Unit 10

Thermal Expansion

This unit online, with its interactives:
science-playground.com/s/11f10

The steel deck of the Mugling bridge on the Prithvi Highway is 125 m long. Between a cold January dawn and a hot June afternoon it grows by about 3.5 cm, and nothing in the world can stop it: hold its ends fast and the steel pushes on them with millions of newtons. So the bridge is built to let it grow.

Nearly everything gets bigger when it is heated and smaller when it cools. This unit measures that growth exactly: in length, in area and in volume; for solids and for liquids; and the forces it sets up when it is stopped. You will see why a pendulum clock loses time in summer, why a thermostat clicks, why a full flask of mercury overflows when warmed, and how a U-tube of mercury measures a liquid's expansion without caring how the glass expands.

10.1 Introduction

A STORY FROM SCIENCE

Lord Kelvin, who found the bottom of the temperature scale

William Thomson (1824–1907) was born in Belfast and became Professor of Natural Philosophy at the University of Glasgow at 22; he held the post for 53 years. In 1848 he showed that there must be a **lowest possible temperature**, about −273 °C, where a body has given up all the thermal energy it can, and he built a temperature scale that starts from it: the **absolute** (Kelvin) scale, whose unit, the kelvin (K), you use in this unit.

He was as much an engineer as a physicist. He worked out how signals travel along very long undersea cables, invented the instruments that could detect their faint currents (a mirror galvanometer and a recorder), and sailed on the ships that laid the first lasting telegraph cables across the Atlantic in 1866, for which he was knighted. In 1892 he was made Baron Kelvin, taking the name of the river that flows past his university, and so became the first British scientist to sit in the House of Lords for his science.

Two things engineers do every day show that heat changes size:

- the long steel girders of a bridge are laid with **gaps** between them, or rest on rollers at one end, so that they can grow in summer without pushing the piers apart;
- a blacksmith makes the iron rim (tyre) of a wooden cart wheel slightly **smaller** than the wheel, heats it until it grows enough to slip over the wood, and lets it cool: it shrinks and grips the wheel tightly.

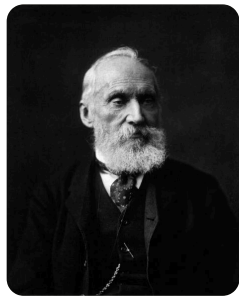


Figure 10.1 *William Thomson, Lord Kelvin, in his later years.*

Photo: "Photo by Messrs. Dickinson, London, New Bond Street" (according to http://www.sil.si.edu/DigitalCollections/hst/
Public domain

When a body is heated, its atoms or molecules vibrate (or move) more vigorously, and on average they sit a little farther apart. So the body grows in **length**, in **area** and in **volume**. This is **thermal expansion**. It happens in solids, liquids and gases. How much a substance expands depends on what it is made of: its structure and the strength of the forces between its particles. Gases expand most, liquids less, solids least.

Water is the famous exception: heated from 0 °C to 4 °C it **contracts**, and only above 4 °C does it expand as other liquids do.

Engineers, architects and instrument makers need to know exactly how much things expand: a railway, a bridge, a dam, a power line, a pendulum clock, a dental filling, a glass flask that must survive boiling water. This unit gives them the numbers.

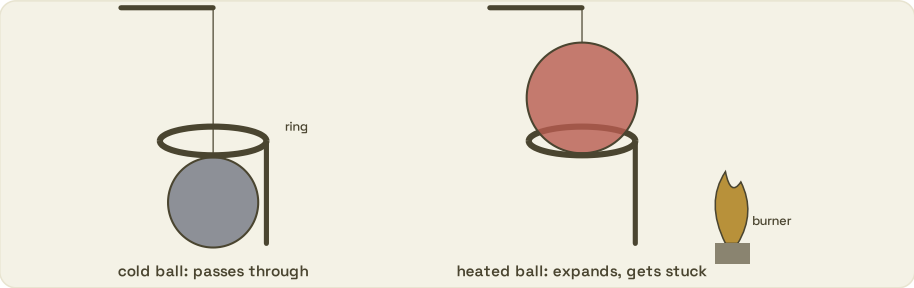


Figure 10.2 *The ball and ring. Cold, the ball just passes through the ring; heated, it expands and won't go through. (The expansion is drawn far bigger than it really is: a fraction of a millimeter is enough.)*

A STORY FROM SCIENCE

's Gravesande's ball

The Dutch physicist Willem Jacob 's Gravesande (1688–1742) of Leiden University was one of the first to teach physics by experiment, at a time when it was mostly taught from books. He had a workshop make dozens of pieces of apparatus to show the laws of nature to his students, and the ball and ring was one of them. Three hundred years later, the same demonstration is in nearly every school laboratory.

10.2 Thermal expansion

The two demonstrations show that when heat is given to a body, its temperature rises **and** its size increases: the ball grew, and both the flask and the water in it grew.

Why? In a solid, each atom is held in place by its neighbors, as if joined to them by springs, and it vibrates about that place. Heating gives the atoms more kinetic energy, so they vibrate with bigger amplitude. The "springs" are lopsided: squeezing two atoms closer than their usual spacing takes a large force, while pulling them apart takes much less. So an atom swinging harder can't go much closer to its neighbor but goes a good deal farther away, and on average the atoms sit a little farther apart. Every gap between neighbors grows a little, and the whole body grows. In a liquid or a gas, the particles also move faster and push each other farther apart.

The increase in the size of a body when it is heated is called **thermal expansion**. (When it cools, it shrinks: **thermal contraction**.)

CAREFUL

How to write it in an exam

The usual one-line answer: "On heating, the molecules gain kinetic energy and vibrate with larger amplitude, so the average distance between them increases and the body expands." It is right as far as it goes. The fuller reason is the lopsided force between atoms: if it pushed back equally both ways, the atoms would only swing harder about the same middle place, and the solid would not expand at all.

Check 1

A metal plate with a round hole in it is heated. The hole:

a gets smaller, because the metal expands into it
b gets bigger, in the same proportion as the metal
c stays the same size
d gets bigger in area but its diameter stays the same

Test yourself

10.3 Linear expansion

The increase in the **length** of a solid when it is heated is called **linear expansion**. It matters for long, thin things (wires, rods, rails, pipes, bridges), and it can also be seen in the diameter of a heated plate or coin.

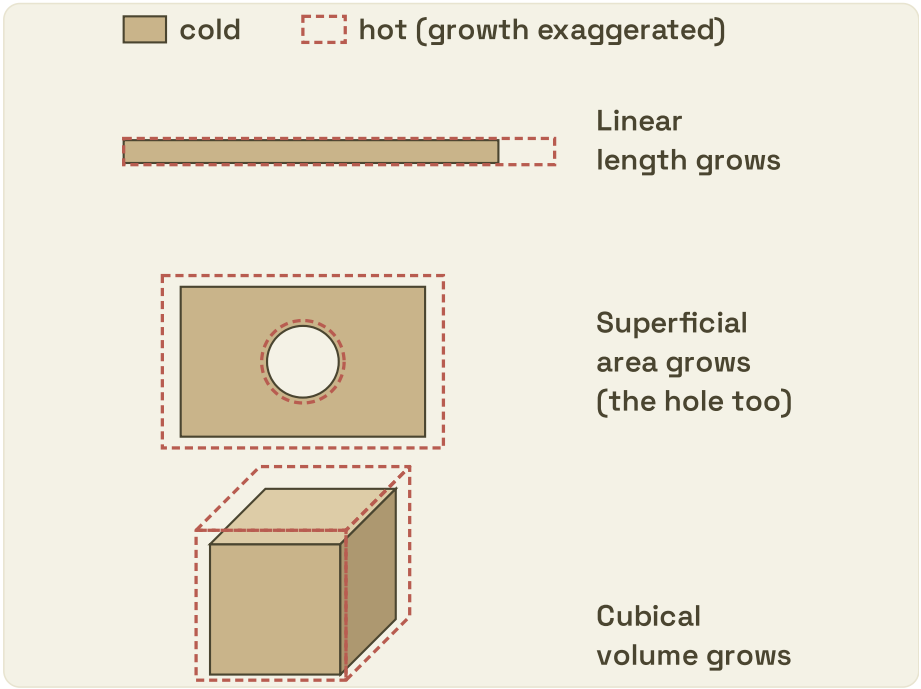


Figure 10.3 *The three kinds of thermal expansion: a rod gets longer (linear), a plate grows in area (superficial), a block grows in volume (cubical). The hot shapes are drawn far bigger than they really get: heating steel by 100 °C makes it only about 0.1% bigger each way.*

Take a metal rod of length l_1 at temperature θ_1, and heat it to θ_2; its length becomes l_2. Careful measurements show that the increase in length $l_2 - l_1$ is

1. **proportional to the original length**: $(l_2 - l_1) \propto l_1$. A rod twice as long grows twice as much, because each half grows by the same amount as the short rod;
2. **proportional to the rise in temperature**: $(l_2 - l_1) \propto (\theta_2 - \theta_1)$, as long as the rise is not very large.

Combining the two,

$$(l_2 - l_1) \propto l_1(\theta_2 - \theta_1) \quad \Rightarrow \quad l_2 - l_1 = \alpha\, l_1(\theta_2 - \theta_1)$$

$$\alpha = \frac{l_2 - l_1}{l_1(\theta_2 - \theta_1)} = \frac{\text{change in length}}{\text{original length} \times \text{change in temperature}} \tag{10.1}$$

The constant α (alpha) is the **coefficient of linear expansion**, or **linear expansivity**, of the material. Put $l_1 = 1$ unit and $\theta_2 - \theta_1 = 1$ degree, and $\alpha = l_2 - l_1$. So:

The linear expansivity of a solid is the increase in length per unit original length per degree rise in temperature.

- Its unit is $\mathbf{K^{-1}}$ (per kelvin), the same number as $°C^{-1}$, because a change of 1 K is a change of 1 °C. The unit of length cancels, so α is the same in meters or centimeters.

- It depends on the **material**, not on the rod's length or the shape of its cross-section. (It changes a little with temperature, so a table gives its value near room temperature.)

Rearranging (10.1) gives the length after heating:

$$l_2 - l_1 = \alpha\, l_1\, \Delta\theta \quad \Rightarrow \quad l_2 = l_1(1 + \alpha\, \Delta\theta) \tag{10.2}$$

where $\Delta\theta = \theta_2 - \theta_1$. For cooling, Δθ is negative and the same formula gives the shorter length.

Substance	α (K⁻¹ or °C⁻¹)
Quartz (fused)	0.05×10^{-5}
Invar (an iron–nickel alloy)	0.12×10^{-5}
Glass	$(0.3 – 0.9) \times 10^{-5}$
Steel	1.1×10^{-5}
Iron	1.2×10^{-5}
Gold	1.4×10^{-5}
Copper	1.7×10^{-5}
Brass	1.9×10^{-5}
Silver	1.9×10^{-5}
Aluminium	2.3×10^{-5}

The numbers are tiny: steel grows by about 11 millionths of its length for every degree. But multiplied by a long length and a large temperature change, they add up. (Values for one metal differ a little from book to book, since alloys vary; use the value a question gives.)

DID YOU KNOW?

The tower that grows in summer

The Eiffel Tower is about 300 m of wrought iron. Between a freezing winter day and a hot summer one it changes height by about 15 cm, and on a sunny day it leans slightly away from the sun, because its sunny side grows more than its shady side.

WORKED EXAMPLE

Mugling bridge

The Mugling bridge on the Prithvi Highway has a single steel span 125 m long. Find how much its steel deck grows when the temperature rises from 10 °C to 33 °C (α for steel = 1.2×10^{-5} °C⁻¹).

Answer

$l_1 = 125$ m, $\Delta\theta = 33 - 10 = 23$ °C (= 23 K), $\alpha = 1.2 \times 10^{-5}$ °C⁻¹.

$$\Delta l = l_1\, \alpha\, \Delta\theta = 125 \times 1.2 \times 10^{-5} \times 23 = 0.0345 \text{ m}$$

The deck grows by about **3.5 cm**: so its joints must leave at least that much room, and more, since the steel in the sun gets much hotter than the air.

Thermal expansion and the pendulum clock

A pendulum clock counts the swings of its pendulum. A simple pendulum of length l has a time period (one complete swing, there and back)

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Numerical problems

22. The distance between two points A and B on a railway is 625 km. Find the total space that must be left between the rails to allow for a change of temperature from 36 °F in winter to 117 °F in summer. (Linear expansivity of iron = 12×10^{-6} °C⁻¹.)

23. A brass rod 0.40 m long and a steel rod 0.60 m long, both at 0 °C, are heated to 85 °C. Both grow by the same amount. Find the linear expansivity of brass (α for steel = 12×10^{-6} °C⁻¹).

24. Find the temperature at which wood will just sink in benzene. At 0 °C the density of benzene is 9×10^2 kg/m³ and its cubical expansivity 1.2×10^{-3} K⁻¹; the density of the wood is 8.8×10^2 kg/m³ and its cubical expansivity 1.5×10^{-4} K⁻¹.

25. The markings on an aluminium ruler and a brass ruler are exactly in line at 0 °C. How far apart will their 20.0 cm marks be at 100 °C, if their left-hand ends are kept exactly in line? (Take α from the table in 10.3: aluminium 2.3×10^{-5} °C⁻¹, brass 1.9×10^{-5} °C⁻¹.)
The textbook prints no expansivities for this question, so the book's numbers don't work as given; with the table's values it gives the book's answer, 0.008 cm.

26. An underground tank with a capacity of 1700 liters (1.70 m³) is filled with ethanol at 19 °C. The ethanol cools to the temperature of the tank and the ground, 10 °C. How much air space will there be above the ethanol? (Take the tank's volume as unchanged; γ for ethanol = 75×10^{-5} K⁻¹.)

27. A wire 1.5 m long at 20 °C gets 1.9 cm longer when warmed to 420 °C.
    a. Find its average coefficient of linear expansion over this range.
    b. The wire is stretched just taut (zero tension) at 420 °C. Find the stress in it if it is cooled to 20 °C without being allowed to contract ($Y = 2 \times 10^{11}$ Pa).

28. A steel tape measure is accurate at 20 °C. It is used at 10 °C to measure a distance of exactly 1 km. What will the tape read? (α for steel = 12×10^{-6} K⁻¹)

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.

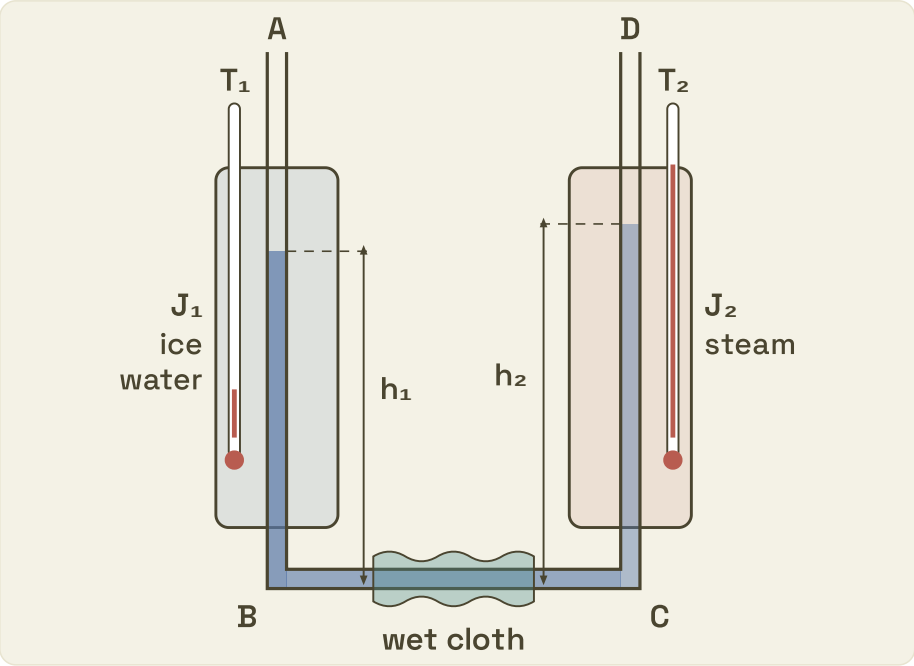


Figure 10.15 *For question 19: Dulong and Petit's apparatus.*

WORKED EXAMPLE

A rod measured by a scale that also expands

The length of an iron rod is measured with a brass scale. When both are at 10 °C, the reading is 50.0 cm. What will the brass scale read for the rod when both are at 40 °C? (α for brass $= 24 \times 10^{-6}$ °C^{-1}, α for iron $= 16 \times 10^{-6}$ °C^{-1})

Answer

The scale's divisions are correct (1 cm each) at 10 °C. Between 10 °C and 40 °C, $\Delta\theta = 30$ °C, and both the rod and the scale grow.

The rod's real length at 40 °C:

$$l_{40} = 50.0\,(1 + 16 \times 10^{-6} \times 30) = 50.0 \times 1.00048 = 50.024 \text{ cm}$$

One "centimeter" of the brass scale at 40 °C:

$$1 \times (1 + 24 \times 10^{-6} \times 30) = 1.00072 \text{ cm}$$

The reading is how many of these longer divisions fit along the rod:

$$\frac{50.024}{1.00072} = 49.988 \text{ cm} \approx \mathbf{49.99\ cm}$$

Another way: the scale and the rod both start at 50.0 cm. The scale grows by $50.0 \times 24 \times 10^{-6} \times 30 = 0.036$ cm and the rod by $50.0 \times 16 \times 10^{-6} \times 30 = 0.024$ cm. The scale has grown 0.012 cm more than the rod, so it **reads short** by about 0.012 cm: $50.0 - 0.012 = 49.988$ cm.

A scale that expands more than the thing it measures reads too little when hot; one that expands less reads too much.

YOUR TURN

A steel rail

A steel rail is 12 m long at 5 °C. How much longer is it at 45 °C, in the Tarai sun? Take α for steel from the table.

Answer

$\Delta l = l\,\alpha\,\Delta\theta = 12 \times 1.1 \times 10^{-5} \times 40 = 5.3 \times 10^{-3}$ m, about **5 mm**. That is why short rails were laid with gaps of a few millimeters between their ends.

where g is the acceleration due to gravity. The clock's gears are set so that each swing advances the hands by exactly T seconds; a **seconds pendulum**, with $T = 2$ s (one second each way), is about 99.4 cm long.

The pendulum rod is metal, so its length depends on the temperature:

- in **hot** weather the rod is longer, the period is **longer**, the pendulum makes fewer swings in a day, and the clock **loses** time (runs slow);
- in **cold** weather the rod is shorter, the period is shorter, and the clock **gains** time (runs fast).

How much? Let the clock keep correct time at θ_1, with length l and period T. At $\theta_2 = \theta_1 + \Delta\theta$ its length is $l' = l(1 + \alpha\Delta\theta)$ and its period

$$T' = 2\pi\sqrt{\frac{l(1+\alpha\Delta\theta)}{g}} = T(1+\alpha\Delta\theta)^{1/2} \approx T\left(1 + \tfrac{1}{2}\alpha\Delta\theta\right)$$

using $(1+x)^n \approx 1 + nx$ for a very small x. So each swing takes longer by a fraction $\dfrac{T'-T}{T} = \tfrac{1}{2}\alpha\Delta\theta$. The clock still counts T seconds for each swing, so it falls behind by that same fraction of every stretch of time:

$$\text{time lost (or gained)} = \tfrac{1}{2}\,\alpha\,\Delta\theta \times \text{the time it runs} \tag{10.3}$$

In one day (86 400 s), the loss is $\tfrac{1}{2}\alpha\Delta\theta \times 86\,400$ s. A rise in temperature makes it lose; a fall makes it gain.

WORKED EXAMPLE

A brass pendulum in summer

A seconds pendulum made of brass keeps correct time at 10 °C. How many seconds will it lose or gain per day when the temperature rises to 35 °C? (α for brass = $2 \times 10^{-5}\ \text{K}^{-1}$)

Answer

The temperature rises, so the pendulum gets longer and the clock **loses** time. $\Delta\theta = 35 - 10 = 25\ \text{K}$.

$$\text{loss per day} = \tfrac{1}{2}\,\alpha\,\Delta\theta \times 86\,400 = \tfrac{1}{2} \times 2 \times 10^{-5} \times 25 \times 86\,400 = 21.6\ \text{s}$$

It loses about **22 s a day**, more than two and a half minutes a week.

A STORY FROM SCIENCE

Clocks that ignore the weather

For two hundred years, the best clocks in the world were pendulum clocks, and their makers fought thermal expansion. In 1721 the English clockmaker George Graham put a jar of **mercury** at the bottom of his pendulum: when the rod grew longer and lowered the bob, the mercury expanded upward in its jar and lifted it back. A few years later John Harrison built the **gridiron** pendulum: rods of steel and brass side by side, joined so that the steel rods' expansion lowers the bob and the brass rods' expansion raises it. Their lengths were chosen so that the two expansions cancel exactly (Section 10.9 shows how). In 1896 the Swiss physicist Charles-Édouard Guillaume found **invar**, an alloy of iron and 36% nickel that hardly expands at all, and simple invar pendulums made the problem disappear. He won the 1920 Nobel Prize in Physics for it.

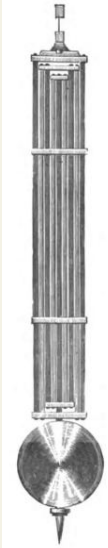


Figure 10.4 *A gridiron pendulum: its rods of two metals are arranged so that their expansions cancel, and the bob stays at the same distance below the pivot whatever the temperature.*
Photo: Unknown author, Public domain

each 1 m long with a cross-section of 1 m². To make them reach the same final length, which must be heated through more degrees?

e. A glass tube 133 cm long, of uniform cross-section, is to be partly filled with mercury so that the volume of the tube **not** occupied by mercury stays the same at all temperatures. The cubical expansivities of glass and mercury are 0.000 026 °C⁻¹ and 0.000 182 °C⁻¹. Find the length of the mercury column.

19.

a. A physics teacher asks a student to measure the real expansivity of mercury. What experiment can measure it without taking the expansion of the vessel into account?

b. Using the figure above, show that $\gamma = \dfrac{h_2 - h_1}{h_1\,\Delta\theta}$.

c. In such an apparatus, the liquid column is 65 cm high at 0 °C and 66.18 cm high at 100 °C. Find the real expansivity of the liquid.

20.

a. A brass rod measured with an aluminium scale reads shorter on a hot summer day than on a cold winter day. Why?

b. A disc has a hole at its center. When the disc is heated, what happens to the diameter of the hole?

c. A one-rupee coin has a diameter of 2.0 cm at 20 °C. It is made of an alloy (mostly copper) whose linear expansivity is $1.76 \times 10^{-5}\ \text{K}^{-1}$. What is its diameter i. on a hot day in Nepalgunj (48 °C), ii. on a cold night in the mountains of the Annapurna range (−23 °C)?

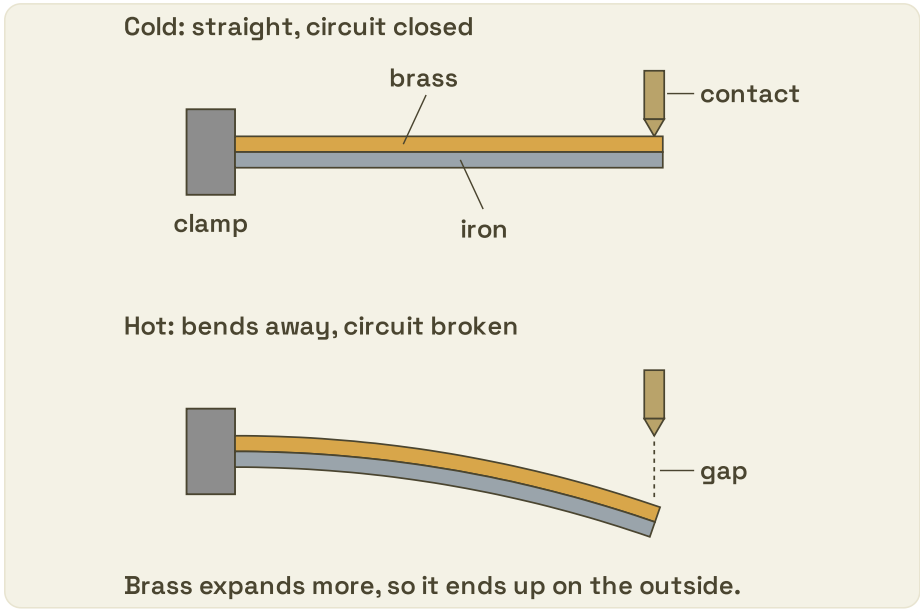


Figure 10.14 *For question 18d: a bimetallic strip used as a thermostat, cold and hot.*

Check 2

A pendulum clock that keeps correct time in spring is moved to a much colder room. It will:

a keep correct time, since the period does not depend on temperature
b gain time, because its pendulum gets shorter
c lose time, because its pendulum gets shorter
d stop, because the metal contracts

Sample questions

10.4 Superficial expansion

The increase in the **area** of a solid (a sheet, a plate, a disc) when it is heated is called **superficial** (or **areal**) **expansion**.

Take a metal sheet of area A_1 at θ_1, heated to θ_2, where its area is A_2. The increase in area is found to be proportional to the original area and to the rise in temperature:

$$(A_2 - A_1) \propto A_1(\theta_2 - \theta_1) \quad \Rightarrow \quad A_2 - A_1 = \beta\, A_1(\theta_2 - \theta_1)$$

$$\beta = \frac{A_2 - A_1}{A_1(\theta_2 - \theta_1)} = \frac{\text{change in area}}{\text{original area} \times \text{change in temperature}} \tag{10.4}$$

β (beta) is the **coefficient of superficial expansion**, or **superficial expansivity**.

The superficial expansivity of a solid is the increase in area per unit original area per degree rise in temperature. Its unit is K^{-1}. Like α, it depends on the material, not on the original area or the shape of the sheet.

The area after heating:

$$A_2 = A_1(1 + \beta\,\Delta\theta) \tag{10.5}$$

Check 3

A metal plate of area 2.000 m^2 is heated by 50 K. Its superficial expansivity is $4 \times 10^{-5}\ \text{K}^{-1}$. Its new area is:

a 2.002 m^2
b 2.004 m^2
c 2.040 m^2
d 2.400 m^2

10.5 Cubical expansion

The increase in the **volume** of a body when it is heated is called **cubical** (or **volume**) **expansion**. It is the only kind a liquid or a gas can have, since they take the shape of their container.

Take a metal cube of volume V_1 at θ_1, heated to θ_2, where its volume is V_2. The increase is proportional to the original volume and to the rise in temperature:

$$(V_2 - V_1) \propto V_1(\theta_2 - \theta_1) \quad \Rightarrow \quad V_2 - V_1 = \gamma\, V_1(\theta_2 - \theta_1)$$

$$\gamma = \frac{V_2 - V_1}{V_1(\theta_2 - \theta_1)} = \frac{\text{change in volume}}{\text{original volume} \times \text{change in temperature}} \tag{10.6}$$

γ (gamma) is the **coefficient of cubical expansion**, or **cubical expansivity**.

The cubical expansivity of a substance is the increase in volume per unit original volume per degree rise in temperature. Its unit is K^{-1}; it depends on the substance, not on its volume or shape.

The volume after heating:

$$V_2 = V_1(1 + \gamma\,\Delta\theta) \tag{10.7}$$

CAREFUL

A hollow body expands like a solid one

A hollow metal sphere and a solid one of the same metal and the same outside size expand **by the same amount** when heated equally. The hollow inside grows exactly as if it were filled with the same metal, as the hole in a plate does. So the same γ gives the volume inside a flask or a tank: it grows as if the glass or steel were solid.

Sample questions

10.6 Relation among α, β and γ

If you know α for a solid, can you find β and γ without another experiment? Yes: area and volume are made of lengths, and every length grows by the same fraction.

β = 2α

Take a square metal sheet of side l_1 at θ_1, so its area is $A_1 = l_1^2$. Heat it by $\Delta\theta$. Each side becomes $l_2 = l_1(1 + \alpha\Delta\theta)$, and the area becomes $A_2 = l_2^2$.

1. From the definition of β (10.5): $A_2 = A_1(1 + \beta\Delta\theta)$, that is, $l_2^2 = l_1^2(1 + \beta\Delta\theta)$.
2. Put in $l_2 = l_1(1 + \alpha\Delta\theta)$: $l_1^2(1 + \alpha\Delta\theta)^2 = l_1^2(1 + \beta\Delta\theta)$.
3. Cancel l_1^2 and expand the square: $1 + 2\alpha\Delta\theta + \alpha^2\Delta\theta^2 = 1 + \beta\Delta\theta$.
4. α is tiny (about $10^{-5}\ \mathrm{K}^{-1}$), so $\alpha^2\Delta\theta^2$ is far smaller than $2\alpha\Delta\theta$ (their ratio is $\alpha\Delta\theta/2$, about 1/2000 for steel heated by 100 K): neglect it. Then $2\alpha\Delta\theta = \beta\Delta\theta$, and

$$\boxed{\beta = 2\alpha} \tag{10.8}$$

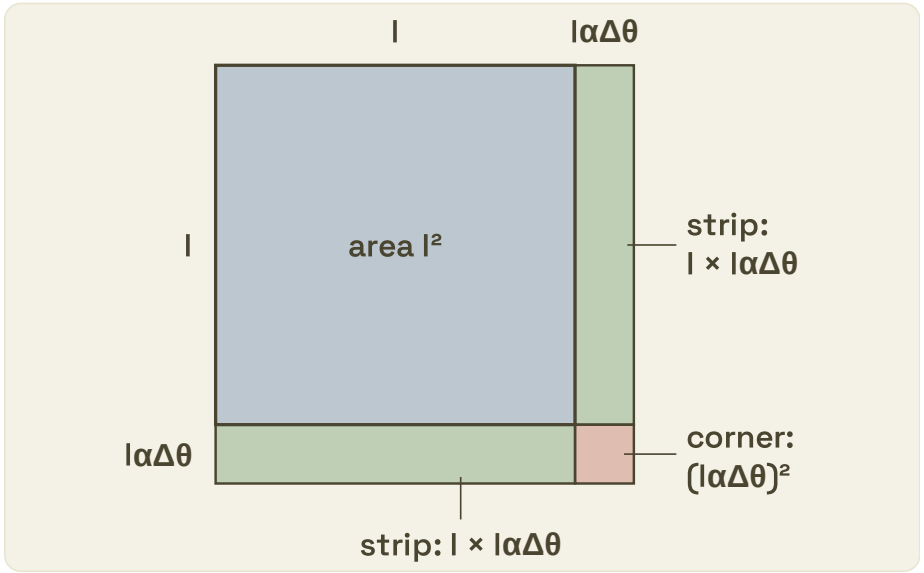


Figure 10.5 *Why β = 2α. The square of side l grows by a thin strip lαΔθ along two of its sides: two strips, each of area $l^2\alpha\Delta\theta$, add $2\alpha\Delta\theta$ to every unit of area. The tiny corner square, $(l\alpha\Delta\theta)^2$, is smaller than a strip by the factor αΔθ: about a thousand times smaller for steel heated by 100 K, too small to count. (The growth is drawn about 150 times bigger than it is for 100 K.)*

c. A hole 2 cm in diameter is drilled through a piece of steel at 30 °C. What happens to the diameter of the hole when the steel is heated to 80 °C? (α for steel = $1.2 \times 10^{-5}\ ^\circ\mathrm{C}^{-1}$)

17.

a. Define the coefficients of linear and cubical expansion of a material.

b. A solid cube of metal is heated. Derive the relation between its coefficient of linear expansion and its coefficient of cubical expansion.

c. Would the coefficient of cubical expansion change if the same solid cube were made into a hollow cube?

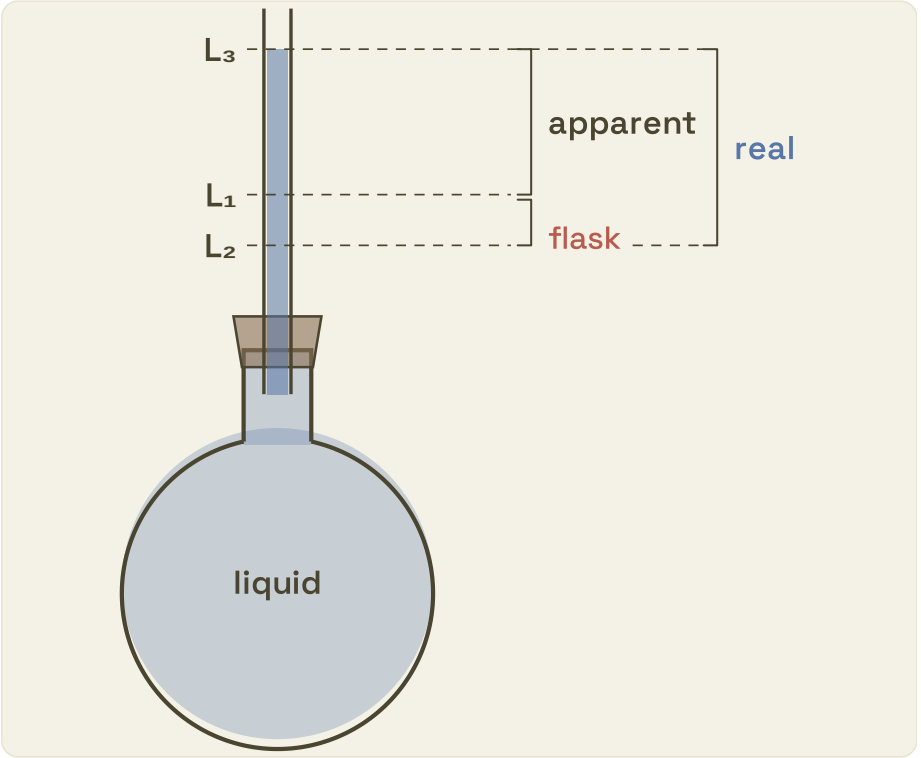


Figure 10.13 *For question 18a: the levels L_1, L_2, L_3 in the tube of a heated flask.*

Check 17

9. A disc ($\alpha = 1.9 \times 10^{-5}\ \text{K}^{-1}$) has a hole of diameter 1.5 cm at 20 °C. What happens to the diameter of the hole when the disc is heated to 150 °C?

a. it increases by 0.002 cm
b. it decreases by 0.002 cm
c. it increases by 0.004 cm
d. it decreases by 0.004 cm

Check 18

10. The coefficient of apparent expansion of mercury in a glass vessel is $153 \times 10^{-6}\ \text{K}^{-1}$, and in a steel vessel $144 \times 10^{-6}\ \text{K}^{-1}$. If the linear expansivity of steel is $12 \times 10^{-6}\ °\text{C}^{-1}$, the linear expansivity of glass is:

The textbook prints $114 \times 10^{-6}\ K^{-1}$ for the steel vessel, but then glass would have a negative expansivity, so the book's numbers don't work; $144 \times 10^{-6}\ K^{-1}$ gives its answer.

a. $6 \times 10^{-6}\ °\text{C}^{-1}$
b. $9 \times 10^{-6}\ °\text{C}^{-1}$
c. $27 \times 10^{-6}\ °\text{C}^{-1}$
d. $36 \times 10^{-6}\ °\text{C}^{-1}$

Check 19

11. A glass flask of volume 1 liter is full of mercury at 0 °C. Both are heated to 100 °C. How much mercury spills out? (Cubical expansivity of mercury $1.82 \times 10^{-4}\ °\text{C}^{-1}$, linear expansivity of glass $0.1 \times 10^{-4}\ °\text{C}^{-1}$.)

a. 1.52 cm³
b. 2.12 cm³
c. 15.2 cm³
d. 21.2 cm³

Check 20

12. A liquid of cubical expansivity γ is heated in a container whose linear expansivity is $\gamma/3$. The level of the liquid will:

a. rise
b. fall
c. stay where it is
d. it is difficult to say

Check 21

13. The coefficient of linear expansion of a substance is $18 \times 10^{-5}\ °\text{C}^{-1}$. What is it on the Fahrenheit scale?

a. $32 \times 10^{-5}\ °\text{F}^{-1}$
b. $18 \times 10^{-5}\ °\text{F}^{-1}$
c. $9 \times 10^{-5}\ °\text{F}^{-1}$
d. $1 \times 10^{-4}\ °\text{F}^{-1}$

Check 22

14. An iron rod of radius 2.4 cm ($Y = 2 \times 10^{12}$ dyne/cm², $\alpha = 1.2 \times 10^{-5}\ °\text{C}^{-1}$) is heated from 0 °C to 50 °C. What force does it exert if it is prevented from expanding?

a. 4.34×10^{5} N
b. 2.17×10^{5} N
c. 3.12×10^{5} N
d. 2.17×10^{4} N

Check 23

15. The side of a cube increases by 0.50% when its temperature changes by 100 °C. By how much does its volume increase?

a. 0.05%
b. 1.0%
c. 1.5%
d. 2%

Answer these questions

16. a. Why does a bimetallic strip bend to one side when heated?
b. A solid sphere is heated from θ_1 to θ_2. Find a relation between its final and initial diameters.

γ = 3α

Take a metal cube of side l_1, volume $V_1 = l_1^3$. Heated by $\Delta\theta$, its side becomes $l_2 = l_1(1 + \alpha\Delta\theta)$ and its volume $V_2 = l_2^3$.

1. From the definition of γ (10.7): $l_2^3 = l_1^3(1 + \gamma\Delta\theta)$.
2. Put in l_2: $l_1^3(1 + \alpha\Delta\theta)^3 = l_1^3(1 + \gamma\Delta\theta)$.
3. Expand the cube, $(1 + x)^3 = 1 + 3x + 3x^2 + x^3$: $1 + 3\alpha\Delta\theta + 3\alpha^2\Delta\theta^2 + \alpha^3\Delta\theta^3 = 1 + \gamma\Delta\theta$.
4. Neglect the terms in α^2 and α^3, which are far smaller: $3\alpha\Delta\theta = \gamma\Delta\theta$, and

$$\boxed{\gamma = 3\alpha} \tag{10.9}$$

The picture is the same as for the square: the cube grows by three thin slabs, one on each of three faces; the thin edge pieces and the corner are too small to count.

Together:

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}, \qquad \alpha : \beta : \gamma = 1 : 2 : 3$$

These hold for an **isotropic** solid, one that expands equally in every direction (metals, glass). A crystal like quartz or a piece of wood expands differently along different directions, and then γ is the sum of its three linear expansivities.

WORKED EXAMPLE

A vessel that never overflows

A cylindrical vessel of uniform inside diameter and linear expansivity α is filled with a liquid. On heating, the liquid does not overflow. Find the largest possible cubical expansivity of the liquid.

Answer

The space inside grows with the vessel's cubical expansivity, 3α. The liquid doesn't overflow, so its volume grows no more than the space does: $\gamma_{\text{liquid}} \le 3\alpha$. The largest possible value is **3α**.

Sample questions

Check 4

3. Which of the following is correct for an isotropic solid?

a. $\gamma = \frac{3}{2}\alpha$
b. $\gamma = \frac{2}{3}\alpha$
c. $\gamma = \frac{3}{2}\beta$
d. $\gamma = \frac{2}{3}\beta$

Check 5

4. The relation among the three coefficients of thermal expansion of a solid is:

a. $\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$
b. $\alpha = 2\beta = 3\gamma$
c. $\frac{1}{\alpha} = \frac{1}{\beta} = \frac{1}{\gamma}$
d. $\frac{1}{\alpha} = 2\beta = \frac{1}{3\gamma}$

Check 6

5. The volume of a metal block increases by 0.12% when it is heated through 20 °C. The linear expansivity of the metal is:

a. $1 \times 10^{-5}\ °\text{C}^{-1}$
b. $1.5 \times 10^{-5}\ °\text{C}^{-1}$
c. $2 \times 10^{-5}\ °\text{C}^{-1}$
d. $2.5 \times 10^{-5}\ °\text{C}^{-1}$

10.7 Determination of linear expansivity of a solid: Pullinger's apparatus

A rod 1 m long heated by 80 °C grows by only about a millimeter. To measure α, we must measure that millimeter to a hundredth of a millimeter, and Pullinger's apparatus does it with a **spherometer**, a screw gauge that reads to 0.01 mm.

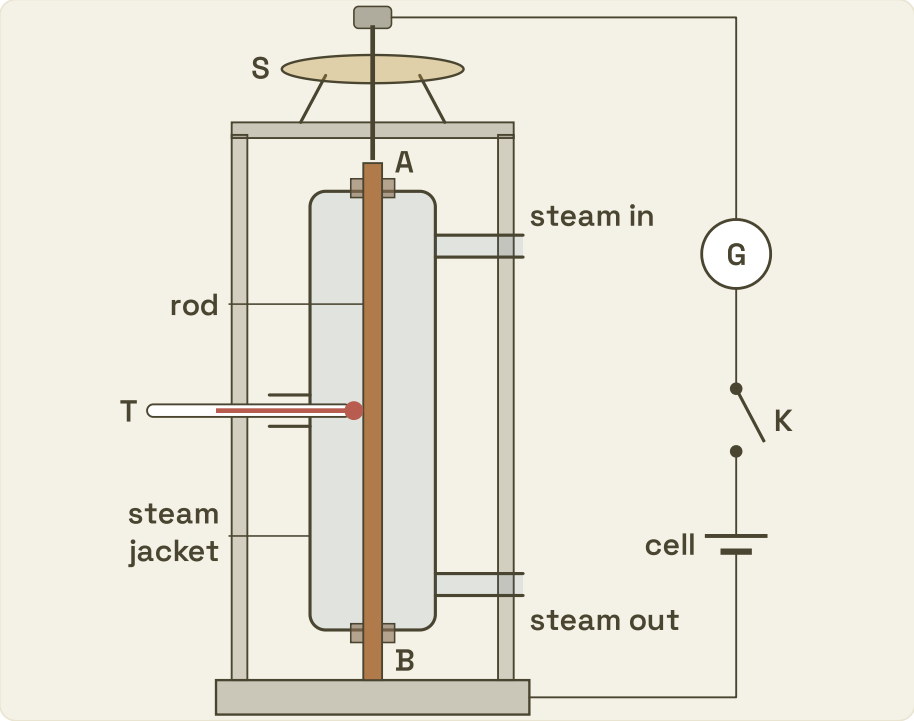


Figure 10.6 *Pullinger's apparatus. The rod AB stands inside a steam jacket on a fixed base, its lower end B against the base. A spherometer S sits on top; its central screw is brought down until it just touches the rod's top A, which closes a circuit (cell, key K and galvanometer G) through the rod, so G deflects at the moment of contact. Steam enters at the top and leaves at the bottom; a thermometer T through the middle opening reads the rod's temperature.*

YOUR TURN

A brass rod in Pullinger's apparatus

A brass rod 80.0 cm long at 20 °C is heated by steam to 99 °C. The spherometer readings are 3.250 mm and 4.452 mm. Find α for brass.

Answer

$\Delta l = 4.452 - 3.250 = 1.202 \text{ mm} = 0.1202 \text{ cm}$; $\Delta\theta = 99 - 20 = 79$ °C.

$$\alpha = \frac{0.1202}{80.0 \times 79} = 1.9 \times 10^{-5} \text{ °C}^{-1}$$

Check 9

1. What happens to the density of a liquid when its temperature rises?

a it increases
b it decreases
c it stays the same
d it can't be predicted

Check 10

2. A metal piece is weighed while hanging in a liquid whose temperature is raised steadily. Its apparent weight:

a decreases
b increases
c stays the same
d changes erratically

Check 11

3. When does 5 liters of benzene weigh more?

a in winter
b in summer
c the same in summer and in winter
d it can't be predicted

Check 12

4. Two rods of different materials expand by the same amount for the same rise in temperature. The condition is:

a $l_1\alpha_1 = l_2\alpha_2$
b $l_1\alpha_2 = l_2\alpha_1$
c $l_1^2\alpha_1 = l_2^2\alpha_2$
d $l_2^2\alpha_1 = l_1^2\alpha_2$

Check 13

5. Two spheres of the same size and material, one solid and one hollow, are heated through the same range of temperature. How do they expand?

a both expand equally
b the solid one expands more
c the hollow one expands more
d it can't be predicted

Check 14

6. A bimetallic thermostat is made of strips A and B with linear expansivities $\alpha_A < \alpha_B$. On heating, the strip:

a bends with A on the outer side
b bends with B on the outer side
c does not bend at all
d twists about its length

Check 15

7. A clock with a metal pendulum ($\alpha = 2 \times 10^{-5}$ K^{-1}) keeps correct time at 20 °C. The temperature falls to 10 °C. How many seconds does the clock lose or gain per day?

a loses 8.64 s
b gains 8.64 s
c loses 19.28 s
d gains 19.28 s

Check 16

8. A thin copper wire of length L gets 1% longer when heated from T_1 to T_2. By what percentage does the area of a thin copper plate measuring 2L × L change when heated from T_1 to T_2?

a 1%
b 2%
c 3%
d 4%

A height of liquid is in inverse proportion to its density. From (10.16), $\rho_2 = \dfrac{\rho_1}{1+\gamma\Delta\theta}$, with $\Delta\theta = \theta_2 - \theta_1$ and γ the **real** expansivity:

$$\frac{h_1}{h_2} = \frac{1}{1+\gamma\Delta\theta} \quad\Rightarrow\quad h_1(1+\gamma\Delta\theta) = h_2 \quad\Rightarrow\quad h_1\gamma\Delta\theta = h_2 - h_1$$

$$\boxed{\gamma = \frac{h_2 - h_1}{h_1\,\Delta\theta}} \tag{10.17}$$

Every quantity on the right is measured. And because the pressure of a column doesn't depend on its width, the expansion of the glass (which widens the tube) doesn't enter at all: this gives the **real** expansivity directly.

WORKED EXAMPLE

Two limbs at 0 °C and 100 °C

A U-tube containing a liquid has its two limbs kept at 0 °C and 100 °C. When everything is steady, the liquid columns are 60 cm and 65 cm high. Find the liquid's coefficient of real expansion.

Answer

$h_1 = 60$ cm (cold, 0 °C), $h_2 = 65$ cm (hot, 100 °C), $\Delta\theta = 100$ °C.

$$\gamma = \frac{h_2 - h_1}{h_1\Delta\theta} = \frac{65 - 60}{60 \times 100} = \mathbf{8.3 \times 10^{-4}\ °C^{-1}}$$

DID YOU KNOW?

Dulong and Petit's other law

The same two physicists found, in 1819, that one mole of almost any solid element needs about the same heat, near 25 J, to warm it by 1 K. The law of Dulong and Petit let chemists check atomic masses for decades, and its failures at low temperatures were one of the first clues, a century later, to quantum physics.

IN SHORT

Key ideas of Unit 10

- Heated, a body expands: its particles vibrate harder and sit farther apart on average.
- Linear: $\alpha = \dfrac{\Delta l}{l\Delta\theta}$, $l_2 = l_1(1+\alpha\Delta\theta)$. Superficial: $A_2 = A_1(1+\beta\Delta\theta)$. Cubical: $V_2 = V_1(1+\gamma\Delta\theta)$. Units $\mathrm{K^{-1}}$ (= $\mathrm{°C^{-1}}$).
- For an isotropic solid $\beta = 2\alpha$, $\gamma = 3\alpha$: $\alpha : \beta : \gamma = 1 : 2 : 3$.
- A pendulum clock loses $\frac{1}{2}\alpha\Delta\theta$ of its time when warmer (gains when colder).
- Pullinger's apparatus: $\alpha = \dfrac{S_2 - S_1}{l(\theta_2 - \theta_1)}$.
- Thermal stress $Y\alpha\Delta\theta$; thermal force $F = YA\alpha\Delta\theta$, independent of length.
- Bimetallic strip bends with the more expanding metal outside; constant difference in length when $l_1\alpha = l_1'\alpha'$.
- Liquids: $\gamma_r = \gamma_a + \gamma_g$; density $\rho_2 = \dfrac{\rho_1}{1+\gamma\Delta\theta}$; Dulong and Petit: $\gamma = \dfrac{h_2 - h_1}{h_1\Delta\theta}$.

Exercise

Choose the correct option

10.8 Force set up by expansion or contraction: thermal stress

Heat a metal bar and it expands. But hold its ends between two rigid walls so that it **can't** expand, and the bar pushes on the walls with a large force. That force is exactly the force needed to squeeze the bar back from the length it wanted to have to the length it is allowed. In the same way, a metal wire stretched between two rigid poles and then **cooled** tries to contract, can't, and pulls on the poles.

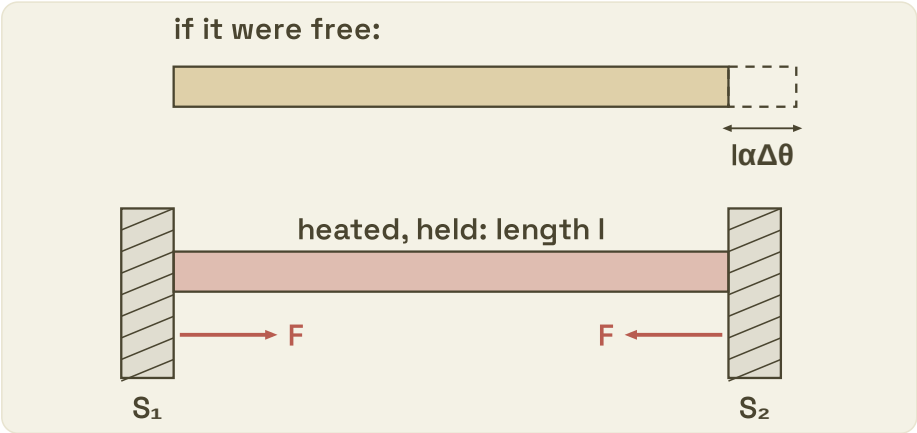


Figure 10.7 *A bar of length l clamped between two rigid supports S_1 and S_2 and heated from θ_1 to θ_2. It would grow by $l\alpha\Delta\theta$ if it were free (dashed); held, it pushes outward on the supports, and they push back on it with an equal force F.*

The force set up in a body when it is prevented from expanding on heating, or from contracting on cooling, is called the force set up by expansion (or contraction), or the thermal force.

How big? From Unit 8, Young's modulus is $Y = \dfrac{\text{stress}}{\text{strain}} = \dfrac{F/A}{\Delta l/l}$. The supports stop a growth $\Delta l = l\alpha\Delta\theta$, so they must compress the bar by a **strain** of

$$\frac{\Delta l}{l} = \alpha\Delta\theta \quad \text{(the thermal strain)}$$

and the stress needed is

$$\text{thermal stress} = \frac{F}{A} = Y\alpha\Delta\theta, \qquad F = YA\alpha\Delta\theta \tag{10.10}$$

where A is the area of cross-section. The same expressions give the tension in a wire that is held when it cools.

Notice that the force does **not depend on the length**: a long rail and a short one, equally heated and held, push equally hard. A longer rail would grow more, but it also needs more squeezing to shorten it by that much: the two cancel.

WORKED EXAMPLE

A steel wire cooled between two supports

A steel wire 4 m long and 2 mm in diameter is fixed tightly to two rigid supports. Find the increase in its tension when its temperature falls by 10 °C (α for steel $= 1.2 \times 10^{-5}$ K^{-1}, Y for steel $= 2 \times 10^{11}$ N/m^2).

Answer

Area of cross-section: $A = \dfrac{\pi d^2}{4} = \dfrac{3.14 \times (2 \times 10^{-3})^2}{4} = 3.14 \times 10^{-6}$ m^2.

$$F = Y A \alpha \Delta\theta = 2 \times 10^{11} \times 3.14 \times 10^{-6} \times 1.2 \times 10^{-5} \times 10 = \mathbf{75\ N}$$

That is about the weight of a 7.7 kg mass, from a fall of only 10 °C. The 4 m length isn't needed: the force doesn't depend on it.

DID YOU KNOW?

A force you can't stop

A steel rail with a cross-section of 75 cm^2 (a typical rail), held so it can't expand, heated by 40 °C: $F = Y A \alpha \Delta\theta = 2 \times 10^{11} \times 75 \times 10^{-4} \times 1.1 \times 10^{-5} \times 40 \approx 660\,000$ N. That is the weight of about 66 tonnes, pushing on whatever holds the rail's ends. When the hold fails, the rail bends sideways: a buckled track, a **sun kink**.



Figure 10.8 *Rails buckled by the heat. Held so that they couldn't grow, they pushed themselves sideways out of line.*
Photo: The original uploader was Trainwatcher at English Wikipedia ., Public domain

10.9 Differential expansion

The bimetallic strip

Weld or rivet together, along their whole length, two thin strips of different metals, brass and steel, say: a **bimetallic strip**. Brass expands more than steel. Heated, the brass gets longer than the steel, but the two are joined, so the strip **bends into an arc with the brass on the outside** (the longer strip goes round the bigger curve). Cooled below the temperature at which it is straight, it bends the other way, with the steel on the outside.

A strip like this, with one end fixed and the other touching a contact, is a **thermostat**: when the temperature rises past a set value, the strip bends away and **breaks the circuit**; as it cools, it straightens and closes it again. Thermostats keep the temperature steady in electric irons, rice cookers, water heaters, room heaters, ovens and aquarium heaters, and act as a safety cut-out against overheating.

The difference in the expansions of two bodies of different expansivities, heated through the same range of temperature, is called **differential expansion**.

10.12 Dulong and Petit's experiment: the real expansivity of a liquid

In 1817 the French physicists Pierre Dulong and Alexis Petit found a way to measure a liquid's **real** expansivity with no correction at all for its container. The trick: compare two columns of the same liquid by their **pressure**, which depends only on height and density, never on the width of the tube.

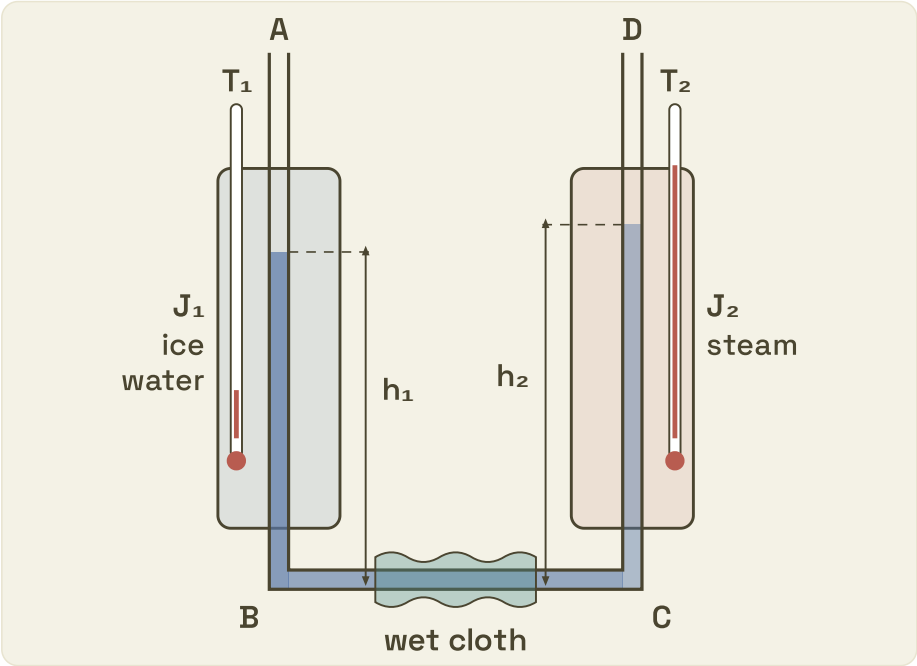


Figure 10.12 *Dulong and Petit's apparatus. A U-tube ABCD holds the liquid. Limb AB is kept cold (ice water in jacket J_1, thermometer T_1) and limb CD hot (steam through jacket J_2, thermometer T_2); the cross-tube BC is kept at one temperature by a wet cloth. The hot, less dense liquid stands higher: h_2 against h_1. (Drawn with the heights of the example below, 60 cm and 65 cm; for mercury between 0 °C and 100 °C the difference is under 2%.)*

The method:

1. Fill a U-shaped glass tube ABCD with the liquid (mercury, say).
2. Surround the limbs AB and CD with glass jackets J_1 and J_2.
3. Pass ice water through J_1, to keep the liquid in AB at a low temperature θ_1.
4. Pass steam through J_2, to keep the liquid in CD at a high temperature θ_2.
5. Read the two temperatures on the thermometers T_1 and T_2 in the jackets.
6. Keep the horizontal tube BC cool and at one temperature with a wet cloth over which cold water is sprayed, so heat doesn't pass from one limb to the other.
7. Wait until everything is steady: the hot liquid in CD has expanded and stands higher, and the cold liquid in AB lower.
8. Measure the heights of the two columns above BC, h_1 (cold) and h_2 (hot), and the temperatures.

The theory. The liquid in BC is at rest, so the pressure at B equals the pressure at C. Both columns have the same air pressure on top, so their own pressures are equal:

$$\rho_1 g h_1 = \rho_2 g h_2 \quad \Rightarrow \quad \frac{h_1}{h_2} = \frac{\rho_2}{\rho_1}$$

WORKED EXAMPLE

Mercury overflows

A glass flask of volume 400 cm³ is just filled with mercury at 0 °C. How much mercury overflows when the flask and the mercury are heated to 80 °C? (Cubical expansivity of glass = 2.7×10^{-5} °C^{-1}, of mercury = 18×10^{-5} °C^{-1}.)

Answer

$V_0 = 400$ cm³, $\Delta\theta = 80$ °C.

Volume of the mercury at 80 °C: $V_{\text{Hg}} = 400(1 + 18 \times 10^{-5} \times 80) = 400 \times 1.0144 = 405.760$ cm³.

Volume inside the flask at 80 °C: $V_{\text{flask}} = 400(1 + 2.7 \times 10^{-5} \times 80) = 400 \times 1.00216 = 400.864$ cm³.

$$\text{overflow} = 405.760 - 400.864 = \mathbf{4.9\ cm^3}$$

Shortcut: the overflow is the apparent expansion, $V_0\gamma_a\Delta\theta = V_0(\gamma_r - \gamma_g)\Delta\theta = 400 \times 15.3 \times 10^{-5} \times 80 = 4.9$ cm³.

Check 8

A liquid shows no change of level at all when heated in a vessel. Then:

a the liquid does not expand

b the liquid's real expansivity equals the vessel's cubical expansivity

c the liquid's apparent expansivity equals its real expansivity

d the vessel does not expand

Sample questions

10.11 Variation of density with temperature

When a substance is heated, its **mass** stays the same while its **volume** increases. Density is mass ÷ volume, so the density **decreases**.

Let a mass m have volume V_1 and density $\rho_1 = m/V_1$ at θ_1, and volume $V_2 = V_1(1 + \gamma\Delta\theta)$ at θ_2. Then

$$\rho_2 = \frac{m}{V_2} = \frac{m}{V_1(1 + \gamma\Delta\theta)} \quad \Rightarrow \quad \rho_2 = \frac{\rho_1}{1 + \gamma\Delta\theta} \tag{10.16}$$

Because $\gamma\Delta\theta$ is small, this is very nearly $\rho_2 \approx \rho_1(1 - \gamma\Delta\theta)$: the density falls by the same fraction as the volume grows.

The density of a substance falls as its temperature rises. (Water between 0 °C and 4 °C is the exception: there its volume falls and its density rises, to its greatest at 4 °C.)

WORKED EXAMPLE

Silver at 100 °C

The density of silver at 0 °C is 10 310 kg/m³, and its linear expansivity is 1.9×10^{-5} °C^{-1}. Find its density at 100 °C.

Answer

$\gamma = 3\alpha = 3 \times 1.9 \times 10^{-5} = 5.7 \times 10^{-5}$ °C^{-1}; $\Delta\theta = 100$ °C.

$$\rho_2 = \frac{\rho_1}{1 + \gamma\Delta\theta} = \frac{10\,310}{1 + 5.7 \times 10^{-5} \times 100} = \frac{10\,310}{1.0057} = \mathbf{10\,252\ kg/m^3}$$

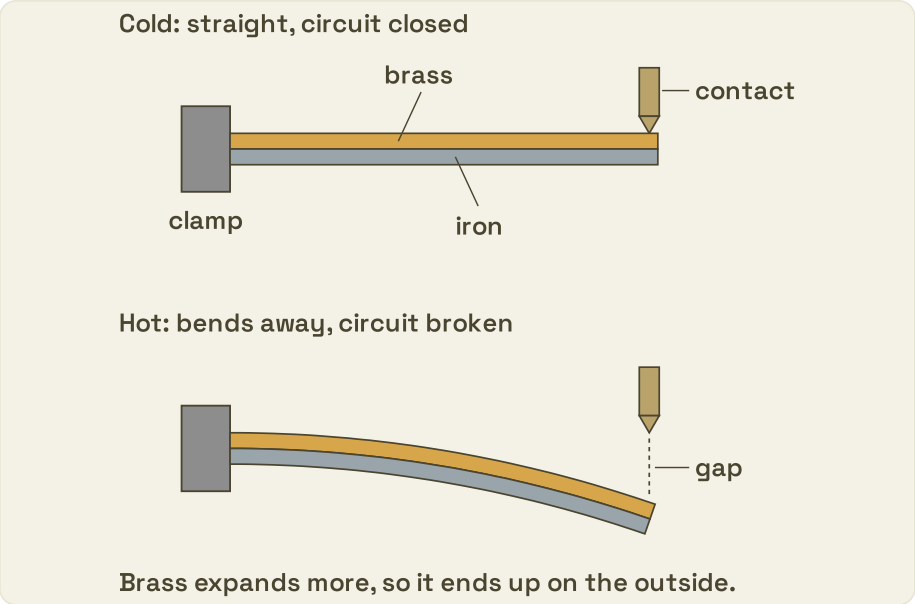


Figure 10.9 *A bimetallic strip used as a thermostat. Cold, it is straight and its tip touches the contact, so current flows. Hot, the brass grows more than the iron (or steel), the strip bends with the brass on the outside, and the contact breaks.*

Check 7

A bimetallic strip is made of strips A and B, with $\alpha_A < \alpha_B$. When it is heated, it:

a bends with A on the outside

b bends with B on the outside

c stays straight

d twists about its length

Zero differential expansion

Can two rods of different metals keep the **same difference in length** at every temperature? Yes, if their **growths are equal**.

Rod A has length l_1 and expansivity α; rod B has length l_1' and expansivity α'. Heated by $\Delta\theta$:

$$l_2 = l_1(1 + \alpha\Delta\theta), \qquad l_2' = l_1'(1 + \alpha'\Delta\theta)$$

The difference in their lengths before heating is $d_1 = l_1 - l_1'$, and after heating

$$d_2 = l_2 - l_2' = (l_1 - l_1') + (l_1\alpha - l_1'\alpha')\Delta\theta$$

so the **differential expansion** is

$$d_2 - d_1 = (l_1\alpha - l_1'\alpha')\,\Delta\theta$$

For the difference to stay the same ($d_2 = d_1$, with Δθ not zero):

$$l_1\alpha = l_1'\alpha' \quad \text{or} \quad \frac{l_1}{l_1'} = \frac{\alpha'}{\alpha} \tag{10.11}$$

The lengths must be in the inverse ratio of the expansivities: the rod of the metal that expands less must be longer. This is the principle of the **compensated pendulum** (Harrison's gridiron, the story in 10.3): steel rods carry the bob down and brass rods carry it up, with lengths in the inverse ratio of their α's, so the bob stays at the same distance below the pivot.

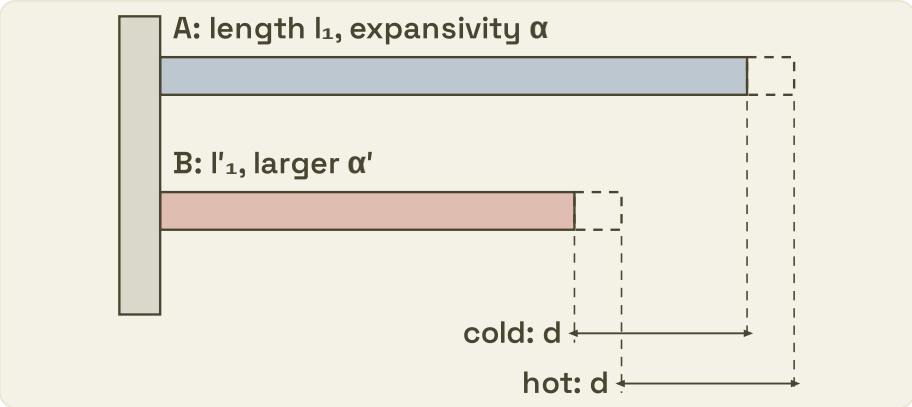


Figure 10.10 *Two rods side by side, A of length l_1 and expansivity α, and B, shorter, of length l'_1 and larger expansivity α', heated by the same $\Delta\theta$. Each grows by its length × expansivity × $\Delta\theta$. When $l_1\alpha = l'_1\alpha'$, both grow by the same amount (dashed), so the difference d between their lengths stays the same.*

WORKED EXAMPLE

An iron rod and a copper rod

An iron rod and a copper rod lie side by side. When heated, the difference in their lengths stays 10 cm. Find their lengths at 0 °C (α for copper $= 1.7 \times 10^{-5}$ °C^{-1}, α for iron $= 1.2 \times 10^{-5}$ °C^{-1}).

Answer

Let the lengths at 0 °C be l_{Cu} and l_{Fe}. For the difference to stay the same, the growths are equal:

$$l_{\text{Cu}}\,\alpha_{\text{Cu}} = l_{\text{Fe}}\,\alpha_{\text{Fe}} \quad \Rightarrow \quad \frac{l_{\text{Fe}}}{l_{\text{Cu}}} = \frac{\alpha_{\text{Cu}}}{\alpha_{\text{Fe}}} = \frac{1.7}{1.2} = \frac{17}{12}$$

Iron expands less, so the iron rod is the **longer** one: $l_{\text{Fe}} - l_{\text{Cu}} = 10$ cm.
Put $l_{\text{Fe}} = \frac{17}{12}\,l_{\text{Cu}}$: $\frac{17}{12}\,l_{\text{Cu}} - l_{\text{Cu}} = \frac{5}{12}\,l_{\text{Cu}} = 10$, so $l_{\text{Cu}} = 24$ cm and $l_{\text{Fe}} = 34$ cm.
Copper rod 24 cm, iron rod 34 cm. Check: growths per degree, $24 \times 1.7 = 40.8$ and $34 \times 1.2 = 40.8$ ($\times 10^{-5}$ cm): equal.

Sample questions

10.10 Thermal expansion of liquids

A liquid has no shape of its own, so it is always held in a container, and the container expands too. What we **see** is the liquid's expansion **minus** the container's.

So a liquid has **two coefficients** of expansion.

Coefficient of real (absolute) expansion of a liquid: the real increase in volume per unit original volume per degree rise in temperature,

$$\gamma_r = \frac{\Delta V_r}{V_0\,\Delta\theta} \tag{10.12}$$

Coefficient of apparent expansion of a liquid: the apparent increase in volume (the increase seen in its container) per unit original volume per degree rise in temperature,

$$\gamma_a = \frac{\Delta V_a}{V_0\,\Delta\theta} \tag{10.13}$$

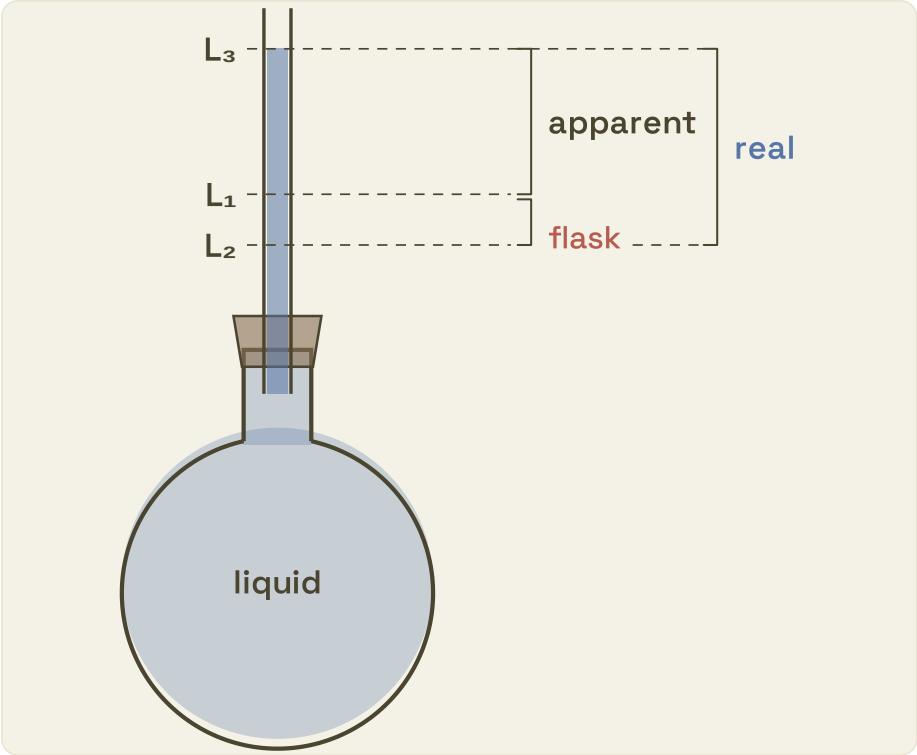


Figure 10.11 *Real and apparent expansion. The liquid starts at L_1. The flask expands first, and the level falls to L_2; then the liquid expands and rises to L_3. L_2 to L_3 is the real expansion of the liquid; L_1 to L_3 is its apparent expansion; L_1 to L_2 is the expansion of the flask. (The tube is drawn wide and the levels far apart, to be seen.)*

And the container's **coefficient of volume expansion**, the increase in the volume inside the vessel per unit original volume per degree rise:

$$\gamma_g = \frac{\Delta V_g}{V_0\,\Delta\theta} \tag{10.14}$$

The relation between γ_r, γ_a and γ_g

From the levels: $L_2L_3 = L_1L_3 + L_2L_1$. Multiply by the tube's cross-section a:

$$\Delta V_r = \Delta V_a + \Delta V_g$$

Real expansion of the liquid = apparent expansion + expansion of the vessel. Put in (10.12) to (10.14):

$$\gamma_r V_0 \Delta\theta = \gamma_a V_0 \Delta\theta + \gamma_g V_0 \Delta\theta \quad \Rightarrow \quad \boxed{\gamma_r = \gamma_a + \gamma_g} \tag{10.15}$$

If the vessel is glass of linear expansivity α_g, then $\gamma_g = 3\alpha_g$ and $\gamma_r = \gamma_a + 3\alpha_g$.

- γ_r belongs to the liquid alone; γ_a depends on the liquid **and** the container. The same mercury has a smaller apparent expansivity in a glass vessel than in a less expanding quartz one.
- If $\gamma_g = \gamma_r$, the apparent expansion is zero: the level never moves. If $\gamma_g > \gamma_r$, the level falls on heating.