melting (fusion)** solid → liquid, and **freezing (solidification)** liquid → solid;
- **vaporization** liquid → gas (by evaporation or boiling), and **condensation** gas → liquid;
- **sublimation** solid → gas directly, and **deposition** gas → solid directly (hoar frost forming on a cold night).

Heating through the changes. Heat a block of ice from well below 0 °C with a steady heater, and its temperature rises steadily to the **melting point**, 0 °C. Then the temperature **stops rising**: the ice melts, and ice and water stay at 0 °C, side by side, until all the ice has melted, although heat is going in all the time. Then the water warms steadily to the **boiling point**, 100 °C at sea level, and stops again until all of it has boiled away. On cooling, the same happens in reverse: the temperature stops at the freezing point until all the liquid has frozen.

Latent heat. The heat taken in or given out by a substance while it changes state at a constant temperature is called its latent heat.

- The latent heat during melting or freezing is the **latent heat of fusion**.
- The latent heat during vaporization or condensation is the **latent heat of vaporization**.

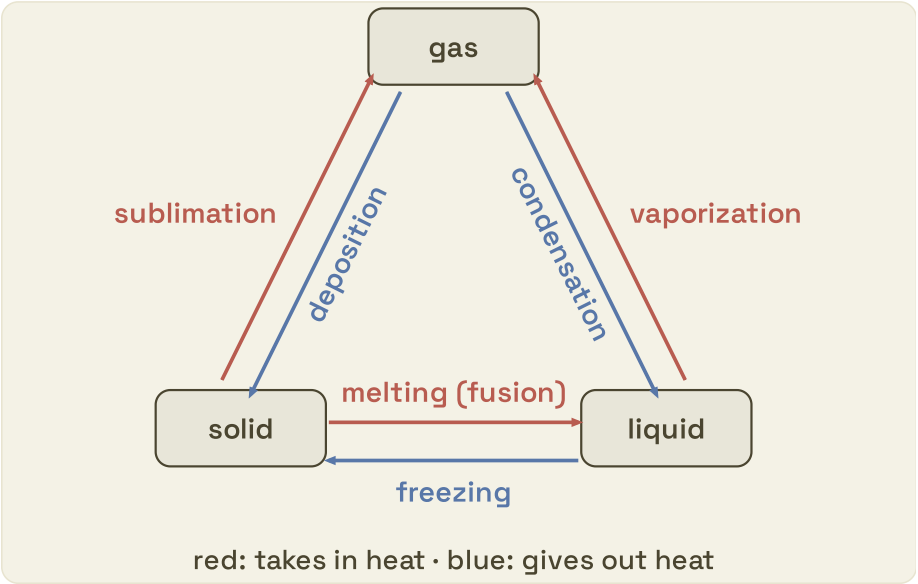


Figure 11.4 *The six changes of state. Melting (fusion), vaporization (evaporation and boiling) and sublimation take in latent heat; freezing (solidification), condensation and deposition give it out.*

"Latent" means hidden: this heat doesn't show on a thermometer. It is also called the **heat of transformation**.

Where does it go? During a change of state, the heat does two things:

1. **Internal latent heat:** most of it goes into pulling the particles apart against their attraction for each other, raising their **potential energy**. Their average kinetic energy, and so the temperature, stays the same.
2. **External latent heat:** a small part does work pushing back the atmosphere as the substance expands (steam takes about 1700 times the volume of the water it came from).

On vaporization, the particles are pulled right apart and the expansion is huge; on melting they only loosen. So both parts are far larger for vaporization: **the latent heat of vaporization of a substance is always much greater than its latent heat of fusion** (for water, 2.26×10^6 J/kg against 3.36×10^5 J/kg, nearly 7 times).

11.11 Specific latent heat of fusion

The heat needed to melt a solid at its melting point is proportional to its mass: $Q \propto m$, or

$$Q = mL, \qquad L = \frac{Q}{m} \tag{11.7}$$

The constant L is the **specific latent heat of fusion** of the substance. Put $m = 1$ kg and $L = Q$:

The specific latent heat of fusion of a substance is the heat needed to change unit mass (1 kg) of it from solid to liquid at its melting point, without a change of temperature.

Its SI unit is **J/kg**. For ice, $L_f = 3.36 \times 10^5$ J/kg: 336 000 J changes 1 kg of ice at 0 °C into 1 kg of water at 0 °C. (The same heat is given out when 1 kg of water at 0 °C freezes.)

11.12 Specific latent heat of vaporization

In the same way, the heat needed to change a liquid into vapor at its boiling point is $Q = mL$.

The specific latent heat of vaporization of a substance is the heat needed to change unit mass (1 kg) of it from liquid to vapor at its boiling point, without a change of temperature.

Its SI unit is **J/kg**. For water, $L_v = 2.26 \times 10^6$ J/kg: 2 260 000 J changes 1 kg of water at 100 °C into steam at 100 °C. That is more than five times the heat that warms the same water from 0 °C to 100 °C (420 000 J).

> DID YOU KNOW?
> **Joseph Black and the hidden heat**
>
> In Glasgow around 1761, the Scottish chemist Joseph Black noticed that ice taken into a warm room sits at its melting point for hours while it melts, though heat must be pouring into it all the time; and that water on a fire takes far longer to boil away than to come to the boil. He called the heat that disappears without warming anything **latent heat**. His friend James Watt, the university's instrument-maker, met the same hidden heat in a model steam engine he was repairing: much of its fuel went into turning water into steam, and much of that steam was wasted heating the cylinder again at every stroke. His separate condenser (1765) saved it and made the engine far more efficient.

Good to know

1. While a substance is heated, it either changes its **temperature** or its **state**, not both at once.
2. While a solid melts, the solid and the liquid **coexist** at the melting point, in thermal equilibrium; while a liquid boils, the liquid and its vapor coexist; during sublimation, the solid and the vapor coexist.
3. Specific heat capacity is different in different states of the same substance: water 4200 J/(kg K), ice 2100 J/(kg K).
4. On a graph of temperature against heat supplied, the flat part for boiling is much longer than the flat part for melting: the latent heat of vaporization is much larger than that of fusion.
5. The sloping parts for the solid, the liquid and the gas have different slopes: different specific heat capacities in each state (the steeper the slope, the smaller the specific heat capacity). They also differ from one substance to another.

Sample questions

11.13 Specific latent heat of fusion of ice by the method of mixtures

The specific latent heat of fusion of ice is the heat needed to change 1 kg of ice at 0 °C into water at 0 °C. It can be measured with a calorimeter: melt a known mass of ice in warm water and see how much the water cools.

Method:

1. Weigh the clean, dry calorimeter with its stirrer: m_1.
2. Fill it about two-thirds with water warmed a little above room temperature (say 10 °C above), and weigh it again: m_2.
3. Put it in its insulating box with a thermometer, and note the temperature: θ_1.
4. Dry some pieces of **melting** ice (so they are at 0 °C) with blotting paper and drop them in.
5. Stir until all the ice has melted, and note the lowest steady temperature of the mixture: θ_2.
6. Weigh the calorimeter and its contents again: m_3.

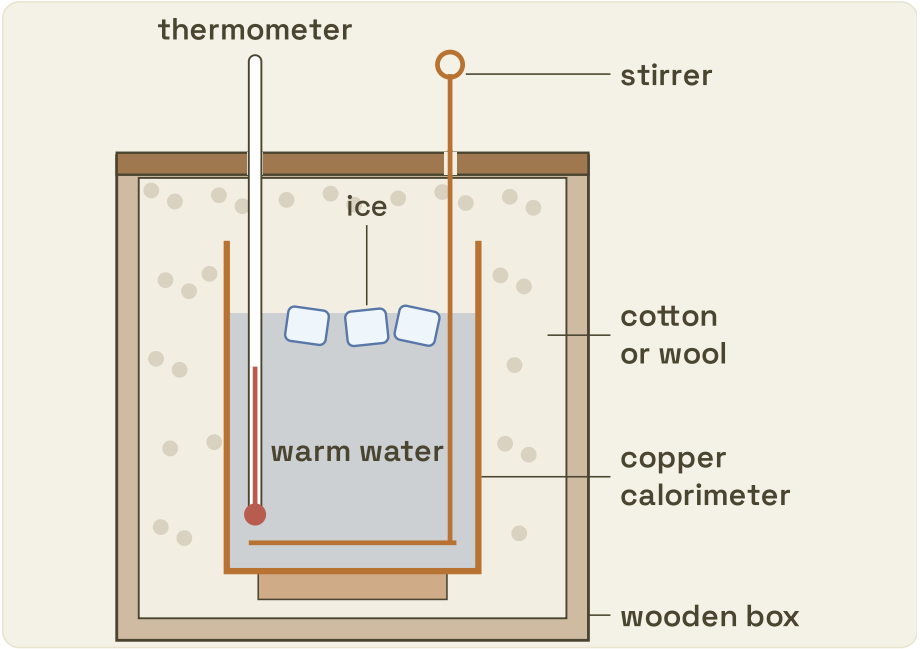


Figure 11.5 *Measuring the latent heat of fusion of ice: pieces of dry, melting ice in the warm water of the calorimeter.*

Calculation. Mass of water $m_w = m_2 - m_1$; mass of ice $m_i = m_3 - m_2$.

- Heat gained by the ice: it melts, $m_i L_i$, then the water from it warms from 0 °C to θ_2, $m_i s_w(\theta_2 - 0)$.
- Heat lost by the calorimeter and the warm water as they cool from θ_1 to θ_2: $(m_1 s_c + m_w s_w)(\theta_1 - \theta_2)$.

By the principle of calorimetry:

$$m_i L_i + m_i s_w \theta_2 = (m_1 s_c + m_w s_w)(\theta_1 - \theta_2)$$

$$L_i = \frac{(m_1 s_c + m_w s_w)(\theta_1 - \theta_2)}{m_i} - s_w \theta_2 \tag{11.8}$$

Every quantity on the right is measured or from tables, so L_i is found.

Why start the water about as much **above** room temperature as it will end **below** it? Then the mixture gains heat from the room during the second half of the experiment about as much as it lost in the first, and the two errors nearly cancel.

WORKED EXAMPLE

How much must evaporate?

Evaporating sweat is one of the main ways a warm-blooded animal controls its temperature. What mass of water must evaporate from the skin of an 80 kg person to cool the body by 1 °C? (Average specific heat capacity of the human body about 3500 J/(kg K); latent heat of vaporization of water at body temperature 2.4×10^6 J/kg.)

Answer

Heat to be lost: $Q_1 = mc\Delta\theta = 80 \times 3500 \times 1 = 280\,000$ J.

Heat carried off by evaporating a mass m_w: $Q_2 = m_w L = 2.4 \times 10^6\, m_w$.

$$Q_1 = Q_2 \quad \Rightarrow \quad m_w = \frac{280\,000}{2.4 \times 10^6} = 0.117 \text{ kg} \approx \mathbf{0.12\ kg}$$

About 120 g of sweat, a small glass of water, to cool the whole body by one degree. (The body's specific heat capacity is a little less than water's, since it is about 60% water. At body temperature, water's latent heat is larger than at 100 °C.)

WORKED EXAMPLE

From ice at −10 °C to steam

How much heat is needed to change 10 g of ice at −10 °C into steam at 100 °C? (Specific heat capacity of ice 2100 J/(kg K), of water 4200 J/(kg K); latent heat of fusion of ice 3.36×10^5 J/kg; of vaporization of water 2.26×10^6 J/kg.)

Answer

$m = 10$ g $= 0.010$ kg. Four steps:

ice at −10 °C → ice at 0 °C → water at 0 °C → water at 100 °C → steam at 100 °C.

$Q_1 = mc_i\Delta\theta = 0.010 \times 2100 \times 10 = 210$ J (warming the ice)

$Q_2 = mL_f = 0.010 \times 3.36 \times 10^5 = 3360$ J (melting)

$Q_3 = mc_w\Delta\theta = 0.010 \times 4200 \times 100 = 4200$ J (warming the water)

$Q_4 = mL_v = 0.010 \times 2.26 \times 10^6 = 22\,600$ J (boiling)

$$Q = 210 + 3360 + 4200 + 22\,600 = 30\,370 \text{ J} \approx \mathbf{3.04 \times 10^4\ J}$$

Three-quarters of it goes into the boiling alone.

WORKED EXAMPLE

Ice that freezes water

50 g of ice at −6 °C is dropped into water at 0 °C. How many grams of the water freeze? (Specific heat capacity of ice 2000 J/(kg K), latent heat of fusion of ice 3.36×10^5 J/kg.)

Answer

The ice can only warm to 0 °C, the water's temperature; the heat it takes in comes from water freezing, which gives out its latent heat while staying at 0 °C.

Heat gained by the ice: $Q_1 = 0.050 \times 2000 \times 6 = 600$ J.

Heat given out by a mass m_w of water freezing: $Q_2 = m_w \times 3.36 \times 10^5$.

$$m_w = \frac{600}{3.36 \times 10^5} = 1.79 \times 10^{-3} \text{ kg} \approx \mathbf{1.8\ g}$$

Everything ends at 0 °C, with about 51.8 g of ice.

WORKED EXAMPLE

Ice dropped from a height

From what height must a block of ice at 0 °C be dropped to melt completely, if 20% of the energy of its fall goes into the ice? (Latent heat of fusion of ice 3.36×10^5 J/kg; g = 10 m/s².)

Answer

The heat to melt mass m must come from 20% of the potential energy lost: $mL = 0.20\,mgh$.

$$h = \frac{L}{0.20\,g} = \frac{3.36 \times 10^5}{0.20 \times 10} = 1.68 \times 10^5 \text{ m} \approx \mathbf{170\ km}$$

The mass cancels. This shows how large a latent heat is: 170 km is far above the atmosphere (and g would not stay 10 m/s² over such a height), and a real falling block would reach a steady speed in the air long before. Melting ice takes a great deal of energy.

WORKED EXAMPLE

Ice and water in a vessel

10 g of ice at 0 °C is put into 15 g of water at 20 °C in a vessel of mass 100 g and specific heat capacity 378 J/(kg K). What is the result? (Latent heat of fusion of ice 3.36×10^5 J/kg, specific heat capacity of water 4200 J/(kg K).)

Answer

First see whether there is enough heat to melt all the ice.
Heat the water and the vessel can give out in cooling from 20 °C to 0 °C:

$$Q_l = 0.015 \times 4200 \times 20 + 0.100 \times 378 \times 20 = 1260 + 756 = 2016 \text{ J}$$

Heat needed to melt all 10 g of ice: $Q_r = 0.010 \times 3.36 \times 10^5 = 3360$ J.
$Q_l < Q_r$: not all the ice can melt, and everything ends at **0 °C**. The ice that melts:

$$m = \frac{Q_l}{L_i} = \frac{2016}{3.36 \times 10^5} = 6.0 \times 10^{-3} \text{ kg} = 6 \text{ g}$$

The result: **4 g of ice and 21 g of water, all at 0 °C.**
In such questions, always check first whether all the ice can melt. If you write “heat lost = heat gained” with an unknown final temperature, you get an answer below 0 °C, which is impossible for a mixture of water and ice.

Sample questions

YOUR TURN

1. Steam into ice and water

10 g of steam at 100 °C is passed into a mixture of 100 g of water and 10 g of ice at 0 °C. Find the final temperature. (Specific heat capacity of water 4200 J/(kg K), latent heat of fusion of ice 3.36×10^5 J/kg, of vaporization of water 2.26×10^6 J/kg.)

Answer

Check first: 10 g of steam condensing gives out $0.010 \times 2.26 \times 10^6 = 22\,600$ J, more than the 3360 J that melts the ice, so all the ice melts and the final temperature θ is above 0 °C.
Heat lost by the steam: $22\,600 + 0.010 \times 4200 \times (100 - \theta) = 26\,800 - 42\theta$.
Heat gained by the ice and water: $3360 + 0.110 \times 4200 \times \theta = 3360 + 462\theta$.
$26\,800 - 42\theta = 3360 + 462\theta \Rightarrow 504\theta = 23\,440 \Rightarrow \theta = \mathbf{46.5\ °C}$.

YOUR TURN

2. How much steam?

A copper calorimeter of mass 200 g holds a mixture of 500 g of water and 100 g of ice at 0 °C. How much steam at 100 °C must be passed in to bring the mixture to 40 °C? (Water 4200 J/(kg K), copper 386.4 J/(kg K); latent heat of fusion of ice 3.36×10^5 J/kg, of vaporization of water 2.26×10^6 J/kg.)

Answer

Heat gained: melting the ice, $0.100 \times 3.36 \times 10^5 = 33\,600$ J; then all 600 g of water and the calorimeter from 0 °C to 40 °C, $(0.600 \times 4200 + 0.200 \times 386.4) \times 40 = (2520 + 77.28) \times 40 = 103\,891$ J. Total 137 491 J.

Heat lost by a mass m of steam condensing and cooling to 40 °C: $m(2.26 \times 10^6 + 4200 \times 60) = 2\,512\,000\,m$.

$m = \dfrac{137\,491}{2\,512\,000} = 0.0547$ kg ≈ **54.7 g**.

11.14 Boiling

Boiling is the fast, bubbling change of a liquid into vapor that happens throughout the liquid at one fixed temperature, the **boiling point**. While it boils, the liquid takes in its latent heat of vaporization and its temperature stays constant. Each liquid has its own boiling point; water boils at 100 °C under a pressure of 760 mm of mercury (one standard atmosphere).

Effect of pressure on the boiling point

Bubbles of vapor can form and grow inside a liquid only when the pressure of the vapor in them (the liquid's **saturated vapor pressure**, which rises steeply with temperature) is as large as the pressure pressing on the liquid. So:

A liquid boils at the temperature at which its saturated vapor pressure equals the pressure on its surface.

- **Raise the pressure** and the liquid must get hotter before it boils: the boiling point **rises**.
- **Lower the pressure** and it boils at a lower temperature: the boiling point **falls**.

Air pressure falls with height, so water boils at a lower temperature the higher you go: at about 100 °C in Birgunj (about 90 m), 95 °C in Kathmandu (1400 m), 89 °C in Namche Bazaar in Solukhumbu (3440 m), and about 71 °C on top of Everest.



Figure 11.6 *A pressure cooker, open. Its oval lid goes in from inside and is pulled up against the rim, sealed by a rubber ring; a weight on the vent keeps the steam in until the pressure inside is high enough to lift it.*
Photo: Cantons-de-l'Est, CC BY-SA 4.0

11.15 Evaporation

Spray perfume on your arm and the spot feels cold; a menthol sweet such as a Halls gives a cool feeling in the mouth. The perfume's cold is the **cooling effect of evaporation**. (Menthol's coolness is a trick of the nerves: it sets off the nerve endings that sense cold, without any actual cooling.)

Evaporation is the slow, quiet change of a liquid into vapor from its **surface**, at **any temperature** below its boiling point. In a liquid, the molecules move with a range of speeds. The fastest ones near the surface can break free of the others' attraction and escape. They take more than their share of energy with them (each needs its latent heat to leave), so the **average** kinetic energy of the molecules left behind falls, and the liquid **cools**: the **cooling effect of evaporation**.

Examples:

a. After sweating we feel cool: the sweat evaporates, taking its latent heat from the skin.
b. Wet clothes dry: the water in them evaporates, faster in the sun and in wind.
c. Spirit (alcohol), which evaporates fast at room temperature, is rubbed on the skin before an injection or a blood test: it kills germs and evaporates quickly, leaving the skin clean and dry, and feeling cold.
d. In the water cycle, water evaporates from seas, lakes, rivers and leaves into the air; the vapor cools as it rises, condenses into clouds, and falls as rain.
e. After a bath we feel cold when we step into the open air but not inside the bathroom: outside, the air is drier and moving, so the water on the skin evaporates much faster.

Evaporation	Boiling
Happens only at the **surface** of the liquid.	Happens **throughout** the liquid (bubbles form inside).
Happens at **all temperatures**.	Happens only at the **boiling point** (for the pressure).
Slow and quiet.	Fast and bubbling.
Cools the liquid that is left.	The temperature stays **constant**: the heat supplied is taken as latent heat.
Depends on the surface area, wind and dampness of the air.	Depends on the pressure on the liquid.

Sample questions

11.16 Effect of pressure on the melting point

Pressure changes the melting point of a solid, in a direction that depends on whether the solid **expands or contracts** on melting.

- **Ice** (and cast iron, bismuth and a few others) **contracts** on melting: water takes less room than the ice it came from. Squeezing helps it melt, so **its melting point falls as the pressure rises**, by about 0.0075 °C for each extra atmosphere.
- **Wax**, **lead** and most other substances **expand** on melting. Squeezing resists melting, so **their melting point rises with pressure**.

Regelation

Press two pieces of ice firmly together, and then let go: they come away **joined** into one piece. Where they touch, the pressure is high, the melting point there falls below 0 °C, and a little ice melts. When the pressure is released, the melting point returns to 0 °C, and the water, now below its melting point, **freezes again**, joining the pieces.

Regelation is the melting of ice under pressure and its freezing again when the pressure is removed.

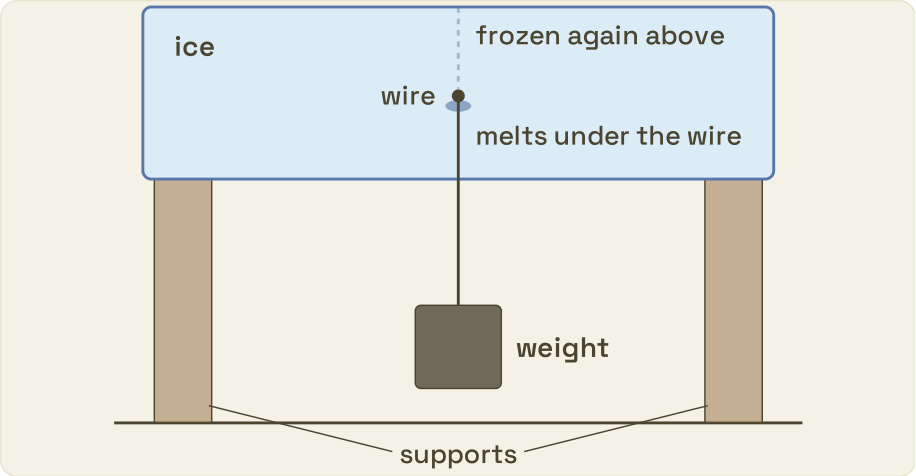


Figure 11.7 *Regelation, seen from the side. A block of ice rests with its ends on two supports, and a thin wire looped over its middle carries a heavy weight. Under the wire the pressure is high, so the ice melts; the water flows round to the top of the wire, where the pressure is normal, and freezes again. The wire sinks slowly through the block and falls out below, leaving the block in one piece.*

CAREFUL

How much does pressure do?

Pressure lowers ice's melting point only slightly: a pressure of 100 atmospheres lowers it by less than 1 °C. Under a fine wire with heavy weights the pressure is that large, and the wire's metal also carries heat from the water above to the ice below, so the experiment works. But a person standing on ice exerts far less than that: a skater's blade lowers the melting point by only a fraction of a degree. That ice is slippery is mainly because of a thin layer of water on its surface, made by friction and by the loosely held molecules at the surface itself.

Sample questions

11.17 Phase diagram and triple point

A graph of **pressure against temperature** showing the conditions in which a substance is solid, liquid or gas, separated by three curves, is called a **phase diagram** or **P–T diagram**.

The three curves:

a. **Vaporization curve (steam line):** how the **boiling point** changes with pressure. Above (and to the left of) the curve the substance is a **liquid**; below (and to the right) it is a **vapor (gas)**. Along the curve, liquid and vapor coexist. It ends at the **critical point** (374 °C and 218 atm for water), above which liquid and vapor can no longer be told apart.

b. **Fusion curve (ice line):** how the **melting point** changes with pressure. It separates the **solid** (to the left) from the **liquid** (to the right). For water it leans slightly to the **left** as it rises: more pressure, lower melting point. For most other substances, which expand on melting, it leans to the right.

c. **Sublimation curve (hoar-frost line):** how the **sublimation point** changes with pressure, at low pressures and temperatures. Above it the substance is a **solid**, below it a **vapor**. Solid carbon dioxide (dry ice) lies on this

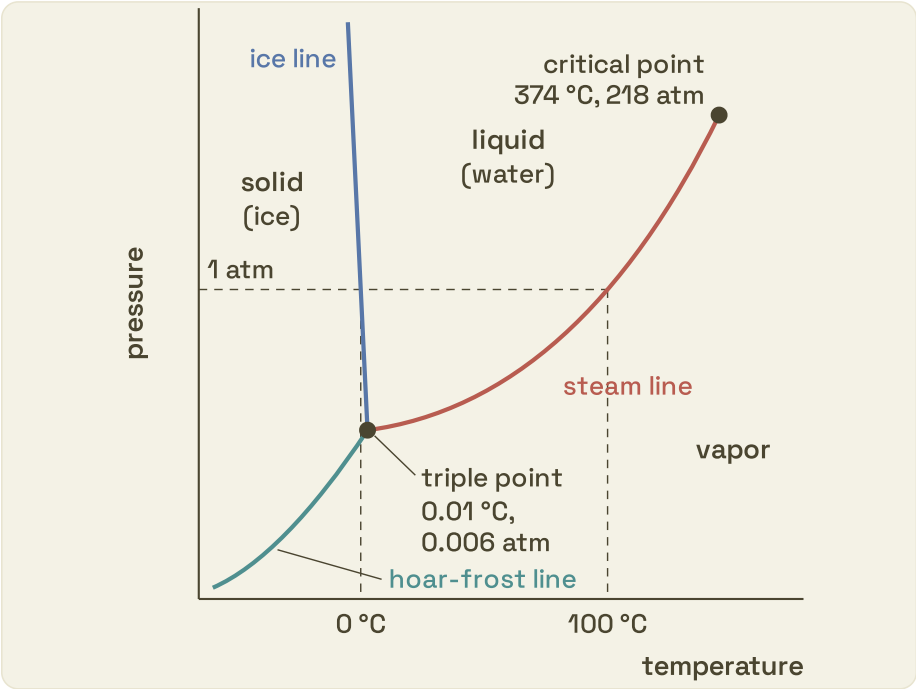


Figure 11.8 *The phase diagram of water (not to scale). The fusion curve (ice line) separates solid from liquid, the vaporization curve (steam line) liquid from vapor, and the sublimation curve (hoar-frost line) solid from vapor. All three meet at the triple point, 0.01 °C and 0.006 atm. The steam line ends at the critical point, 374 °C and 218 atm.*

side even at atmospheric pressure: at 1 atm it changes straight from solid to gas at −78.5 °C, which is why it is called "dry". Frost forming on a cold night is deposition, water vapor turning straight into ice.

Triple point

The triple point of a substance is the one temperature and pressure at which its solid, liquid and vapor can all exist together in equilibrium. On the phase diagram, it is the point where the vaporization, fusion and sublimation curves meet.

- Water: **0.006 atm** (611.657 Pa, or 4.58 mm of mercury) and **0.01 °C** (273.16 K).
- Carbon dioxide: **5.11 atm** and **−56.6 °C**.



Figure 11.9 *Dry ice (solid carbon dioxide) sublimes at room pressure: it turns straight into gas without melting. The white fog is water vapor from the air, condensed by the cold gas.*
Photo: Christopher from Salem, OR, USA, CC BY-SA 2.0

Because CO_2's triple-point pressure is above atmospheric pressure, liquid carbon dioxide can't exist at 1 atm: it only sublimes. Water's triple point can be set up in a sealed glass cell in any laboratory and gives exactly the same temperature every time; until 2019 it defined the kelvin, as 1/273.16 of the triple point's temperature.

Check 3

At the top of a very high mountain, water in an open pot boils at 70 °C. Compared with sea level, the temperature at which water freezes there is:

a much lower, about −30 °C
b almost the same, a tiny fraction of a degree higher
c much higher, about 30 °C
d the same as its boiling point

IN SHORT

Key ideas of Unit 11

- Heat equation $Q = ms\Delta\theta$. Heat capacity ms (J/K) belongs to a body; specific heat capacity s (J/(kg K)) to a substance. 1 cal = 4.2 J.
- For equal heat and mass, $\Delta\theta \propto 1/s$: water warms and cools slowly (coolant, hot-water bag); mercury, copper and oil quickly.
- Newton's law of cooling: $-\dfrac{dQ}{dt} = K(\theta - \theta_0)$; $\ln(\theta - \theta_0) = -\dfrac{K}{ms}t + C$, a straight line. For small temperature differences.
- Principle of calorimetry: heat lost = heat gained in an isolated system. Method of mixtures: $s = \dfrac{(m_w s_w + m_1 s_c)(\theta - \theta_1)}{m_s(\theta_2 - \theta)}$.
- Latent heat $Q = mL$: ice 3.36×10^5 J/kg, water 2.26×10^6 J/kg. It changes the potential energy of the particles, not the temperature. $L_v > L_f$.
- Boiling point rises with pressure (pressure cooker) and falls with height. Evaporation cools.
- Ice's melting point falls with pressure (regelation); wax's rises.
- Phase diagram: fusion, vaporization and sublimation curves meet at the triple point (water 0.01 °C, 0.006 atm).

Exercise

Choose the correct option

Check 4

1. Which of these is true of a substance with a low specific heat capacity?

a it heats up quickly and keeps its heat for a long time
b it heats up slowly and keeps its heat for a short time
c it heats up quickly and keeps its heat for a short time
d it heats up slowly and keeps its heat for a long time

Check 5

2. What is the specific heat capacity of water while it is boiling?

a 0
b 0.5
c 1
d infinite

Check 6

3. For Newton's law of cooling to hold, the temperature difference between a body and its surroundings must be:

a small
b large
c very large
d any value

Check 7

4. Which of these gives the most severe burn?

a boiling water **b** steam
c hot air **d** ordinary water

Check 8

5. A 10 kg iron bar (specific heat capacity 462 J/(kg K)) at 80 °C is placed on a block of ice at 0 °C (latent heat of fusion 3.36×10^5 J/kg). How much ice melts?

a 1.1 kg **b** 10 kg
c 16 kg **d** 60 kg

Check 9

6. Three liquids A, B and C are at 10 °C, 25 °C and 40 °C. When equal masses of A and B are mixed, the mixture is at 15 °C; when equal masses of B and C are mixed, it is at 30 °C. What will the temperature be when equal masses of A and C are mixed?

The textbook gives only the temperatures, without "equal masses", so the book's numbers don't work as given; with equal masses they do.

a 16 °C **b** 12 °C
c 8 °C **d** 4 °C

Check 10

7. Equal volumes of two liquids have densities in the ratio 1 : 3 and specific heat capacities in the ratio 3 : 1. The ratio of their heat capacities is:

a 1 : 1 **b** 1 : 3
c 1 : 9 **d** 9 : 1

Check 11

8. What happens to the melting point of ice?

a it decreases with a decrease in pressure **b** it increases with an increase in pressure
c it does not depend on pressure **d** it decreases with an increase in pressure

Check 12

9. The latent heat of fusion of ice is 3.36×10^5 J/kg and its specific heat capacity 2100 J/(kg K). How much heat changes 1 kg of ice at −10 °C into water at 50 °C? (Water 4200 J/(kg K).)

a 5.04×10^5 J **b** 5.67×10^5 J
c 6.51×10^5 J **d** 7.56×10^5 J

Check 13

10. Two blocks of ice pressed together join into one. Why?

a heat is produced **b** their specific heat capacity increases
c the melting point of ice falls **d** the melting point of ice rises

Check 14

11. The specific heat capacity of a substance is s_1 J/(kg K) when measured per kelvin and s_2 J/(kg °F) when measured per degree Fahrenheit. Which is true?

a $s_1 = s_2$
b $s_1 < s_2$
c $s_1 \leq s_2$
d $s_1 > s_2$

Check 15

12. Water falls from a height of 84 m. Half of its kinetic energy turns into heat in the water. What is the rise in temperature of the water when it strikes the bottom? (g = 10 m/s², water 4200 J/(kg K).)

a 0.10 °C
b 0.20 °C
c 0.30 °C
d 0.40 °C

Check 16

13. Two liquids are at 20 °C and 40 °C. When equal masses of them are mixed, the mixture is at 32 °C. What is the ratio of their specific heat capacities (the first liquid's to the second's)?

a 1/3
b 2/3
c 2/5
d 3/2

Check 17

14. 10 g of ice at −20 °C is dropped into a calorimeter holding 10 g of water at 10 °C. The specific heat capacity of water is twice that of ice. Ignoring the calorimeter, what does it contain at equilibrium?

a 10 g of ice and 10 g of water
b 20 g of water
c 5 g of ice and 15 g of water
d 20 g of ice

Check 18

15. A 1 kW immersion heater heats 1 liter of water in a kettle for 5 minutes, raising its temperature by 60 °C. What is the heat capacity of the kettle? (Water 4200 J/(kg K).)

a 400 J/K
b 600 J/K
c 800 J/K
d 1000 J/K

Answer these questions

16.
a. What does the statement "the specific heat capacity of water is 4200 J/(kg K)" mean?
b. Why are raw peanuts fried in a sand bath?
c. An 80 kg man with flu has a fever of 39.0 °C instead of his normal 37.0 °C. Taking the human body as mostly water, how much heat raised his temperature by this amount? (Water 4200 J/(kg K).)

17.
a. State and explain Newton's law of cooling.
b. A body at 100 °C cools to 60 °C in 5 minutes and to 40 °C in 10 minutes. What will its temperature be at the end of 15 minutes?

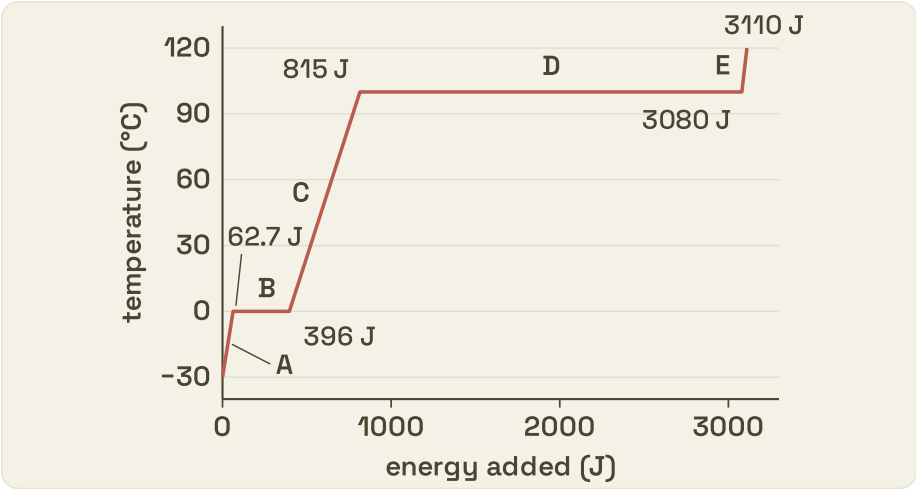


Figure 11.10 *For question 18: the temperature of 1.00 g of water, starting as ice at −30 °C, as energy is added, up to steam at 120 °C. The energy added at the end of each stage is marked.*

19.

a. To raise the temperature of an object, must you add heat to it? If you add heat to an object, must its temperature rise? Explain.

b. A nail driven into a board gets warmer. Ankit strikes an 8.00 g aluminium nail ten times with a 1.80 kg hammer moving at 7.80 m/s. If 60% of the hammer's kinetic energy turns into heat that stays in the nail, by how much does the nail's temperature rise? (Aluminium 910 J/(kg K).)

20.

a. A beaker of water at room temperature is placed in a closed chamber, and the air is slowly pumped out. When the pressure is low enough, the water begins to boil, but its temperature doesn't rise; in fact it falls slightly. Why?

b. 1 g of steam at 100 °C and 5 g of ice at 0 °C are mixed. What is the final result? (Water 4200 J/(kg K); latent heat of fusion of ice 3.36×10^5 J/kg; of vaporization of water 2.26×10^6 J/kg.)

21. a. Explain the role of latent heat in a change of phase, and how it affects the temperature during the change.
    b. Design an experiment to find the energy needed to change 1 kg of ice into water at 0 °C. Give the steps, the materials and equipment, the variables to keep the same, and the measurements to take.
    c. Suggest how the results might be interpreted, and what they mean for materials science.
    d. Discuss how pressure affects the melting point of substances.

Numerical problems

22. A 1 kW immersion heater heats 1 liter of water in a kettle for 5 minutes, and the temperature rises by 60 °C. Find the heat capacity of the kettle. (Water 4200 J/(kg K).)

23. A lead bullet hits a target at 500 m/s. Find the rise in its temperature, if 40% of the heat produced goes into heating the bullet. (Lead 126 J/(kg K).)

24. You are designing an electronic circuit element made of 23 mg of silicon. The current through it adds energy at 7.4×10^{-3} J/s. If the design lets no heat out of the element, how fast does its temperature rise? (Silicon 705 J/(kg K).)

25. Yangchen, a geologist working in the field, drinks her morning coffee from an aluminium cup of mass 0.120 kg at 20.0 °C. She pours in 0.300 kg of coffee at 70.0 °C. What is the final temperature, once the cup and coffee are at the same temperature? (Take coffee's specific heat capacity as water's, 4200 J/(kg K); aluminium 910 J/(kg K); no heat lost to the surroundings.)

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.