Unit 10

Answers: Thermal Expansion

Quick checks

1 b Every part of a heated solid moves away from every other part, as in an enlarged photograph, and the hole is enlarged with the rest. That's why a tight metal lid comes off a jar when it is held under hot water.

2 b A colder rod is shorter, and $T = 2\pi\sqrt{l/g}$ is then shorter: more swings a day, each counted as the same number of seconds, so the clock runs fast and gains. A hot room makes it lose.

3 b $\Delta A = A\,\beta\,\Delta\theta = 2.000 \times 4 \times 10^{-5} \times 50 = 0.004$ m², so $A_2 = 2.004$ m².

4 c $\gamma = 3\alpha$ and $\beta = 2\alpha$, so $\alpha = \beta/2$ and $\gamma = 3 \times \beta/2 = \frac{3}{2}\beta$.

5 a $\beta = 2\alpha$ and $\gamma = 3\alpha$, so $\alpha = \beta/2 = \gamma/3$. The second option has the factors upside down: β is the bigger number, not α.

6 c $\gamma = \dfrac{\Delta V}{V\Delta\theta} = \dfrac{0.0012}{20} = 6 \times 10^{-5}$ °C⁻¹, and $\alpha = \gamma/3 = 2 \times 10^{-5}$ °C⁻¹.

7 b B expands more, so B becomes the longer strip; joined to A, it must take the outside of the curve, the longer path.

8 b No change of level means zero apparent expansion: $\gamma_a = \gamma_r - \gamma_g = 0$, so $\gamma_r = \gamma_g$. Both the liquid and the vessel expand, by equal amounts.

9 b The mass stays the same while the volume grows, so the density $\rho = m/V$ falls: $\rho_2 = \rho_1/(1 + \gamma\Delta\theta)$. (Water between 0 °C and 4 °C is the one common exception.)

10 b Apparent weight = weight − upthrust, and upthrust = (volume of the metal) × (density of the liquid) × g. As both warm, the liquid's density falls much faster (liquids expand far more than metals) than the metal's volume grows, so the upthrust falls and the apparent weight **increases**.

11 a In winter benzene is colder and denser, so 5 liters of it holds more mass and weighs more. (A fixed **mass** of benzene weighs the same in either season; it is the fixed volume that differs.)

12 a Each grows by $l\alpha\Delta\theta$. Equal growths for the same Δθ need $l_1\alpha_1 = l_2\alpha_2$.

13 a A hollow body expands as if its hollow were filled with the same material: every length grows by the same fraction. Both outside volumes grow by $\gamma V\Delta\theta$.

14 b B expands more, and the longer strip takes the outside of the curve.

15 b Colder, the pendulum is shorter and the clock **gains**: $\frac{1}{2}\alpha\Delta\theta \times 86\,400 = \frac{1}{2} \times 2 \times 10^{-5} \times 10 \times 86\,400 = 8.64$ s per day.

16 **b** Both sides of the plate grow by 1%: $(1.01 \times 2L)(1.01 \times L) = 1.0201 \times 2L^2$, about **2%** more ($\beta = 2\alpha$). The plate's shape and size don't matter.

17 **c** The hole grows as if it were made of the disc's metal: $\Delta d = d\,\alpha\,\Delta\theta = 1.5 \times 1.9 \times 10^{-5} \times 130 = 0.0037$ cm, an **increase of about 0.004 cm**.

18 **b** Mercury's real expansivity is the same in both: $\gamma_r = \gamma_a + \gamma$.
In steel: $\gamma_r = 144 \times 10^{-6} + 3 \times 12 \times 10^{-6} = 180 \times 10^{-6}$ K⁻¹.
In glass: $\gamma_g = \gamma_r - \gamma_a = 180 \times 10^{-6} - 153 \times 10^{-6} = 27 \times 10^{-6}$ K⁻¹, so $\alpha_g = \gamma_g/3 = 9 \times 10^{-6}$ °C⁻¹. (27×10^{-6} is glass's cubical expansivity, not its linear one.)

19 **c** $\gamma_g = 3 \times 0.1 \times 10^{-4} = 0.3 \times 10^{-4}$ °C⁻¹. Spilled = apparent expansion = $V(\gamma_r - \gamma_g)\Delta\theta = 1000 \times (1.82 - 0.3) \times 10^{-4} \times 100 = 15.2$ cm³.

20 **c** The container's cubical expansivity is $3 \times \gamma/3 = \gamma$, equal to the liquid's. The space and the liquid grow equally: zero apparent expansion.

21 **d** A Fahrenheit degree is 5/9 of a Celsius degree: $\alpha_F = \frac{5}{9} \times 18 \times 10^{-5} = 10 \times 10^{-5} = 1 \times 10^{-4}$ °F⁻¹.

22 **b** $Y = 2 \times 10^{12}$ dyne/cm² $= 2 \times 10^{11}$ N/m² (1 dyne/cm² = 0.1 N/m²). $A = \pi r^2 = 3.14 \times 0.024^2 = 1.81 \times 10^{-3}$ m².
$F = YA\alpha\Delta\theta = 2 \times 10^{11} \times 1.81 \times 10^{-3} \times 1.2 \times 10^{-5} \times 50 = 2.17 \times 10^5$ N.

23 **c** $\gamma = 3\alpha$, so each 0.50% in length gives $3 \times 0.50\% =$ **1.5%** in volume. (Exactly: $1.005^3 = 1.0151$.)

16. a. Its two metals have different linear expansivities but are joined along their whole length. Heated, one gets longer than the other; they can't slide apart, so the strip curves, the more expanding metal on the outside (the longer arc).

b. The diameter is a length, so it grows with the linear expansivity α of the material:

$$d_2 = d_1\,[1 + \alpha(\theta_2 - \theta_1)]$$

(Its volume grows as $V_2 = V_1(1 + 3\alpha\Delta\theta)$, which agrees: d^3 grows by about $3\alpha\Delta\theta$.)

c. The hole **gets bigger**: $\Delta d = d\alpha\Delta\theta = 2 \times 1.2 \times 10^{-5} \times 50 = 0.0012$ cm, so its diameter becomes **2.0012 cm**.

17. a. **Linear expansivity** α: the increase in length per unit original length per degree rise in temperature, $\alpha = \dfrac{l_2 - l_1}{l_1(\theta_2 - \theta_1)}$. **Cubical expansivity** γ: the increase in volume per unit original volume per degree rise in temperature, $\gamma = \dfrac{V_2 - V_1}{V_1(\theta_2 - \theta_1)}$. Both in K^{-1}.

b. A cube of side l_1 has $V_1 = l_1^3$. Heated by $\Delta\theta$, its side becomes $l_2 = l_1(1 + \alpha\Delta\theta)$, and its volume $V_2 = l_2^3 = V_1(1 + \gamma\Delta\theta)$. So

$$l_1^3(1 + \alpha\Delta\theta)^3 = l_1^3(1 + \gamma\Delta\theta)$$
$$1 + 3\alpha\Delta\theta + 3\alpha^2\Delta\theta^2 + \alpha^3\Delta\theta^3 = 1 + \gamma\Delta\theta.$$

α is very small, so the terms in α^2 and α^3 can be neglected: $3\alpha\Delta\theta = \gamma\Delta\theta$, and $\boldsymbol{\gamma = 3\alpha}$.

c. **No.** γ is a property of the material, and a hollow cube of the same material grows exactly as the solid one does: its hollow expands as if it were filled with the same metal.

18. a. $L_1 \to L_2$, the fall: the expansion of the flask. $L_2 \to L_3$: the **real** expansion of the liquid. $L_1 \to L_3$, the net rise you see: the **apparent** expansion.

b. Let the tube's cross-section be a. From the levels, $L_2L_3 = L_1L_3 + L_2L_1$; multiplying by a, $\Delta V_r = \Delta V_a + \Delta V_g$. With $\Delta V_r = \gamma_r V_0 \Delta\theta$, $\Delta V_a = \gamma_a V_0 \Delta\theta$ and $\Delta V_g = \gamma_g V_0 \Delta\theta$, dividing by $V_0\Delta\theta$ gives $\gamma_r = \gamma_a + \gamma_g$.

c. Inside volume of the flask at 60 °C: $200(1 + 2.7 \times 10^{-5} \times 60) = 200.324$ cm³.

Mercury at 60 °C: $180(1 + 18 \times 10^{-5} \times 60) = 181.944$ cm³.
Space left: $200.324 - 181.944 = \mathbf{18.4\ cm^3}$ (it was 20 cm³ at 0 °C).

d. i. **Brass** ($\alpha \approx 1.9 \times 10^{-5}\ K^{-1}$, against steel's $1.1 \times 10^{-5}\ K^{-1}$).

ii. The brass grows longer than the steel it is joined to. The only way two joined strips of different lengths can fit together is to curve, with the longer brass on the outside of the arc and the shorter steel on the inside: so the strip bends towards the steel side.

iii. **Steel.** For the same increase in length from the same 1 m, $\Delta l = l\alpha\Delta\theta$ needs a larger $\Delta\theta$ where α is smaller. Steel must be heated about $1.9/1.1 \approx 1.7$ times as many degrees as brass. (The cross-section doesn't matter.)

e. Let the mercury column be x cm long, in a tube of cross-section a. The empty part stays the same if the mercury's growth equals the tube's:

$$x\,a\,\gamma\,\Delta\theta = 133\,a\,\gamma_g\,\Delta\theta \quad \Rightarrow \quad x = 133 \times \frac{0.000\,026}{0.000\,182} = 133 \times \frac{1}{7} =$$

19.

a. **Dulong and Petit's balanced-column method** (10.12): a U-tube of the liquid with one limb cold and the other hot. It compares the columns by their pressures, which depend only on height and density, not on the width of the tube, so the glass's expansion doesn't enter.

b. At equilibrium, the pressures at the bottoms of the two columns are equal: $\rho_1 g h_1 = \rho_2 g h_2$, so $\dfrac{h_1}{h_2} = \dfrac{\rho_2}{\rho_1}$. The density at the higher temperature is $\rho_2 = \dfrac{\rho_1}{1+\gamma\Delta\theta}$. So $\dfrac{h_1}{h_2} = \dfrac{1}{1+\gamma\Delta\theta}$, $h_1 + h_1\gamma\Delta\theta = h_2$, and $\gamma = \dfrac{h_2 - h_1}{h_1\Delta\theta}$.

c. $$\gamma = \frac{66.18 - 65}{65 \times 100} = \frac{1.18}{6500} = \times\,\mathbf{10^{-4-1}}$$

That is mercury's own real expansivity.

20.

a. Aluminium expands **more** than brass (about 2.3 against $1.9 \times 10^{-5}\ \text{K}^{-1}$). On a hot day both are longer, but the scale's centimeters have grown more than the rod, so fewer of them fit along it: the reading is smaller. (On a cold day the reverse.)

b. It **increases**, in the same proportion as the disc's metal: the hole grows as if it were filled with the same metal.

c. i. $\Delta\theta = 48 - 20 = 28$ K: $d = 2.0 \times (1 + 1.76 \times 10^{-5} \times 28) = 2.0 \times 1.000\,493 =$ **2.000 99 cm**.

ii. $\Delta\theta = -23 - 20 = -43$ K: $d = 2.0 \times (1 - 1.76 \times 10^{-5} \times 43) = 2.0 \times 0.999\,243 =$ **1.998 49 cm**.

Between Nepalgunj and the Annapurnas, the coin changes by about 0.025 mm, a quarter of the thickness of a sheet of paper.

21.

a. Aluminium has a larger linear expansivity (2.3×10^{-5} against 1.2×10^{-5} K^{-1}): for each degree, each meter of aluminium grows by about twice as much as each meter of steel. The forces between aluminium atoms are weaker and less stiff, and their spacing grows more with the same increase in vibration.

b. They form a bimetallic strip (the figure in 10.9). Heated, the aluminium tries to grow about twice as much as the steel; since they are joined, the pair curves into an arc with the **aluminium on the outside** (convex side) and the steel on the inside. Cooled below the starting temperature, it curves the other way.

c. A **thermostat** in an iron, rice cooker, water heater or aquarium heater: a bimetallic strip bends away from its contact when hot and cuts the current. **Circuit breakers** (MCBs): a strip heated by too much current bends and trips the switch. **Dial thermometers** (in ovens and for meat): a coiled bimetallic strip turns a pointer. **Compensated pendulums** use zero differential expansion to keep a constant length. Its harmful side: a thick glass cracking when hot water is poured in (the inside expands before the outside), and filling materials in teeth must expand like the tooth.

d. **2 m.** After cooling back to the starting temperature, each rod contracts to exactly its original length, 1 m. Joined end to end: 1 m + 1 m = 2 m. (The expansivities don't matter.)

22.

$$\Delta\theta = 117 - 36 = 81\ ^\circ\text{F} = 81 \times \frac{5}{9} = 45\ ^\circ\text{C}.$$

$$\Delta l = l\alpha\Delta\theta = 625 \times 10^3 \times 12 \times 10^{-6} \times 45 =$$

Spread along 625 km of track in thousands of small gaps.

23.

Equal growths: $l_b\alpha_b\Delta\theta = l_s\alpha_s\Delta\theta$, so

$$\alpha_b = \frac{l_s\alpha_s}{l_b} = \frac{0.60 \times 12 \times 10^{-6}}{0.40} = \times\ \mathbf{10^{-6}\ ^{-1}}$$

(The 85 °C isn't needed.)

24.

At θ °C:

$$\rho = \frac{900}{1 + 1.2 \times 10^{-3}\theta}, \qquad \rho = \frac{880}{1 + 1.5 \times 10^{-4}\theta}$$

Set them equal and cross-multiply:

$900(1 + 1.5 \times 10^{-4}\theta) = 880(1 + 1.2 \times 10^{-3}\theta)$

$900 + 0.135\theta = 880 + 1.056\theta$

$20 = 0.921\theta \Rightarrow \theta = \mathbf{21.7\ ^\circ C}$ (about 295 K).

Below this the wood is less dense and floats; above it, the benzene is less dense and the wood sinks.

25. Each 20.0 cm mark moves out by $l\alpha\Delta\theta$:
aluminium: $20.0 \times 2.3 \times 10^{-5} \times 100 = 0.046$ cm;
brass: $20.0 \times 1.9 \times 10^{-5} \times 100 = 0.038$ cm.
They are $0.046 - 0.038 = \mathbf{0.008\ cm}$ apart (the aluminium mark farther out), less than a tenth of a millimeter.

26. $\Delta V = V\gamma\Delta\theta = 1.70 \times 75 \times 10^{-5} \times 9 = \mathbf{0.0115\ m^3}$, about 11.5 liters of air space.

27. a. $$\alpha = \frac{\Delta l}{l\Delta\theta} = \frac{0.019}{1.5 \times 400} = \times\,\mathbf{10^{-5-1}}$$

b. $$= Y\alpha\Delta\theta = 2 \times 10^{11} \times 3.17 \times 10^{-5} \times 400 = \times\,\mathbf{10^{92}}$$

(That is more than most steels can bear: a real wire would stretch permanently or snap.)

28. At 10 °C the tape is colder and shorter, so each of its "meters" is less than a real meter, and more of them fit into the kilometer: it reads **more**.
Length of one tape "meter" at 10 °C: $1 \times (1 - 12 \times 10^{-6} \times 10) = 0.999\,88$ m.
Reading: $\dfrac{1000}{0.999\,88} = 1000.12$ m $= \mathbf{1.000\ 12\ km}$, 12 cm too much.