Unit 6

Circular Motion

This unit online, with its interactives:
science-playground.com/s/11f6

A bus on a hill road swings round a hairpin bend, and every passenger slides towards the window. Nobody pushed them. A stone whirled on a string pulls at your hand; let go, and it does not fly outward from the center, as most people expect, but off along a straight line that just grazes the circle. A satellite falls towards the Earth all the time and never comes any closer.

All three are the same physics. A body moving in a circle is always changing its direction, so it is always **accelerating**, towards the center, and something must be pulling it there. In this unit you will measure turning with angles and radians, derive the acceleration v^2/r that every circle demands, and use it to explain swing rides, a bucket of water whirled over your head, why roads are tilted at bends, and why a cyclist leans into a turn.

6.1 Introduction to circular motion

A STORY FROM SCIENCE

Newton's polygon

How does a steady pull towards one point make a body go round in a circle? Isaac Newton (1642–1727) answered it in his *Principia* (1687) with a trick. Replace the circle by a polygon of many sides. Along each side the body moves in a straight line, as a body with no force on it does; at each corner it gets a short kick **towards the center**, which turns it onto the next side. Let the number of sides grow without limit, and the kicks merge into one continuous pull towards the center, and the polygon into a circle.

So a circle needs **one** force, directed inwards, that keeps turning the body away from the straight line it would otherwise follow. There is no second, outward force balancing it: if the two balanced, there would be no resultant force, and the body would go straight on.

A body is in **circular motion** when it stays at the same distance from a fixed point. The fixed point is the **center** of the circle, and the line from the center to the body is the **radius vector**; its length is the radius r.

At every point of the circle, the body's velocity is along the **tangent** at that point, at right angles to the radius. As the body goes round, the tangent keeps turning, so the **direction of the velocity changes all the time**, even when the speed stays the same:

$$\text{velocity} = \text{speed} + \text{direction}$$

A change in velocity is an acceleration, so **every body moving in a circle is accelerating**, even at a constant speed. By Newton's second law an acceleration needs a resultant force: something must keep pulling the body towards the center.

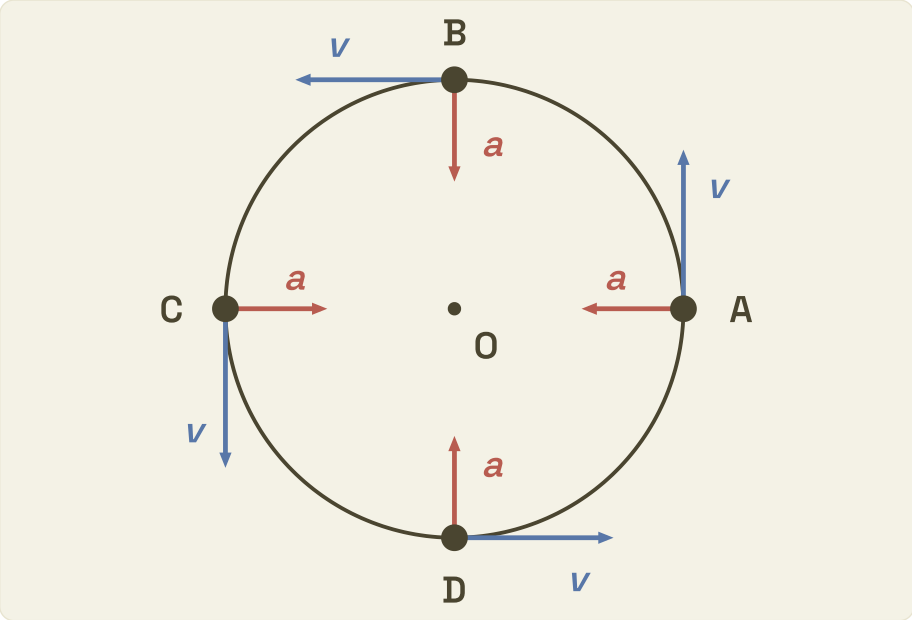


Figure 6.1 *A body going round a circle anticlockwise at a steady speed. At A, B, C and D its velocity (blue) is along the tangent, at right angles to the radius; its acceleration (red) points to the center O. If the string broke at any point, the body would leave along that point's blue arrow.*

Check 1

A stone is whirled in a horizontal circle on a string. The string breaks when the stone is at the north side of the circle, moving east. Just after that, seen from above, the stone moves:

a north, straight away from the center
b east, along the tangent
c south, towards the center
d round the circle as before

6.1.1 Centripetal force, and the "centrifugal force"

The resultant force that keeps a body moving in a circle is the **centripetal force** ("center-seeking"). It always points **towards the center**. It is not a new kind of force: it is whatever force happens to be pulling the body inwards.

- A stone on a string: the **tension** in the string.
- The Moon round the Earth, a satellite round the Earth: **gravity**.
- A car turning on a level road: **friction** between the tyres and the road.
- An electron round the nucleus, in the simple Bohr picture: the **electric attraction** of the nucleus.

Watched from the ground (an **inertial frame**), that inward force is the only sideways force on the body, and it is unbalanced. That is exactly why the body's path keeps bending.

So why does it *feel* as if something throws you outwards on a turning bus? Because of your **inertia**. Your body tends to keep moving in a straight line (Newton's first law). The bus turns under you, and the seat and the side of the bus must push you inwards to turn you with it. Until they do, you slide, relative to the bus, towards the outside of the bend. To someone sitting in the bus, using the bus as their frame, it looks as if a force pushed them outwards. That apparent outward force is the **centrifugal force** ("center-fleeing"). It is a **pseudo-force** (a fictitious force): it appears only when we describe the motion from the rotating frame, it equals mv^2/r in size, and no body exerts it.

The everyday examples, explained correctly:

- **A bucket of water whirled in a vertical circle.** At the top, the bucket is upside down, yet the water stays in. The water must move in a circle, so it needs a downward (towards the center) force of mv^2/r. Its weight provides part of it and the bucket's bottom pushes down for the rest. As long as v^2/r is at least g, the water is pressed against the bucket instead of falling out (Section 6.6).

CAREFUL

Centrifugal force is not a real force on the body

Some books say the centripetal and centrifugal forces act together on the body and balance each other, or that they are an action–reaction pair. Neither is right. If two forces on the body balanced, it would have no resultant force and would move in a straight line, not a circle. Seen from the ground, only the inward (centripetal) force acts on the body. The real *reaction* to it is the body's outward pull on whatever holds it, the stone pulling on the string, which acts on the string, not on the stone. In exams, describe the centrifugal force as a pseudo-force felt in the rotating frame, equal and opposite to the centripetal force.

- **Leaning outwards in a turning bus.** Your inertia keeps you going straight while the bus turns; the seat and the friction on you pull you round. It feels like an outward push only in the bus's frame.
- **A stone whirled on a string.** The string pulls the stone inwards; the stone pulls the string outwards. If the string breaks, the stone goes off along the tangent.
- **Mud flying off a tyre.** The mud needs an inward force to go round with the wheel. When its stickiness can't provide that much, it leaves the tyre along the tangent.
- **A satellite orbiting the Earth.** Gravity provides its centripetal force.
- **The Earth's bulge.** The Earth **spins on its axis**. Matter at the equator moves round a circle of about 6400 km radius once a day and needs an inward force to do so, so a little less of gravity is left to press it down: the spinning Earth settled into a shape slightly flattened at the poles and bulging at the equator. Its equatorial radius is about 21 km greater than its polar radius. (Its yearly orbit round the Sun has nothing to do with this.)

A body moving in a circle at a **constant speed** has **no work done on it**. The centripetal force points to the center and the motion is along the tangent: the angle between the force and the displacement is **90°**, and $W = Fs\cos 90^\circ = 0$. So its speed and kinetic energy don't change; only its direction does.

Check 2

A girl whirls a stone in a horizontal circle on a string. Which force on the **stone** keeps it in its circle?

a the centrifugal force, outwards
b the centripetal and centrifugal forces together
c the tension in the string, inwards
d the stone's weight

Check 3

A car goes round a roundabout at a steady 20 km/h. The work done on it by the resultant force over one full turn is:

a positive, since a force acts
b negative, since it turns
c zero
d equal to its kinetic energy

6.1.2 Angular displacement and angular velocity

Angular displacement

The **angular displacement** θ of a body moving round a circle is the angle swept out at the center by its radius vector: the angle AOB as the body goes from A to B.

Angles in circular motion are measured in **radians** (rad). An angle in radians is the arc length divided by the radius:

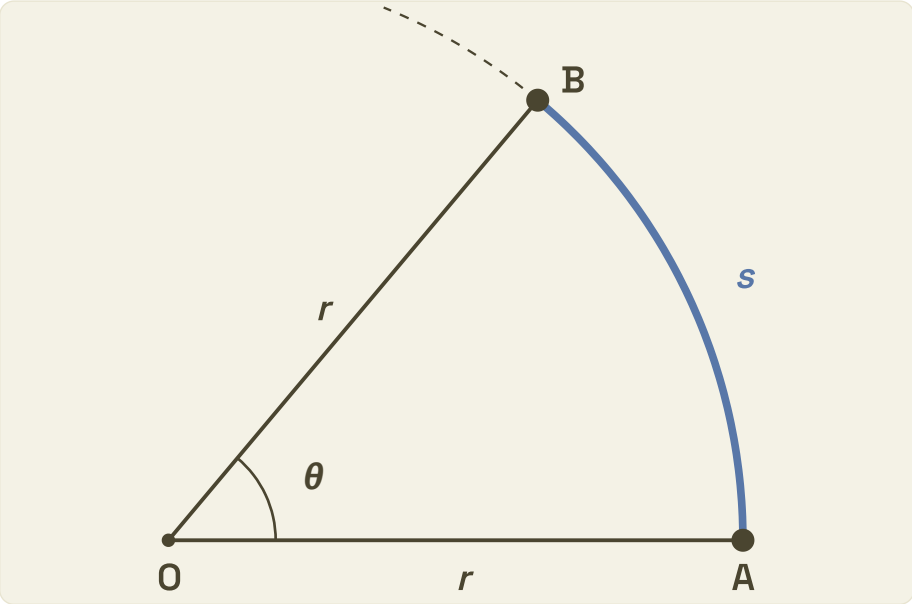


Figure 6.2 *A body goes from A to B along the circle. Its angular displacement is the angle θ at the center; the arc AB has length s. By the definition of the radian, θ = s/r.*

$$\theta = \frac{s}{r} \quad \Rightarrow \quad s = r\theta$$

So 1 radian is the angle whose arc is as long as the radius (about 57.3°). A whole circle has an arc of $2\pi r$, so

$$2\pi \text{ rad} = 360^\circ, \qquad \pi \text{ rad} = 180^\circ, \qquad 1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.3^\circ$$

A radian is a ratio of two lengths, so it has no dimensions; we write “rad” to remind ourselves that an angle is meant.

Angular velocity

The **angular velocity** ω (omega) is the rate of change of angular displacement:

$$\omega = \frac{\theta}{t} \quad \text{(average)}, \qquad \omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \quad \text{(instantaneous)}$$

Its SI unit is the **radian per second**, rad/s. Machines often give it in **revolutions per minute** (rpm): one revolution is 2π rad, so $1 \text{ rpm} = \dfrac{2\pi}{60}$ rad/s.

Angular acceleration

The **angular acceleration** α (alpha) is the rate of change of angular velocity, in rad/s². If the angular velocity changes uniformly from ω_0 to ω in a time t,

$$\alpha = \frac{\omega - \omega_0}{t} \quad \Rightarrow \quad \omega = \omega_0 + \alpha t$$

This is the turning version of $v = u + at$. In the same way, for uniform angular acceleration, $\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$ and $\omega^2 = \omega_0^2 + 2\alpha\theta$.

Time period and frequency

The **time period** T is the time for one complete revolution. In one period the body turns through 2π rad, so

$$\omega = \frac{2\pi}{T}$$

The **frequency** f is the number of revolutions made in one second, in hertz (Hz, or rev/s). Since each revolution takes T seconds,

$$f = \frac{1}{T}, \qquad \omega = 2\pi f$$

37. A spaceman in training is whirled round in a seat 5 m from the axis of rotation. He can withstand an acceleration of up to 9g. What is the greatest number of revolutions per second allowed? (g = 10 m/s²)

38. A 0.4 kg mass is whirled on a string at a constant speed v in a vertical circle of radius 1 m. The least tension in the string is 3 N. Find i. v, ii. the greatest tension, iii. the tension when the string is horizontal. (g = 10 m/s²)

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.

22. a. Why is a body in circular motion accelerating, even though it moves at a constant speed?
    b. The positive nucleus of an atom attracts the electrons, yet in Bohr's model the electrons don't fall into it. Why?
    c. Calculate the force needed to keep a 0.2 kg mass moving in a horizontal circle of radius 0.5 m with a period of 0.5 s.

23. a. A particle moves in a circle at a constant speed. Is its acceleration constant?
    b. What provides the centripetal force on a car going round a level bend?
    c. Two bodies move round circles whose radii are in the ratio 2 : 3. If their accelerations are the same, find the ratio of the squares of their speeds.

24. a. Why does an airplane tilt while flying round a curve?
    b. A racing car at a constant speed of 50 m/s takes 40 s for one lap of a circular track. Find the size of its acceleration.

25. a. Define angular displacement.
    b. Derive an expression for the force towards the center on a body of mass m moving at a constant speed v round a circle of radius r.
    c. The tip of the hour hand of a clock moves 1/20 as fast as the tip of the minute hand. If the minute hand is 10 cm long, how long is the hour hand?

26. a. What is a conical pendulum?
    b. Show that the period of a conical pendulum of length l whose string makes an angle θ with the vertical is $2\pi\sqrt{\frac{l\cos\theta}{g}}$.
    c. A 10 kg object is whirled round a horizontal circle of radius 4 m by a string inclined to the vertical, at a steady 5 m/s. Find i. the tension in the string and ii. the angle of the string to the vertical (g = 10 m/s²).

27. a. What is meant by the banking of a track?
    b. Find the angle through which the outer edge of a road must be raised at a bend of radius r, for a speed v.
    c. A train of mass 10^5 kg rounds a curve of radius 150 m at 20 m/s. Find the horizontal thrust on the outer rail if the track is not banked. At what angle must the track be banked so that there is no thrust on the rail? (g = 9.8 m/s²)

28. a. What is motion in a vertical circle?
    b. Find the critical velocity a body needs at the top to go safely round a vertical circle.
    c. A construction team wants cars to go round a level bend without skidding even in bad conditions. Find the least radius the bend can have if a car may travel at 110 km/h when the coefficient of static friction between the tyres and the road is only 0.60. (g = 9.8 m/s²)

Numerical problems

29. A 100 g mass hangs on a string 40 cm long. Keeping the string taut, it is made to go round a horizontal circle of radius 10 cm. Find its angular speed. (g = 9.8 m/s²)
30. What is the angular velocity of the minute hand of a clock? What is its angular displacement in 20 minutes? If the minute hand is 5 cm long, what is the linear velocity of its tip?
31. A 0.8 kg stone is tied to one end of a string 0.9 m long, which breaks if its tension goes above 600 N. The stone is whirled in a horizontal circle about the fixed other end. Find the greatest speed it can reach without breaking the string. (Ignore its weight.)
32. A 200 g bob is whirled in a horizontal circle of radius 50 cm on a string inclined at 30° to the vertical. Find the tension in the string and the speed of the bob. (g = 9.8 m/s²)
33. On a meter-gauge railway (the rails 1 m apart), a bend has a radius of curvature of 50 m. A train goes round it at 36 km/h. For no side thrust on the rails, how high must the outer rail be above the inner one? (g = 9.8 m/s²)
34. A stone on an 80 cm string is whirled in a horizontal circle at a constant speed, making 14 revolutions in 25 s. Find the size and direction of its acceleration.
35. A car moves round a horizontal circular track of radius 10 m at a constant 10 m/s. A plumb bob hangs from the roof of the car on a light rigid rod. Find the angle the rod makes with the vertical. (g = 10 m/s²)
36. A 500 g stone on a string 50 cm long, which breaks if its tension goes above 20 N, is whirled in a vertical circle whose center is 100 cm above the ground. The angular speed is increased very slowly until the string breaks. Where is the break most likely, at what angular speed, and where does the stone hit the ground? (g = 10 m/s²)

YOUR TURN

Frequency and period

Show from the definitions that $f = \frac{1}{T}$.

Answer

One revolution takes T seconds, so in 1 second the body makes $\frac{1}{T}$ revolutions. The number of revolutions in one second is the frequency, so $f = \frac{1}{T}$. For example, a fan that takes 0.05 s per turn makes $1 \div 0.05 = 20$ turns every second.

WORKED EXAMPLE

The hands of a clock

Find the angular displacement of each hand of a clock in 1.00 h. Take clockwise as negative, as mathematicians measure angles anticlockwise.

Answer

Second hand: it turns once a minute, so 60 times in 1 h: $\theta = -60 \times 2\pi = -120\pi \approx$ **−377 rad**.

Minute hand: once in 1 h: $\theta = -2\pi \approx$ **−6.28 rad**.

Hour hand: once in 12 h, so $\frac{1}{12}$ of a turn in 1 h: $\theta = -\frac{2\pi}{12} = -\frac{\pi}{6} \approx$ **−0.524 rad**.

YOUR TURN

A door

A door 0.52 m wide swings open so that its outer edge moves 46.7 cm along its arc. Through what angle does it turn?

Answer

$\theta = \frac{s}{r} = \frac{0.467}{0.52} =$ **0.90 rad** (about 51°). Both lengths must be in the same unit.

Which way does ω point?

The angular velocity can be given a direction along the **axis** of rotation, by the **right-hand rule**: curl the fingers of your right hand the way the body goes round, and your thumb points along $\vec{\omega}$. For a body going round **anticlockwise** as you look down on it, $\vec{\omega}$ points **up**, towards you; for **clockwise**, down. Its size tells how fast it turns, its direction the plane and sense of the turning. (Small angular displacements can be given a direction the same way; large ones don't add like vectors, as you can see by turning a book through 90° about two different axes in the two orders.)

Check 4

A wheel turns at 120 rpm. Its angular velocity is:

a 2 rad/s
b 2π rad/s
c 4π rad/s (about 12.6 rad/s)
d 754 rad/s

6.2 Relation between linear velocity and angular velocity

The speed along the circle, the **linear velocity** $v = \frac{ds}{dt}$, and the **angular velocity** $\omega = \frac{d\theta}{dt}$ are tied together by the radius.

Derivation. A body moves round a circle of radius r, center O, from A to B in a time t (Figure 6.2). The arc AB, of length s, subtends an angle θ at O:

1. By the definition of the radian, $s = r\theta$.
2. Differentiate with respect to time; r is constant: $\frac{ds}{dt} = r\frac{d\theta}{dt}$.
3. So
$$\boxed{v = r\omega}$$
4. Differentiate once more: $\frac{dv}{dt} = r\frac{d\omega}{dt}$, that is
$$a_t = r\alpha$$

where a_t is the **tangential acceleration**, the rate at which the speed along the circle changes, and α the angular acceleration.

Every point of a turning wheel has the same ω, but the farther a point is from the axis, the faster it moves: the rim of a bicycle wheel goes much faster than a point near the hub.

WORKED EXAMPLE

An angle that grows faster and faster

The angular displacement of a particle is $\theta(t) = 2e^{0.75t} + 3t^2 - t$ (radians, seconds). Find its angular velocity at $t = 2$ s.

Answer

Differentiate term by term (the derivative of e^{kt} is ke^{kt}):

$$\omega = \frac{d\theta}{dt} = 2 \times 0.75\, e^{0.75t} + 6t - 1 = 1.5\, e^{0.75t} + 6t - 1$$

At $t = 2$ s:

$$\omega = 1.5\, e^{1.5} + 12 - 1 = 1.5 \times 4.482 + 11 = 6.72 + 11 = 17.72 \text{ rad/s}$$

Uniform circular motion is circular motion in which the body sweeps out **equal angular displacements in equal intervals of time**, however small: ω is constant, and so is the speed $v = r\omega$. As you saw in 6.1.1, no work is done on the body: the force is at 90° to the motion.

Check 5

A bicycle wheel of radius 0.35 m turns at 10 rad/s. How fast is a point on its rim moving?

a 0.035 m/s
b 3.5 m/s
c 28.6 m/s
d 10 m/s

6.3 Centripetal acceleration

A body in uniform circular motion has a constant speed but a changing velocity, so it has an acceleration. This **centripetal acceleration** always points **towards the center** along the radius, and it is what keeps the body on its circle. Its size is

$$a = \frac{v^2}{r}$$

Derivation

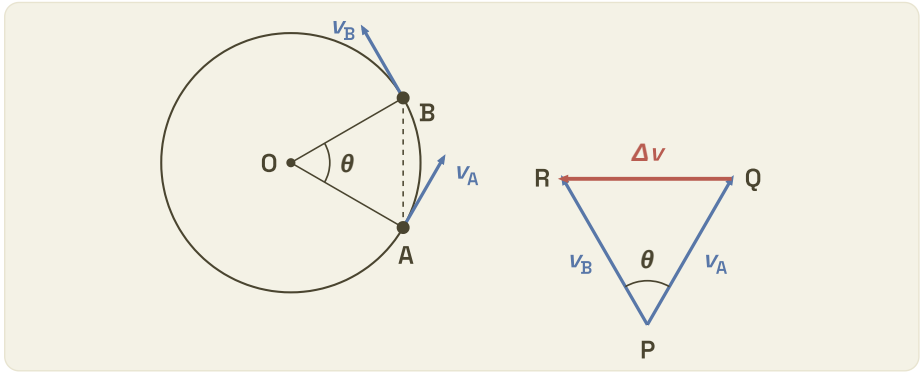


Figure 6.3 *Left: a body goes from A to B round a circle of radius r in a short time Δt; the radius turns through θ. Right: the two velocities drawn from one point P: PQ = $\vec{v}_A$, PR = $\vec{v}_B$, and QR = Δ$\vec{v}$, the change in velocity. The two triangles OAB and PQR are similar.*

1. A body of mass m moves at a steady speed v round a circle of radius r, center O (Figure 6.3). At time 0 it is at A, with velocity $\vec{v}_A$; a short time Δt later it is at B, with velocity $\vec{v}_B$. Both have size v; the radius has turned through θ.
2. Draw the two velocities from one point P: $\vec{PQ} = \vec{v}_A$ and $\vec{PR} = \vec{v}_B$. By the triangle law, $\vec{PQ} + \vec{QR} = \vec{PR}$, so $\vec{QR} = \vec{v}_B - \vec{v}_A = \Delta\vec{v}$, the change in velocity.
3. Each velocity is at right angles to its radius, so the angle between $\vec{v}_A$ and $\vec{v}_B$ is the same θ as between OA and OB. Triangles OAB and PQR are both isosceles (OA = OB = r; PQ = PR = v) with the same angle between the equal sides, so they are

Check 22

9. A cyclist goes round a circle at 15 km/h. If her speed doubles on the same circle, the centripetal force on her is:

a doubled
b halved
c quadrupled
d unchanged

Check 23

10. An ant crawls 22 cm along the arc of a circular track and turns through 45° about its center. The radius of the track is:

a 0.23 m
b 0.28 m
c 0.35 m
d 0.44 m

Check 24

11. In uniform circular motion:

a the acceleration and the speed change
b the acceleration and the speed are constant vectors
c the acceleration and the velocity are constant
d the acceleration and the velocity both change (in direction)

Check 25

12. A particle moves round a circle, describing equal angles in equal times. Its velocity vector:

a changes direction
b remains constant
c changes magnitude
d changes both direction and magnitude

Check 26

13. A 2 kg body moves round a circle of radius 1 m at an angular velocity of 2 rad/s. The centripetal force on it is:

a 9 N
b 8 N
c 10 N
d 7 N

Check 27

14. A point on the rim of a circular object of radius 25 cm turns at 20 rev/s. Its acceleration is:

a $400\pi^2$ m/s²
b $200\pi^2$ m/s²
c $250\pi^2$ m/s²
d $450\pi^2$ m/s²

Check 28

15. By nature, the centrifugal force is a:

a gravitational force
b centripetal force
c fictitious force
d viscous force

Check 29

16. A car races round a circular track of radius 747 m at 65 m/s. The banking angle needed is about (g = 9.8 m/s²):

a 25°
b 30°
c 35°
d 32°

Check 30

17. A body moves round a circle of radius r at a steady frequency f. Its centripetal (radial) acceleration is:

a $4\pi^2 rf^2$
b $4\pi rf$
c $4\pi r^2 f$
d $4\pi rf^2$

Check 31

18. A uniform ring with inner radius R_1 and outer radius R_2 rolls down a slope without slipping. Two small bodies of equal mass sit on its inner and outer surfaces. The ratio of the centripetal forces on them (inner : outer), about the ring's center, is:

a R_2/R_1
b $1 + R_2/R_1$
c $1 - R_2/R_1$
d R_1/R_2

Exercise

Choose the correct option

Check 14

1. What type of motion does the Moon execute around the Earth?

a nearly uniform circular motion
b linear motion
c streamline motion
d rotational motion about its own axis only

Check 15

2. A body goes ¾ of the way round a circular track of radius r at a constant speed v. Which is true?

a its change in kinetic energy is ½mvr
b its change in momentum is mvr
c its change in kinetic energy is zero
d its change in kinetic energy is mv^2

Check 16

3. A fighter plane pulls out of a dive at 900 km/h, moving in a vertical circle of radius 2 km. Its mass is 5000 kg. What force does the air exert on it at the lowest point? ($g = 10\ m/s^2$)

a 2.0625×10^4 N upward
b 2.0625×10^5 N downward
c 2.0625×10^4 N downward
d 2.0625×10^5 N upward

Check 17

4. A 2 kg mass is whirled in a horizontal circle on a string at 5 rpm. Keeping the radius the same, the tension in the string is doubled. Its new speed is about:

a 7 rpm
b 10 rpm
c 14 rpm
d 12 rpm

Check 18

5. A stone on an 80 cm string is whirled in a horizontal circle at a constant speed. It makes 14 revolutions in 25 s. Its acceleration is:

a $9.9\ m/s^2$
b $10.6\ m/s^2$
c $11.2\ m/s^2$
d $12.04\ m/s^2$

Check 19

6. A car travels at 30 m/s on a circular road of radius 500 m. Its acceleration is:

a $1.8\ m/s^2$
b $2.2\ m/s^2$
c $3.2\ m/s^2$
d $4.5\ m/s^2$

Check 20

7. A stone on a 20 cm string is whirled in a horizontal circle with a centripetal acceleration of $9.8\ m/s^2$. Its angular speed is:

a 7 rad/s
b 14 rad/s
c 23 rad/s
d 15 rad/s

Check 21

8. The Earth's diameter is 12,800 km and it turns once on its axis in 24 hours. What is the acceleration of a particle on the equator due to the Earth's rotation?

The textbook's options are 0.34, 0.28, 1.4 and 3.2 m/s^2, and none of them fits: the answer is about 0.034 m/s^2, so the book's options don't work. The options below are corrected.

a $0.034\ m/s^2$
b $0.34\ m/s^2$
c $0.0034\ m/s^2$
d $3.4\ m/s^2$

Answer these questions

19. a. Differentiate between linear and circular motion.

similar:

$$\frac{QR}{PQ} = \frac{AB}{OA} \quad \Rightarrow \quad \frac{\Delta v}{v} = \frac{AB}{r}$$

4. When Δt is very small, the chord AB is almost the arc, which the body covers at speed v: $AB = v\,\Delta t$. So

$$\frac{\Delta v}{v} = \frac{v\,\Delta t}{r} \quad \Rightarrow \quad \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

5. In the limit $\Delta t \to 0$, $\frac{\Delta v}{\Delta t}$ becomes the instantaneous acceleration:

$$\boxed{a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r}}$$

6. **Its direction.** As $\Delta t \to 0$, B moves to A, R moves to Q, and the angle θ shrinks to zero. The base angles of the isosceles triangle PQR then become 90°, so $\Delta \vec{v}$ (QR) ends up **at right angles to** $\vec{v}_A$, pointing along the radius **towards the center**. So the centripetal acceleration is always perpendicular to the velocity, and its direction keeps turning as the body goes round.

Using $v = r\omega$, $\omega = \frac{2\pi}{T}$ and $\omega = 2\pi f$, the same acceleration can be written in four ways:

$$a = \frac{v^2}{r} = r\omega^2 = \frac{4\pi^2 r}{T^2} = 4\pi^2 f^2 r$$

Because it points along the radius, it is also called the **radial acceleration**. It is a vector, always directed towards the center.

WORKED EXAMPLE

Testing a g-suit

In a test of a "g-suit" (a suit that stops a pilot's blood pooling in the legs), a volunteer is whirled round a horizontal circle of radius 7.0 m. Find the period of rotation at which the centripetal acceleration is a. 3g and b. 10g ($g = 9.8\ m/s^2$).

Answer

From $a = \frac{4\pi^2 r}{T^2}$, $T = 2\pi\sqrt{\frac{r}{a}}$.

a. $a = 3 \times 9.8 = 29.4\ m/s^2$: $T = 2\pi\sqrt{\frac{7.0}{29.4}} = 2\pi \times 0.488 = \mathbf{3.07\ s}$.

b. $a = 98\ m/s^2$: $T = 2\pi\sqrt{\frac{7.0}{98}} = 2\pi \times 0.267 = \mathbf{1.68\ s}$.

Faster turning (a shorter period) gives a larger acceleration: $a \propto 1/T^2$.

WORKED EXAMPLE

A helicopter rotor in a wind tunnel

A model helicopter rotor has four blades, each 3.40 m long from the central shaft to the tip. It turns at 550 rpm. Find a. the linear speed of a blade tip, and b. the tip's radial acceleration as a multiple of g.

Answer

$f = \frac{550}{60} = 9.17$ rev/s, $r = 3.40$ m.

a. $v = r\omega = 2\pi f r = 2\pi \times 9.17 \times 3.40 = \mathbf{196\ m/s}$ (over half the speed of sound).

b. $a = \frac{v^2}{r} = \frac{196^2}{3.40} = 1.13 \times 10^4\ m/s^2$. In multiples of g: $\frac{1.13 \times 10^4}{9.8} \approx \mathbf{1150\ g}$. Each gram of the blade tip needs an inward pull of over a kilogram's weight to keep it on its circle.

WORKED EXAMPLE

A child on a merry-go-round

A child of mass 15 kg sits 1.75 m from the center of a merry-go-round turning at 1.97 rad/s. Find her centripetal acceleration and the centripetal force on her.

Answer

$a = r\omega^2 = 1.75 \times 1.97^2 = \mathbf{6.79\ m/s^2}$.

$F = ma = 15 \times 6.79 = \mathbf{102\ N}$, towards the center, provided by the seat and the bar she holds.

When the speed changes too

If the speed is also changing (a car speeding up round a bend, a wheel spinning up), the body has two accelerations at right angles:

- the **centripetal** (radial) acceleration $a_c = \dfrac{v^2}{r}$, towards the center, which changes the direction;
- the **tangential** acceleration $a_t = \dfrac{dv}{dt} = r\alpha$, along the tangent, which changes the speed.

Its total acceleration is their vector sum:

$$a = \sqrt{a_c^2 + a_t^2}$$

In uniform circular motion $a_t = 0$, and only the centripetal acceleration is left.

DID YOU KNOW?

Spinning out the heavy parts

A **centrifuge** spins test tubes at thousands of rpm. A lab centrifuge of radius 10 cm at 3000 rpm gives $a = 4\pi^2 f^2 r = 4\pi^2 \times 50^2 \times 0.1 \approx 9900$ m/s², about 1000 g. Blood cells, being denser than the plasma round them, need more inward force than the liquid can give them to go round, so they sink to the bottom of the tube within minutes. The washing machine's spin cycle works the same way: the drum holds the clothes in their circle, but the holes in it can't hold the water, which leaves along the tangent.

Check 6

A car goes round a bend of radius 50 m at 10 m/s. Its centripetal acceleration is:

a 0.2 m/s² b 2 m/s²
c 5 m/s² d 500 m/s²

Check 7

A stone goes round a circle at a steady speed. If its speed is doubled on the same circle, its centripetal acceleration becomes:

a the same b twice as large
c four times as large d half as large

6.4 Centripetal force

A body of mass m moving round a circle of radius r at speed v has an acceleration v^2/r towards the center. By Newton's second law, the **resultant force** on it must point towards the center too, with size

$$\boxed{F = \frac{mv^2}{r} = mr\omega^2 = \frac{4\pi^2 mr}{T^2} = 4\pi^2 m f^2 r}$$

This is the **centripetal force**: the resultant force needed to keep a body moving in a circle, directed towards its center.

- When a body is whirled on a string, the **tension** in the string provides the centripetal force. If the string breaks, the inward pull is gone, and the body leaves along the tangent at the point of breaking (Figure 6.4), then falls like any projectile.
- On a road with many bends, the friction between the tyres and the road provides the bus's centripetal force. The seat, through friction, pulls the lower part of your body round with the bus, but your upper body, by its inertia, tends to keep going straight, so it tips towards the outside of the bend. To you, in the turning bus, it feels as if an outward centrifugal force acts on you, of size $mr\omega^2$; seen from the road, your body simply isn't being turned as quickly as the bus.
- $F = mr\omega^2$ says that at the **same angular velocity**, a longer string (larger r) needs **more** force: it is harder to whirl a stone at the same number of turns per second on a longer string. (At the same *speed* v, $F = mv^2/r$ is *smaller* for a larger circle: a gentle bend needs less grip than a sharp one.)
- The force is always at right angles to the velocity, $\vec{F} \cdot d\vec{s} = 0$, so it does no work and **does not change the speed**, only the direction.

Circular motion all around

Speed, acceleration and force are the three quantities that describe any circular motion: the speed $v = r\omega$, the acceleration v^2/r towards the center, and the force mv^2/r that must provide it.

Circular, or nearly circular, motion is everywhere: the Moon round the Earth, the Earth round the Sun (both very nearly circles), satellites round the Earth, electrons round the nucleus in Bohr's model, the wheels of every vehicle, the drum of a washing machine, and centrifuges that separate cream from milk and blood cells from plasma. A closed path in nature is rarely a perfect circle, but a circle is often a very good first description.

Three points from this unit sum up the rest:

- **Newton's first law** says a body can't change its direction by itself; in circular motion a centripetal force changes it. The tendency of the body to keep going straight is what is felt, in the rotating frame, as a centrifugal force.
- In **uniform** circular motion the only acceleration is centripetal; in **non-uniform** circular motion there is a tangential acceleration too, and the total is $a = \sqrt{a_c^2 + a_t^2}$.
- The angular velocity has a direction along the axis, by the **right-hand rule**: up for anticlockwise rotation seen from above, down for clockwise.

IN SHORT

Key ideas of Unit 6

- A body moving in a circle has a velocity along the tangent, always changing direction, so it is accelerating, even at a steady speed.
- Angular displacement $\theta = s/r$ (radians); angular velocity $\omega = d\theta/dt$; $\omega = 2\pi/T = 2\pi f$; $f = 1/T$; $v = r\omega$; $a_t = r\alpha$.
- Centripetal acceleration $a = \dfrac{v^2}{r} = r\omega^2 = 4\pi^2 f^2 r$, towards the center; centripetal force $F = \dfrac{mv^2}{r}$, the resultant of the real forces. Centrifugal force is a pseudo-force, felt only in the rotating frame.
- Conical pendulum: $\tan\theta = \dfrac{v^2}{rg}$, period $2\pi\sqrt{\dfrac{l\cos\theta}{g}}$, tension $\dfrac{mg}{\cos\theta}$.
- Vertical circle: $T = \dfrac{mv^2}{r} + mg\cos\theta$; greatest at the bottom; critical speed at the top $\sqrt{gr}$; least speed at the bottom $\sqrt{5gr}$.
- Level bend: $v_{\max} = \sqrt{\mu g r}$. Banked road, leaning cyclist, banking plane: $\tan\theta = \dfrac{v^2}{rg}$.

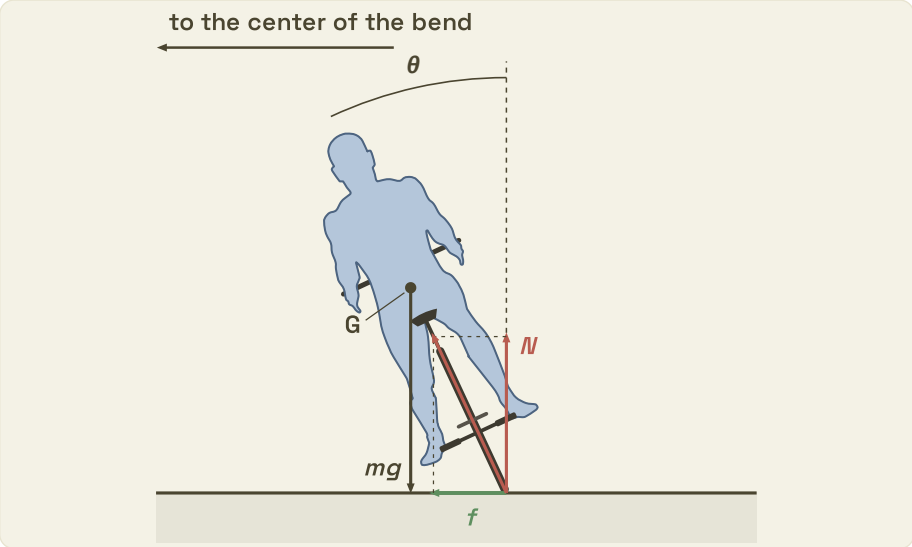


Figure 6.11 *A cyclist leaning into a bend, seen from behind; the center of the bend is to the left. The road pushes up with the normal reaction N and inwards with friction f. Their resultant, the total contact force, points along the line from the tyre to the center of gravity G, at θ to the vertical.*

3. The resultant makes an angle θ with the vertical, with $\tan\theta = \dfrac{f}{N}$. So

$$\boxed{\tan\theta = \frac{v^2}{rg}}, \qquad \theta = \tan^{-1}\frac{v^2}{rg}$$

The faster the cyclist, or the sharper the bend, the more they must lean. And the lean needs friction: $\tan\theta = f/N$ can be at most μ, which is why a cyclist who leans hard on gravel or a wet road slides out.

Some books describe the road's force on a leaning cyclist as a single reaction R along the line of lean; that R is this total contact force, the normal reaction and friction added together, and its parts R cos θ and R sin θ are N and f.

An **airplane** turns the same way: it banks, tilting its wings so that the lift on them, at right angles to the wings, leans towards the center of the turn. The lift's vertical part holds the plane up and its horizontal part is the centripetal force, with $\tan\theta = v^2/(rg)$ again.

Check 12

A road bend of radius 200 m is to be banked for a speed of 20 m/s. The angle of banking is about (g = 9.8 m/s²):

a 2.0° b 11.5°
c 45° d 78.5°

Check 13

On a level road with μ = 0.5, the greatest speed for a bend of radius 20 m is about (g = 10 m/s²):

a 5 m/s b 10 m/s
c 14 m/s d 100 m/s

DID YOU KNOW?

Hairpins on the hills

Nepal's hill roads, the Rajpath climbing from Naubise up to Daman or the Siddhartha Highway between Butwal and Tansen, are full of hairpin bends of radius 15 m or less. With a dry-road friction coefficient of about 0.7, $v_{\max} = \sqrt{0.7 \times 9.8 \times 15} \approx 10$ m/s, about 36 km/h; on a wet road with μ about 0.4 it drops to under 30 km/h. A loaded bus with a high center of gravity can tip over before it slides. That is why the speed limits on such bends are so low, and why the outer edge of a well-built bend is raised.

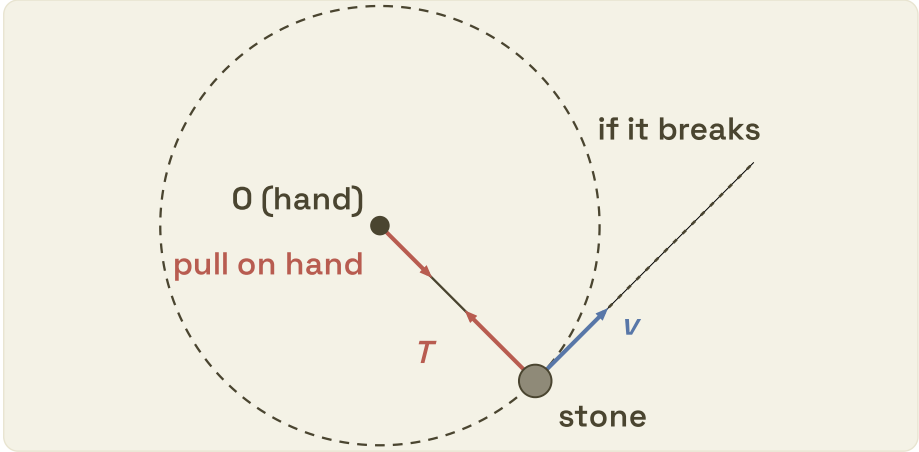


Figure 6.4 *A stone whirled on a string, seen from above. The string's tension T pulls the stone towards the center; the stone pulls the string, and the hand, outwards. If the string breaks, the stone leaves along the tangent (dashed).*

CAREFUL

The centripetal force is not an extra force

Don't draw "centripetal force" as one more arrow beside the string's tension or the friction. In a force diagram, draw the real forces (tension, weight, normal reaction, friction, gravity); then the **resultant** of them, towards the center, must equal mv^2/r. That equation is how every problem in this unit is solved.

Check 8

A 0.5 kg ball is whirled in a horizontal circle of radius 1 m at 4 m/s (ignore its weight). The tension in the string is:

a 2 N b 4 N
c 8 N d 16 N

6.5 Motion in a horizontal circle: the conical pendulum

A **conical pendulum** is a bob on a string that, instead of swinging to and fro, goes round in a **horizontal circle** at a steady speed. The string keeps a constant angle θ with the vertical and sweeps out a cone.

Real-life conical pendulums: a chair-swing ride at a fair (above); a ball whirled on a string at an angle; and the **centrifugal governor** (flyball governor) that controlled the speed of steam engines in the 1800s: two heavy balls on arms spun by the engine swing outwards as it speeds up, and the rising arms close the steam valve.



Figure 6.5 *A chair-swing ride. Each seat on its chain is a conical pendulum: the faster the top spins, the farther out the chains swing.*
Photo: Tila Monto, CC BY 4.0

Derivation. A bob of mass m on a string of length l moves round a horizontal circle of radius r at speed v, the string at an angle θ to the vertical (Figure 6.7). Two forces act on it: its weight mg, down, and the tension T, along the string.

1. **Vertically** the bob doesn't move, so the forces balance:

$$T\cos\theta = mg \tag{6.1}$$

2. **Horizontally** the resultant is the centripetal force:

$$T\sin\theta = \frac{mv^2}{r} \tag{6.2}$$

3. Divide (6.2) by (6.1):

$$\tan\theta = \frac{v^2}{rg}$$

4. From the figure, $\tan\theta = \dfrac{r}{h}$, where $h = l\cos\theta$ is the height of the cone. So

$$\frac{r}{h} = \frac{v^2}{rg} \quad \Rightarrow \quad \frac{g}{h} = \frac{v^2}{r^2} = \omega^2 = \left(\frac{2\pi}{\tau}\right)^2$$

where τ (tau) is the time for one revolution (we use τ here, because T is the tension).

5. So the **time period** of a conical pendulum is

$$\boxed{\tau = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{l\cos\theta}{g}}}$$

6. **The tension.** Square (6.1) and (6.2) and add; $\cos^2\theta + \sin^2\theta = 1$:

$$T^2 = (mg)^2 + \left(\frac{mv^2}{r}\right)^2 \quad \Rightarrow \quad T = mg\sqrt{1 + \left(\frac{v^2}{rg}\right)^2}$$

which is the same as $T = \dfrac{mg}{\cos\theta}$.

Notice what the period depends on: only h, the depth of the cone below the point of support. Two conical pendulums with different strings but the same h go round in step. And a conical pendulum with a small angle has nearly the same period as a simple pendulum of the same length, $2\pi\sqrt{l/g}$, because then $\cos\theta \approx 1$.

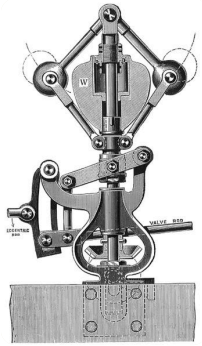


Figure 6.6 *A centrifugal governor (from a book on heat engines, 1913). As the engine speeds up, the balls swing out and up, lifting the collar, which closes the steam valve.*

Photo: Andy Dingley (scanner), Public domain

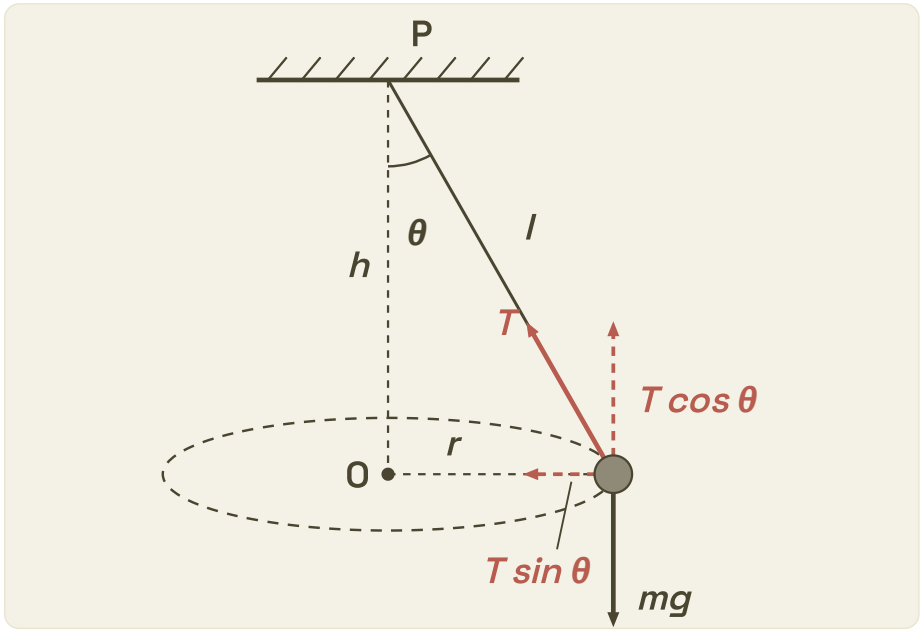


Figure 6.7 *A conical pendulum. A bob of mass m on a string of length l, hanging from P, goes round a horizontal circle of radius r, center O, at speed v; the string makes an angle θ with the vertical, and O is a height h below P. The tension T has a vertical part T cos θ, which balances the weight mg, and a horizontal part T sin θ, towards O, which is the centripetal force.*

In a horizontal circle at a steady speed, the **speed**, the **kinetic energy** and the **angular momentum** about the center of the circle stay constant; the velocity, the acceleration and the force keep changing direction.

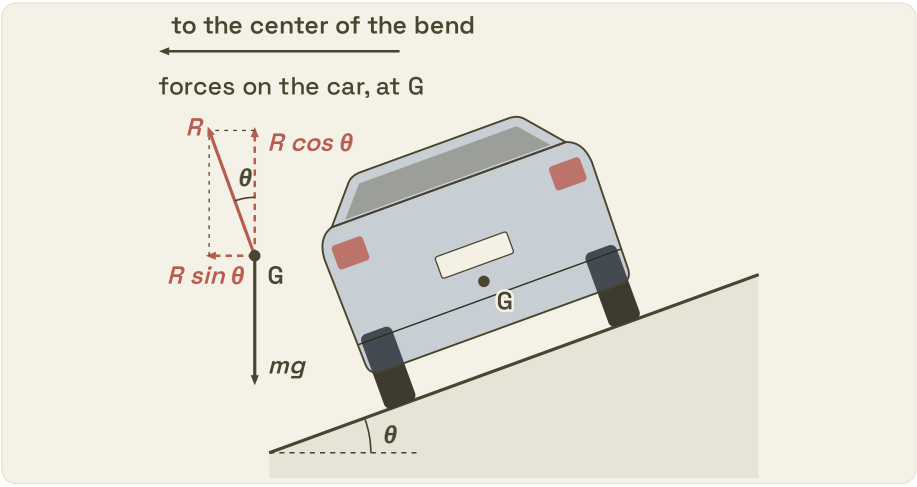


Figure 6.10 *A car on a road banked at θ, seen from behind; the center of the bend is to the left. The total normal reaction R (on all four tyres) is at θ to the vertical. Its vertical part R cos θ balances the weight mg; its horizontal part R sin θ, towards the center, is the centripetal force.*

A road banked at this angle is just right for one speed v, its **design speed**. Slower than that, a car tends to slide down the bank, and friction must act up the slope; faster, it tends to slide up and out, and friction acts down the slope. Within limits, friction covers both.

WORKED EXAMPLE

A racing car on a banked track

A 1000 kg racing car goes round a banked track at a steady 108 km/h. The horizontal radius of the track is 100 m, and the total reaction at the wheels is normal to the track (no friction needed). Find the angle of banking and the reaction at the wheels (g = 10 m/s²).

Answer

$v = \dfrac{108 \times 1000}{3600} = 30$ m/s.

$$\tan\theta = \frac{v^2}{rg} = \frac{30^2}{100 \times 10} = \frac{900}{1000} = 0.9 \quad \Rightarrow \quad \theta = \tan^{-1} 0.9 = 42^\circ$$

From $R\cos\theta = mg$:

$$R = \frac{mg}{\cos\theta} = \frac{1000 \times 10}{\cos 42^\circ} = \frac{10\,000}{0.743} = 13\,460 \text{ N}$$

The angle is **42°** and the reaction about **13,500 N**, a third more than the car's weight.

On a **railway**, the bend is banked by raising the outer rail above the inner one by a height h. With the rails a distance d apart (the **gauge**), $\sin\theta = h/d$, and for small angles $h \approx d\tan\theta = \dfrac{dv^2}{rg}$. Without the raised rail, the flanges of the wheels would press on the outer rail, and the rail's sideways push on them would supply the centripetal force.

6.7.3 A cyclist rounding a bend

A cyclist going round a bend **leans inwards**, towards the center, by an angle θ from the vertical (Figure 6.11). Why?

On a level road, the road gives the tyre two forces: the normal reaction N, upwards, and friction f, inwards, which is the centripetal force. Their resultant, the total contact force, is tilted inwards. If the line of that force did not pass through the cyclist's center of gravity G, it would have a turning effect and topple the cyclist. Leaning so that G lies on that line keeps the cyclist balanced:

1. Vertically: $N = mg$.
2. Horizontally, towards the center: $f = \dfrac{mv^2}{r}$.

6.7.1 A car on a level curved road

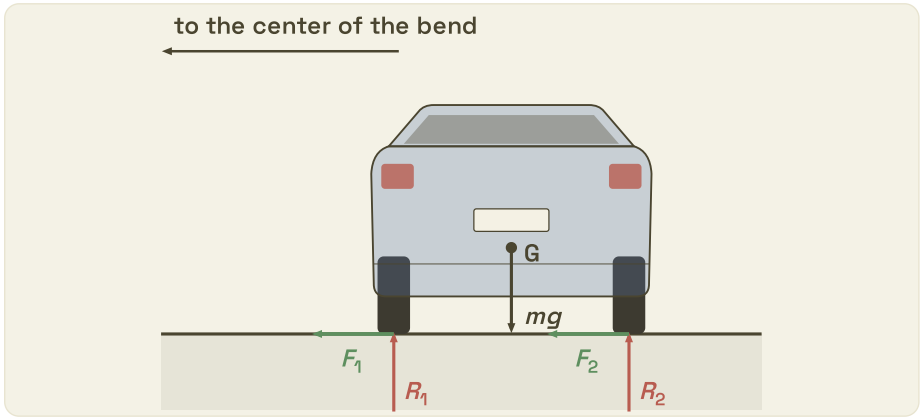


Figure 6.9 *A car going round a level bend, seen from behind; the center of the bend is to the left. The road pushes up on the tyres with normal reactions R_1 and R_2, and friction F_1 and F_2 on the tyres points towards the center of the bend.*

A car of mass m goes round a level bend of radius r at speed v (Figure 6.9). Without enough grip, its tyres would slide outwards, away from the center; friction on the tyres opposes that slide, so it points **towards the center**, and it is the only horizontal force on the car. So friction provides the centripetal force.

1. Let F_1 and F_2 be the friction on the inner and outer tyres, and R_1 and R_2 the normal reactions. The most friction the road can give is μ times the normal reaction ($F = \mu R$):
$$F_1 + F_2 \le \mu(R_1 + R_2)$$
2. Vertically the car doesn't move, so $R_1 + R_2 = mg$. The greatest total friction is $F = \mu mg$.
3. The friction must provide the centripetal force: $\dfrac{mv^2}{r} \le \mu mg$. So
$$\boxed{v_{\max} = \sqrt{\mu g r}}$$

This is the greatest speed at which the car can take the bend. Faster than this, friction can't supply mv^2/r, and the car **skids** outwards, off the road. Notice that $v_{\max}$ doesn't depend on the car's mass: a heavy truck and a small car on the same road have the same limit. A wet or icy road (small μ) and a tight bend (small r) both lower it.

6.7.2 Banking of roads

On highways and race tracks, the outer edge of the road at a bend is raised above the inner edge. This tilting is called **banking**, and the angle θ that the road makes with the horizontal is the **angle of banking**.

Why? On a tilted road, the normal reaction is no longer vertical: it leans towards the center of the bend, so its horizontal part supplies centripetal force. The car can then take the bend without relying on friction, which is a great help on a wet or icy road, and it saves the tyres from wear.

Derivation. A car of mass m goes round a bend of radius r at speed v, on a road banked at θ, and we want friction not to be needed at all (Figure 6.10). Let $R = R_1 + R_2$ be the total normal reaction, at right angles to the road, so at θ to the vertical.

1. Vertically, no acceleration:
$$R\cos\theta = mg \tag{6.3}$$
2. Horizontally, the resultant is the centripetal force:
$$R\sin\theta = \frac{mv^2}{r} \tag{6.4}$$
3. Divide (6.4) by (6.3):
$$\boxed{\tan\theta = \frac{v^2}{rg}}, \qquad \theta = \tan^{-1}\frac{v^2}{rg}$$

WORKED EXAMPLE

How many turns before it breaks?

A 0.5 kg object is whirled in a horizontal circle on a string 1 m long. The string breaks if its tension goes above 50 N. What is the greatest number of revolutions per second it can make? (The speed is high, so the string is almost horizontal: ignore the weight and take r = 1 m.)

Answer

$T = mr\omega^2 = 4\pi^2 mf^2 r$, so
$$f = \frac{1}{2\pi}\sqrt{\frac{T}{mr}} = \frac{1}{2\pi}\sqrt{\frac{50}{0.5\times 1}} = \frac{10}{2\pi} = 1.59 \text{ rev/s}$$
At most about **1.6 revolutions per second**.

WORKED EXAMPLE

Revolutions per minute

A string breaks when the tension in it reaches 25 N. A 500 g mass is tied to a 1 m length of it and whirled in a horizontal circle. Find the greatest number of revolutions per minute it can make without breaking the string. (Ignore the weight, as above.)

Answer

$$f = \frac{1}{2\pi}\sqrt{\frac{T_{\max}}{mr}} = \frac{1}{2\pi}\sqrt{\frac{25}{0.5\times 1}} = \frac{7.07}{2\pi} = 1.125 \text{ rev/s}$$
In a minute: $1.125 \times 60 =$ **67.5 rev/min**.

Check 9

A conical pendulum is made to go round faster. Its string:

a hangs closer to the vertical
b swings out to a larger angle from the vertical
c keeps the same angle
d becomes horizontal at any speed

6.6 Motion in a vertical circle

In a **vertical** circle, gravity acts along the path, so the tension in the string changes from point to point, and (unless a motor keeps it steady) the speed changes too: the body slows down as it climbs and speeds up as it falls. A Ferris wheel, a roller-coaster loop, a bucket of water whirled over your head and a pilot looping the loop all move in vertical circles.

Derivation of the tension. A body of mass m on a string of length r goes round a vertical circle, center O (Figure 6.8). At a point P, where the string makes an angle θ with the **downward** vertical, its speed is v. Two forces act on it: the tension T, along the string towards O, and its weight mg.

1. Resolve the weight: $mg\cos\theta$ along the string, **away** from O; $mg\sin\theta$ along the tangent (this part changes the speed).
2. Along the string, the resultant towards the center is the centripetal force:
$$T - mg\cos\theta = \frac{mv^2}{r} \quad\Rightarrow\quad \boxed{T = \frac{mv^2}{r} + mg\cos\theta}$$

Three special points. Here v means the speed **at that point**.

- **At the lowest point A** ($\theta = 0^\circ$, $\cos\theta = 1$): the tension must hold the body up and pull it round, so it is greatest:
$$T_A = T_{\max} = \frac{mv_A^2}{r} + mg$$
The string is most likely to break here.
- **At the highest point C** ($\theta = 180^\circ$, $\cos\theta = -1$): the weight itself points to the center and does part of the job, so the tension is least:
$$T_C = T_{\min} = \frac{mv_C^2}{r} - mg$$
- **At B and D, level with the center** ($\theta = 90^\circ$, $\cos\theta = 0$): the weight is along the tangent, so the tension alone is the centripetal force:

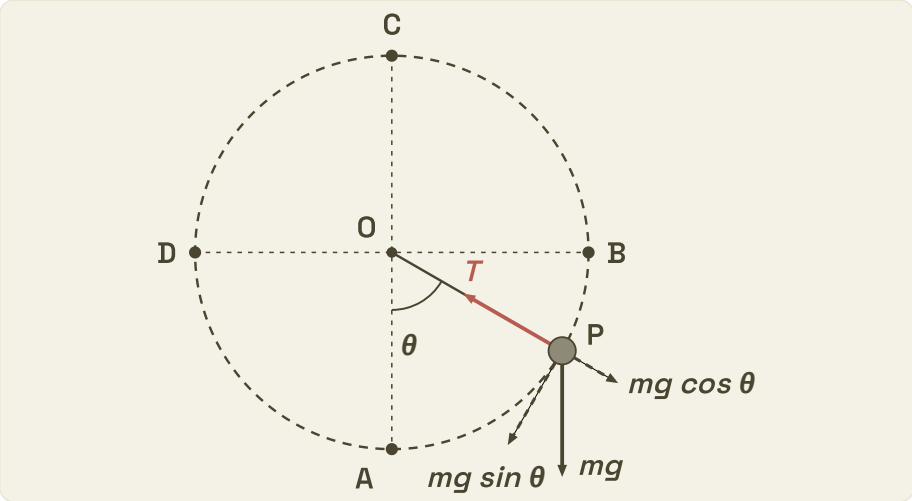


Figure 6.8 *A body of mass m whirled on a string of length r in a vertical circle, center O. At P the string makes an angle θ with the downward vertical. The weight mg is split into mg cos θ, along the string away from O, and mg sin θ, along the tangent. A is the lowest point, C the highest, B and D the points level with O.*

The critical velocity at the top

A string can pull but never push: the tension can't be negative. As the speed at the top falls, T_C falls, and the slowest the body can go at the top while the string stays tight is when $T_C = 0$:

$$0 = \frac{mv_C^2}{r} - mg \quad \Rightarrow \quad \boxed{v_C = \sqrt{gr}}$$

This is the **critical velocity**: the least speed at the highest point for the body to go round the vertical circle. Then gravity alone provides the centripetal force. Below it, the string goes slack before the top, and the body leaves the circle and falls inside it as a projectile.

The least speed at the bottom

If nothing but the string and gravity act (no motor driving it, no air resistance), the body's mechanical energy is conserved, and it slows as it climbs. To arrive at the top C with at least $\sqrt{gr}$, how fast must it go at the bottom A? C is $2r$ higher than A:

$$(\text{K.E.} + \text{P.E.})_A = (\text{K.E.} + \text{P.E.})_C$$

$$\tfrac{1}{2}mv_A^2 + 0 = \tfrac{1}{2}mv_C^2 + mg(2r) = \tfrac{1}{2}m(gr) + 2mgr$$

$$v_A^2 = gr + 4gr = 5gr \quad \Rightarrow \quad \boxed{v_A = \sqrt{5gr}}$$

This is the least speed at the lowest point for the body to go safely round the whole circle. With it, the tension at the bottom is $T_A = \dfrac{m(5gr)}{r} + mg = 6mg$: six times the weight.

At any point P, energy conservation also gives the speed: P is $r(1 - \cos\theta)$ above A, so $v^2 = v_A^2 - 2gr(1 - \cos\theta)$.

CAREFUL

Constant speed, or free?

Some problems say the body is whirled at a **constant speed** (a motor, or a hand, keeps it going): then use the same v at every point in $T = \dfrac{mv^2}{r} + mg\cos\theta$. In others the body **moves freely** under gravity: then its speed changes, and you must find the speed at each point by energy conservation first. Read which one the question means.

WORKED EXAMPLE

Tensions in a vertical circle

An 8 kg object is whirled in a vertical circle of radius 2 m at a constant speed of 6 m/s. Find the greatest and least tensions in the string (g = 10 m/s²).

Answer

$\dfrac{mv^2}{r} = \dfrac{8 \times 6^2}{2} = 144$ N, and $mg = 80$ N.
Greatest, at the bottom: $T_{\max} = 144 + 80 =$ **224 N**.
Least, at the top: $T_{\min} = 144 - 80 =$ **64 N**.

WORKED EXAMPLE

The string that breaks

A 500 g stone on a string 50 cm long is whirled in a vertical circle whose center is 100 cm above the ground. The string breaks if its tension goes above 20 N. The angular speed is increased very slowly until the string breaks. Where is it most likely to break, at what angular speed, and where does the stone hit the ground? (g = 10 m/s²)

Answer

Where: the tension is greatest at the **lowest point**, so the string breaks there, when the stone is 100 − 50 = 50 cm above the ground, moving horizontally.

Angular speed: at the bottom $T_{\max} = \dfrac{mv^2}{r} + mg$:

$$20 = \frac{0.5\,v^2}{0.5} + 0.5 \times 10 \quad \Rightarrow \quad v^2 = 15, \quad v = 3.87 \text{ m/s}$$

$$\omega = \frac{v}{r} = \frac{3.87}{0.5} = 7.75 \text{ rad/s}$$

Where it lands: it leaves horizontally at 3.87 m/s from 0.50 m up, a projectile fired horizontally (Unit 3). Time to fall:

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 0.50}{10}} = 0.316 \text{ s}$$

Horizontal distance: $x = vt = 3.87 \times 0.316 =$ **1.22 m** from the point directly below where it broke, in the direction it was moving.

DID YOU KNOW?

Why loops aren't circles

The first circular roller-coaster loops, around 1900, gave riders a hard jolt: to clear the top safely, the car had to enter the bottom so fast that riders were pressed into their seats with at least 6 times their weight there, as $T_A = 6mg$ says, and more with a margin for safety. Modern loops are tall teardrops (clothoid loops): tight at the top, where the car is slow, and wide at the bottom, where it is fast, so v^2/r stays more even and the push on the riders is far gentler.

Check 10

A bucket of water is whirled in a vertical circle of radius 1 m. The least speed at the top for the water to stay in (g = 9.8 m/s²) is about:

a 1 m/s
b 3.1 m/s
c 7.0 m/s
d 9.8 m/s

Check 11

In a vertical circle, the tension in the string is greatest:

a at the lowest point
b at the highest point
c at the points level with the center
d the same everywhere

6.7 Applications of centripetal force

Every vehicle that takes a bend is moving on an arc of a circle, and something must supply its centripetal force. On a flat road it is **friction** between the tyres and the road; on a tilted (banked) road, part of it is the road's normal reaction.