Unit 5

Work, Energy and Power

This unit online, with its interactives:
science-playground.com/s/11f5

A porter climbs from Lukla towards Namche with a 30 kg load on his back. A pump lifts water to a tank on a roof in Lalitpur. Water from the Kulekhani reservoir rushes down a pipe and spins a turbine. A cricket ball hits the bat and flies off faster than it came. In each, something is pushed through a distance, and something called **energy** moves from one place to another.

In Unit 4 you learned how forces change motion. This unit follows the **energy** that forces carry: how much **work** a force does, how fast it does it (**power**), the energy of motion and of position, the great law that energy is never made or lost, and what happens to energy when bodies collide.

5.1 Introduction

Our everyday words *work*, *energy* and *power* became exact ideas of physics only about two hundred years ago.

- The word **energy** comes from the Greek *ἐνέργεια* (energeia), "activity", first used by Aristotle in the 4th century BCE.
- In the 1680s Gottfried **Leibniz** argued that a moving body carries a "living force", *vis viva*, which he measured by its mass times the square of its speed, mv^2. It was the first step towards kinetic energy.
- In his lectures at the Royal Institution in London, published in 1807, Thomas **Young** used the word *energy* in its modern sense, in place of *vis viva*.
- In 1826–1829 the French engineer Gaspard-Gustave **Coriolis** gave *work* (French *travail*) its exact meaning, the force times the distance moved along it, "weight lifted through a height" as in the steam engines that pumped water out of flooded mines. He also put the factor ½ into Leibniz's "living force", making it $\frac{1}{2}mv^2$, so that it equals the work done to produce it. About twenty years later William Thomson (Lord Kelvin) gave it the name we use, **kinetic energy**.
- In 1853 the Scottish engineer William **Rankine** introduced the term **potential energy**.



Figure 5.1 *Thomas Young (1773–1829), after a portrait by Thomas Lawrence. Young was the first to use "energy" in its modern sense; he also showed that light is a wave.*
Photo: Wikimedia Commons, Public domain

5.2 Work

Work is done when a force acts on a body and the body moves in the direction of the force. It is measured by the force times the displacement **in the direction of the force**. If a constant force F moves a body through a displacement d along it,

$$W = Fd$$

If the force makes an angle θ with the displacement (Figure 5.2), only its part along the displacement, $F\cos\theta$, does work; the part $F\sin\theta$, at right angles to the motion, does none:

$$W = (F\cos\theta)\,d = Fd\cos\theta = \vec{F}\cdot\vec{d}$$

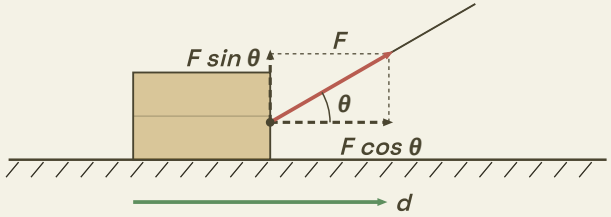


Figure 5.2 *A box pulled along the floor by a rope at an angle θ above the horizontal. The pull F splits into F cos θ along the floor, which does the work, and F sin θ upward, at right angles to the motion, which does none. The box moves a displacement d.*

Work is the **scalar (dot) product** of two vectors, force and displacement (Unit 2), so work itself is a **scalar**: it has a size and a sign, but no direction.

- **θ = 0°:** force and displacement in the same direction, $W = Fd$, the most work this force can do. (Pushing a car whose engine won't start.)
- **0° < θ < 90°:** positive work, less than Fd. (Pulling a desk across the classroom by its edge, at an angle.)
- **θ = 90°:** $\cos 90^\circ = 0$, so $W = 0$. A force at right angles to the motion does no work.
- **90° < θ ≤ 180°:** $\cos\theta$ is negative, so the work is **negative**: the force acts against the motion. Friction on a sliding box (θ = 180°) does negative work, $W = -fd$.

When is no work done?

For work to be done, there must be a displacement, and the force must have a part along it. So **no work** is done:

1. **If there is no displacement** ($d = 0$). A guard standing all day at a gate, or a man standing still with a load, does no work on the load, however tired he gets. Static friction on a box that stays at rest does no work, because the box doesn't move.
2. **If the force is at right angles to the displacement** (θ = 90°).
   - A man walking along a level road with a bucket of water does no work on the bucket: his upward force holds it up, but it moves sideways.
   - A porter with a load on his head walking along a level platform does no work on the load, for the same reason.
   - The centripetal force on a body moving in a circle, or the tension in a string whirling a stone round, does no work: it always points to the center, at right angles to the motion (Unit 6).
3. **If there is no force** along the motion. A puck gliding over smooth ice at a steady speed needs no force, and none does any work on it.

But a **porter climbing stairs** with a suitcase on his head does work on it: he pushes it upward, and it moves upward, in the direction of his push.

> CAREFUL
> **Tired doesn't mean work done**
>
> In physics, holding a heavy bag still does no work on the bag, though your arm aches. Your muscles are not like a table, which can hold the bag up for ever without effort: muscle fibers keep contracting and relaxing to hold a load, and that uses energy from your food, which turns into heat. The energy is spent inside you, not on the bag.

Units and dimensions of work

The SI unit of work is the **joule** (J), named after the English physicist James Prescott Joule. **One joule is the work done when a force of 1 N moves its point of application through 1 m in the direction of the force:**

$$1\ \text{J} = 1\ \text{N} \times 1\ \text{m} = 1\ \text{N m}$$

Older units are still met:

- the **gravitational unit**, the kilogram-force meter (kgf m), the work done in lifting 1 kg through 1 m: 1 kgf m = 9.8 J;
- the CGS unit, the **erg** (1 dyne × 1 cm): $1\ \text{J} = 10^7$ erg.

Dimensions:

$$[W] = [F][d] = [\text{MLT}^{-2}][\text{L}] = [\text{ML}^2\text{T}^{-2}]$$

38. A tandem (two-person) bicycle team must overcome a force of 165 N to keep a speed of 9.00 m/s. Find the power each rider must give, if they share equally, in watts and in horsepower.

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.

Check 1

A girl pulls a sledge 20 m across level snow with a rope at 60° to the ground, with a force of 50 N. How much work does her pull do?

a 1000 J
b 866 J
c 500 J
d 0 J

5.3 Work done by a variable force

So far the force has been **constant**. Often it isn't: its size, its direction, or both change as the body moves. The force needed to stretch a spring grows with the stretch; the pull of gravity, and the electric and magnetic forces, change with distance. How do we find the work then?

The analytical method

Split the path into tiny steps $d\vec{s}$, so small that over each step the force is practically constant. The work done in one step is

$$dW = \vec{F} \cdot d\vec{s} = F \cos\theta \, ds$$

and the total work from x_1 to x_2 is the sum of all these small bits, an **integral**:

$$W = \int_{x_1}^{x_2} \vec{F} \cdot d\vec{s} = \int_{x_1}^{x_2} F \cos\theta \, ds$$

Pushing a wheelbarrow along a curving path, both your push and the angle between your push and the path keep changing; the work is found by adding up $F \cos\theta \, ds$ step by step along the path.

The graphical method

Let a force $F(x)$ act along the x-axis, and plot it against x from x_i to x_f (Figure 5.3). Divide the displacement into narrow strips of width Δx, so narrow that F is nearly constant across each. The work done over one strip is $F\Delta x$: the **area** of that strip. Adding all the strips,

$$W \approx \sum_{x_i}^{x_f} F \, \Delta x$$

and as the strips become infinitely narrow ($\Delta x \to 0$) the sum becomes exact:

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F \, \Delta x = \int_{x_i}^{x_f} F \, dx$$

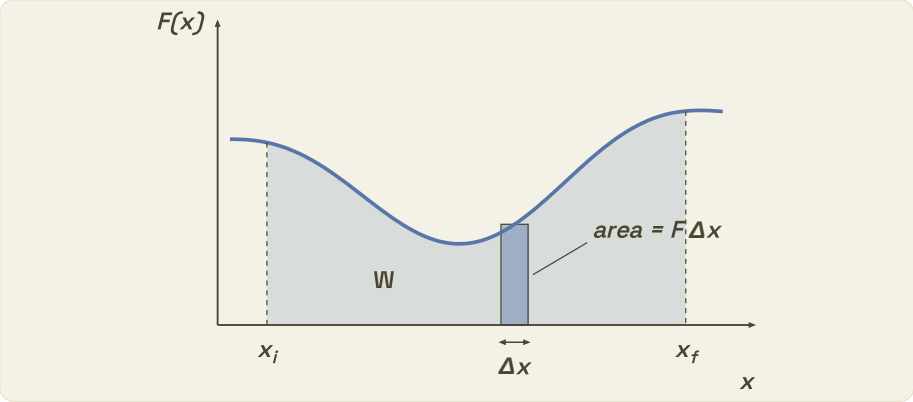


Figure 5.3 *A force that changes with position, F(x), plotted from x_i to x_f. One narrow strip of width Δx has area FΔx, the work done over that small step. The work done over the whole displacement is the whole shaded area under the curve.*

The work done by a force is the area under its force–displacement graph. (Compare: in Unit 3 the displacement was the area under the velocity–time graph.)

WORKED EXAMPLE

A block on a spring

A block of mass 3 kg is attached to a spring of spring constant 5 N/m. It is pulled 2 m from its equilibrium position and released. How much work does the spring do on it as it returns to the equilibrium position for the first time?

Answer

At a displacement x the spring pulls the block back with $F = -kx$ (towards the equilibrium position). The block moves from $x_i = 2$ m to $x_f = 0$:

$$W = \int_2^0 (-5x)\,dx = -5\left[\frac{x^2}{2}\right]_2^0 = -\frac{5}{2}(0-4) = \mathbf{10}\ \text{J}$$

The work is **positive**: the spring pulls the block towards the equilibrium position, and the block moves that way. (The block's mass doesn't come into it. It is also the area of the triangle under the graph of kx: $\tfrac{1}{2}\times 2\times 10 = 10$ J.)

YOUR TURN

A force that grows

A force $F = (10 + 0.50x)$ N acts on a particle along the x-axis (x in meters). Find the work it does from $x = 0$ to $x = 2$ m.

Answer

$W = \int_0^2 (10 + 0.50x)\,dx = \left[10x + 0.25x^2\right]_0^2 = 20 + 1 = \mathbf{21}$ **J**. (On the graph: a trapezium with sides 10 N and 11 N and width 2 m, area $\tfrac{1}{2}(10+11)\times 2 = 21$ J.)

Work done on a body and by a body

- **Work done on a body** by a force from outside is **positive** when the force acts along the body's motion. The body **gains** energy. Lifting a bucket, you do positive work on it, and it gains potential energy.
- **Work done by a body** on its surroundings means the body pushes something through a distance. It **loses** energy doing it. A moving hammer does work on a nail and slows down; the surroundings do negative work on it.

Every push has a pair (Newton's third law): if A does +50 J of work on B, B does −50 J on A. So the sign of work tells us which way energy flows: **positive work on a body carries energy into it, negative work carries energy out.**

5.4 Power

Two porters carry the same 30 kg load up the same 300 m of trail. Both do the same work, about 88 kJ. But the one who does it in one hour is more **powerful** than the one who takes two. **Power** is the **rate of doing work**, or of transferring energy:

$$P = \frac{W}{t}$$

This is the **average power** over the time t. For the same work, the shorter the time, the greater the power; a very long time means a very small power.

If a constant force $\vec{F}$ moves a body at a velocity $\vec{v}$, in a short time dt it moves $d\vec{s} = \vec{v}\,dt$ and does work $dW = \vec{F}\cdot d\vec{s}$, so the power at that instant (the **instantaneous power**) is

$$P = \frac{dW}{dt} = \vec{F}\cdot\frac{d\vec{s}}{dt} = \vec{F}\cdot\vec{v} = Fv\cos\theta$$

Power is a scalar. Its SI unit is the **watt** (W), named after the Scottish engineer James Watt, whose improved steam engine powered the Industrial Revolution:

$$1\ \text{W} = 1\ \text{J/s}$$

Larger units are the kilowatt (1 kW = 1000 W) and the megawatt (1 MW = 10^6 W). Watt himself compared his engines with horses: the **horsepower** is still used for vehicles and pumps,

$$1\ \text{hp} = 746\ \text{W}$$

In electricity, the power taken by a device is the voltage times the current, $P = VI$ (Unit 23).

Dimensions: $[P] = \dfrac{[\text{ML}^2\text{T}^{-2}]}{[\text{T}]} = [\text{ML}^2\text{T}^{-3}]$.

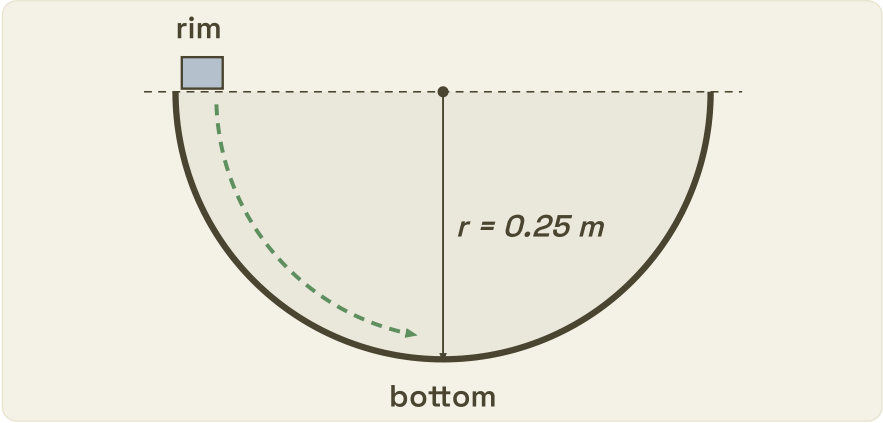


Figure 5.17 *For question 23: a block slides down the inside of a smooth bowl, from its rim to its bottom, a drop equal to the bowl's radius.*

24. An object moves from point A, position vector $\vec{r}_1 = 3\hat{i} + 2\hat{j}$, to point B, $\vec{r}_2 = 2\hat{i} + 3\hat{j}$, under a force $\vec{F} = \hat{i} + \hat{j}$ (in N and m). Find the work done by the force.

25. A particle moves along the x-axis from x = 0 to x = 5 m under a force $F = 3x^2 - 2x + 7$ (F in N, x in m). Find the work done by the force.

26. A car of mass 1000 kg moves at a constant 20 m/s along a level road against a friction force of 200 N. Find the power developed by its engine. The car then climbs a slope at the same constant speed, with the same friction force; $\sin\theta = \frac{1}{20}$. Find the new power.

27. In a football match, player A collides with player B, running towards him from the other side, and they become entangled. Player A's mass is 40 kg and his velocity 4 m/s; player B's mass is 60 kg and his velocity 3 m/s. Find their velocity just after the collision, assuming other forces are negligible.

28. A train of mass 2.0×10^5 kg moves at a constant 72 km/h up a straight incline against a friction force of 1.28×10^4 N. The track rises 1.0 m for every 100 m along it. Find a. the rate at which the train's potential energy increases, and b. the power the train must develop.

29. A radioactive nucleus of mass 210 units, at rest, decays into an alpha particle of mass 4 units and a residual nucleus of mass 206 units. If the alpha particle's kinetic energy is E, find the kinetic energy of the residual nucleus.

30. A railway truck of mass 4×10^4 kg moving at 3 m/s collides with another truck of mass 2×10^4 kg at rest. They couple together and move off. What fraction of the first truck's initial kinetic energy remains as kinetic energy of the two trucks?

31. A bullet of mass 0.010 kg is fired vertically upward at 100 m/s into a wooden block of mass 0.190 kg hanging by a long string above the gun. The bullet stays embedded in the block. Through what height does the block rise?

32. A vehicle of mass 2000 kg traveling at 10 m/s on a level road is stopped in 12.5 m by its brakes. Find the average retarding force. What power must its engine develop to take it up a slope of 1 in 10 at a constant 10 m/s, if the frictional resistance is 200 N?

33. Anisha pushes her physics book 1.50 m along a horizontal table top with a horizontal force of 2.40 N. The opposing force of friction is 0.600 N. Find a. the work done by her 2.40 N force, b. the work done by friction, c. the total work done on the book.

34. How many joules of energy does a 100 W light bulb use in one hour? How fast would a 70 kg person have to run to have that much kinetic energy?

35. A ball of mass 0.420 kg is moving at 2.00 m/s. Rojina kicks it with a constant force of 40.0 N in the direction of its motion. Over what distance must her foot stay in contact with the ball to raise its speed to 6.00 m/s?

36. A worker pushes a block of ice with a constant horizontal force of 80 N on a smooth level floor (friction negligible). The block starts from rest and moves 11 m in 5 s. a. What is the mass of the block? b. If the worker stops pushing after 5 s, how far does the block move in the next 5 s?

37. A 6.0 kg box moving at 3.0 m/s on a smooth horizontal surface runs into a light spring of force constant 75 N/cm. Use the work–energy theorem to find the greatest compression of the spring.

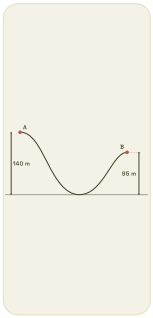


Figure 5.16 *For question 15: a roller coaster track; A is 140 m above the lowest point, and B is 95 m above it.*

16.
a. Can the kinetic energy of a body ever be negative?
b. The momentum of a body is increased by 100 %. What is the percentage increase in its kinetic energy?

17.
a. Explain how the mechanical energy of a freely falling body stays constant until it reaches the ground.
b. When a constant force acts on a body that is speeding up, is the power of the force constant? If not, how would the force have to change with speed for the power to stay constant?

18.
a. Define the coefficient of friction.
b. A 68.5 kg skater moving at 2.40 m/s on rough horizontal ice slows down uniformly and stops in 3.52 s, because of friction from the ice. What force does friction exert on the skater?

19.
a. Show that mechanical energy is conserved for a body falling freely.
b. State the principle of conservation of mechanical energy.
c. A block of weight 150 N is pulled 20 m along a horizontal surface at constant velocity. The coefficient of kinetic friction is 0.20, and the pull makes an angle of 60° with the vertical. Find the work done by the pull.

20.
a. Distinguish between an elastic and an inelastic collision.
b. Explain an elastic collision in one dimension.
c. A vehicle of mass 1.5×10^5 kg with an engine of power 650 kW climbs a slope that rises 1 in 100, at a constant speed of 60 km/h. Find the friction force between its wheels and the slope.

21.
a. State the work–energy theorem.
b. A mass at rest explodes into two parts of 4 units and 40 units. The larger part has 100 J of kinetic energy. Find the kinetic energy of the smaller part.
c. Comment on why the smaller part gets so much more energy.

Numerical problems

22. A ball of mass 0.1 kg is thrown vertically upward at 20 m/s. What is its potential energy at its highest point? What is its potential energy when it is three-quarters of the way up, on its way up?

23. A block of mass 20 kg slides from the rim to the bottom of a smooth bowl of diameter 0.5 m. Find its speed at the bottom.

CAREFUL
The kilowatt-hour is energy, not power

Electricity bills in Nepal charge for **units**: one unit is one **kilowatt-hour** (kWh), the energy used by a 1 kW appliance in one hour. It is a unit of energy: $1\ \text{kWh} = 1000\ \text{W} \times 3600\ \text{s} = 3.6 \times 10^6\ \text{J}$. A watt measures how fast energy is used; a kilowatt-hour measures how much.

WORKED EXAMPLE
The power of a lift motor

A lift of mass 500 kg is to be raised at a constant velocity of 0.4 m/s. What is the least power its motor must have, in watts and in horsepower?

Answer
At constant velocity the motor's upward pull balances the weight: $F = mg = 500 \times 9.8 = 4900$ N. Force and velocity are both upward ($\theta = 0$):

$$P = Fv = 4900 \times 0.4 = \mathbf{1960}\ \text{W} = \frac{1960}{746} = \mathbf{2.63}\ \text{hp}$$

(With g = 10 m/s², 2000 W or 2.68 hp.) A real motor must be more powerful, to start the lift, carry passengers and overcome friction.

YOUR TURN
Bullets into a target

A gun fires 8 bullets a second into a target. Each bullet has a mass of 3 g and a speed of 600 m/s. Find the power delivered to the target.

Answer
Each bullet carries $\frac{1}{2}mv^2 = \frac{1}{2} \times 0.003 \times 600^2 = 540$ J of kinetic energy (Section 5.5), and 8 arrive each second: $P = 8 \times 540 = 4320\ \text{W} = \mathbf{4.32\ kW}$.

DID YOU KNOW?
How powerful is a person?

A fit person can climb stairs at about 0.5 m of height a second. For a 60 kg student that is $mgv = 60 \times 9.8 \times 0.5 \approx 300$ W, nearly half a horsepower, but only for a minute or so. Over a whole day a porter's steady output is nearer 50–100 W. The hydropower stations of Nepal produce thousands of megawatts: millions of porters' worth.

5.5 Energy

Energy is the capacity to do work. A body that has energy can do work, and when it does work it loses that much energy; when work is done on a body, it gains that much. So energy is measured in the same units as work, joules.

Energy comes in many forms. **Mechanical energy**, the energy of motion and of position, is the subject of this unit; there are also heat (thermal) energy, light, sound, chemical, electrical and nuclear energy. Mechanical energy has two kinds: **kinetic energy** and **potential energy**.

5.5.1 Mechanical energy: kinetic energy

A child is about to run into a busy road, and you sprint to catch her. In a race on sports day you run as fast as you can. Walking and chatting with friends, you move slowly. In each case your body has energy **because it is moving**, more the faster you go.

Kinetic energy (KE) is the energy a body has because of its motion. A body of mass m moving with speed v has kinetic energy

$$K = \frac{1}{2}mv^2$$

Proof. A body of mass m, at rest, is pushed by a constant force F along a smooth level surface (Figure 5.4). After a displacement s its velocity is v.

1. Its acceleration is constant, so (from Unit 3) with $u = 0$:

$$v^2 = u^2 + 2as = 2as \quad \Rightarrow \quad as = \frac{v^2}{2}$$

2. The work done on it by the force is

$$W = Fs = (ma)s = m(as) = \frac{1}{2}mv^2$$

3. This work has all gone into the body's motion, so its kinetic energy is

$$\boxed{K = \frac{1}{2}mv^2}$$



Figure 5.4 *A body of mass m starts from rest (u = 0) on a smooth floor. A constant force F pushes it through a displacement s, and it reaches a velocity v. The work done, Fs, is the kinetic energy it gains, ½mv².*

Kinetic energy is a scalar. It depends on the **square** of the speed: double the speed and the kinetic energy is four times as large. That is why a car crash at 80 km/h is far more destructive than one at 40 km/h. And since m and v^2 can never be negative, **kinetic energy is never negative**: it is zero for a body at rest and positive for any moving body.

Check 2

A motorcycle's speed rises from 10 m/s to 20 m/s. Its kinetic energy:

a doubles
b stays the same
c triples
d becomes four times as large

The work–energy theorem

After a long day digging in the field you are tired and hungry: you have spent energy, and the work you did is where it went. Work and energy are linked, and the link is exact.

A body of mass m moving on a smooth surface with velocity u is pushed by a constant resultant force F along its motion, and after a displacement s its velocity is v.

1. The work done on it is $W = Fs = mas$.
2. From $v^2 = u^2 + 2as$:

$$as = \frac{v^2 - u^2}{2}$$

3. So

$$W = m\frac{v^2 - u^2}{2} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = K_f - K_i = \Delta K$$

The work–energy theorem: the work done by the resultant force on a body equals the change in its kinetic energy.

$$W_{\text{net}} = \Delta K \quad \text{or} \quad \frac{1}{2}mv^2 = \frac{1}{2}mu^2 + W_{\text{net}}$$

The final kinetic energy is the initial kinetic energy plus the work done. If the net work is positive, the body speeds up; if negative (friction, braking), it slows down. The theorem holds for a varying force too, with $W = \int F\,dx$.

Check 13

9. The work done by the resultant force on a body changes its:

a total energy
b kinetic energy
c potential energy
d chemical energy

Check 14

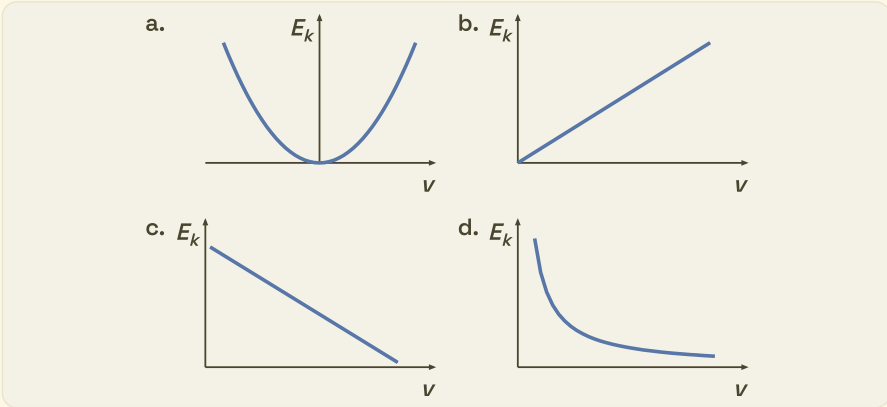


Figure 5.15 *Four graphs of kinetic energy E_k against velocity v, for question 10.*

10. Which graph in Figure 5.15 best shows how a body's kinetic energy depends on its velocity?

a a
b b
c c
d d

Check 15

11. A body of weight W_1 hangs from the ceiling of a room by a chain of weight W_2. What force does the ceiling exert on the chain?

a W_1
b W_2
c $W_1 + W_2$
d $W_1 - W_2$

Check 16

12. Two bodies of masses 3 kg and 4 kg are acted on by the same force. If the lighter body's acceleration is 2 m/s², what is the heavier body's?

a 1 m/s²
b 1.5 m/s²
c 2 m/s²
d 2.5 m/s²

Exercise

Choose the correct option

Check 5

1. A ball of mass m hits a wall normally at speed u and rebounds with the same speed. What is the change in its momentum?

a 2mu
b zero
c mu^2
d $\frac{1}{2}mu^2$

Check 6

2. A man holds a bucket of water of mass 5 kg in his hand and walks 50 m along a level road. How much work does he do on the bucket?

a 250 J
b 10 J
c 0 J
d 5 J

Check 7

3. A water pump has a power of 2 kW. How much water can it raise through 10 m in 1 minute ($g = 10\ m/s^2$)?

a 1000 liters
b 100 liters
c 1200 liters
d 2000 liters

Check 8

4. A body starts from rest and moves in a straight line with constant acceleration. The power delivered to it at time t is proportional to:

a t
b t^2
c $\sqrt{t}$
d $\sqrt[3]{t}$

Check 9

5. A force proportional to the distance moved, s, acts on a body (F = ks). The work done by the force is proportional to:

a s
b s^2
c $\sqrt{s}$
d s^3

Check 10

6. Water from a dam high in the hills is piped down to a town. Which energy change best describes this?

a PE to KE
b KE to PE
c chemical to light
d none of the above

Check 11

7. A meter rule of mass 800 g, pivoted at one end, is turned through 60° from hanging straight down. By how much does its potential energy increase?

a 2 J
b 4 J
c 20 J
d 0.20 J

Check 12

8. Two balls, each of mass 0.06 kg, move in opposite directions at 4 m/s, collide, and rebound at 4 m/s. What impulse does each ball receive?

a 0.48 kg m/s
b 0.53 kg m/s
c 0.81 kg m/s
d 0.92 kg m/s

Answer these questions

13. a. Explain what is meant by work.
b. Obtain an expression for the work done by a constant force.

WORKED EXAMPLE

A bullet through pine boards

A bullet of mass 2.60 g leaves a rifle at 335 m/s. It can go through eight pine boards, each 1.90 cm thick, before it stops. What average force does the wood exert on it?

Answer

$m = 2.60 \times 10^{-3}$ kg, $u = 335$ m/s, $v = 0$, and the stopping distance is $s = 8 \times 1.90 = 15.2$ cm $= 0.152$ m.
The wood's force F acts against the motion, so its work is $-Fs$, and by the work–energy theorem this equals the change in kinetic energy:

$$-Fs = 0 - \frac{1}{2}mu^2 \quad \Rightarrow \quad F = \frac{mu^2}{2s} = \frac{2.60 \times 10^{-3} \times 335^2}{2 \times 0.152} = \mathbf{960}\ \text{N}$$

About 960 N, the weight of a 98 kg mass, on a bullet that weighs less than a coin.

WORKED EXAMPLE

Slowed by a steady force

A body A moving horizontally with 800 J of kinetic energy meets a constant opposing horizontal force of 100 N between two points X and Y, 2 m apart. What is its kinetic energy at Y? How much farther will it go before it stops, if the force keeps acting?

Answer

From X to Y the force does work $-100 \times 2 = -200$ J, so the kinetic energy at Y is $800 - 200 = \mathbf{600\ J}$.
To stop, the body must lose these 600 J: $100 \times s = 600$, so $s = \mathbf{6\ m}$ beyond Y.

WORKED EXAMPLE

Work from the position

A particle of mass 1 kg moves along the x-axis with $x = \frac{1}{2}t^2$ m. Find the work done by the force on it from t = 0 to t = 3 s.

Answer

Its velocity is $v = \dfrac{dx}{dt} = t$ m/s: $v_1 = 0$ at $t = 0$, $v_2 = 3$ m/s at $t = 3$ s.
By the work–energy theorem, $W = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{1}{2} \times 1 \times 9 = \mathbf{4.5\ J}$.

YOUR TURN

Stopping a car

How much work must be done to stop a 1200 kg car moving at 99 km/h?

Answer

$v = 99/3.6 = 27.5$ m/s, so the work needed is its kinetic energy, $\frac{1}{2} \times 1200 \times 27.5^2 = \mathbf{453\ 750\ J}$ (about 454 kJ), done by the brakes and taken away as heat.

Kinetic energy and momentum

Kinetic energy and momentum both depend on mass and velocity. With $p = mv$,

$$K = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m} \qquad \text{and} \qquad p = \sqrt{2mK}$$

An explosion. A body of mass M, at rest, explodes into two parts of masses m_1 and m_2, which fly apart with velocities v_1 and v_2. Its momentum was zero, and momentum is conserved (Unit 4), so

$$0 = m_1 v_1 + m_2 v_2 \quad \Rightarrow \quad p_1 = -p_2$$

The two parts have momenta of equal size, so

$$\frac{K_1}{K_2} = \frac{p^2/2m_1}{p^2/2m_2} = \frac{m_2}{m_1}$$

The lighter part gets the larger share of the kinetic energy, in inverse proportion to the masses. If the total kinetic energy released is $K = K_1 + K_2$, then

$$K_1 = \frac{m_2}{m_1 + m_2}K = \frac{m_2}{M}K, \qquad K_2 = \frac{m_1}{M}K$$

This is why a fired bullet carries almost all the energy of the explosion, while the heavy gun recoils with very little (and why a gun can be held at the shoulder).

WORKED EXAMPLE

Sharing the energy of an explosion

A mass of 20 units, at rest, explodes into two parts of 4 units and 16 units. The larger part has 100 J of kinetic energy. What is the kinetic energy of the smaller part?

Answer

The momenta are equal in size, so $\frac{K_1}{K_2} = \frac{m_2}{m_1}$: $K_{\text{small}} = \frac{16}{4} \times 100 = \mathbf{400\ J}$. Four times lighter, four times the energy.

YOUR TURN

Momentum up by 20 %

The momentum of a body increases by 20 %. By what percentage does its kinetic energy increase?

Answer

$K = p^2/2m$, so $K \propto p^2$: $K_f/K_i = 1.2^2 = 1.44$. The kinetic energy increases by **44 %**.

5.5.2 Potential energy

Potential energy (PE) is energy stored in a body because of its **position** (relative to other bodies) or its **configuration** (its shape). It is stored, ready to be turned into kinetic energy or other forms.

- A **stretched** rubber band or a **compressed** spring has elastic potential energy.
- A **bent bow**, or a catapult pulled back, stores energy that sends the arrow or the stone flying.
- A **compressed gas**, as in a pumped-up tire, stores energy.
- **Water in a dam** and a ball held up high have **gravitational potential energy**, the kind this unit is most concerned with.

Gravitational potential energy. To raise a body of mass m through a height h at a steady speed, you must pull it up with a force equal to its weight, mg. The work you do is

$$W = Fh = mgh$$

This work is stored in the body as potential energy:

$$\boxed{U = mgh}$$

The height is measured from a level we choose as zero (the ground, the floor, a table top). Only **changes** in potential energy matter, and they are the same whatever zero we choose: lifting a 2 kg bag 1 m always adds 19.6 J.

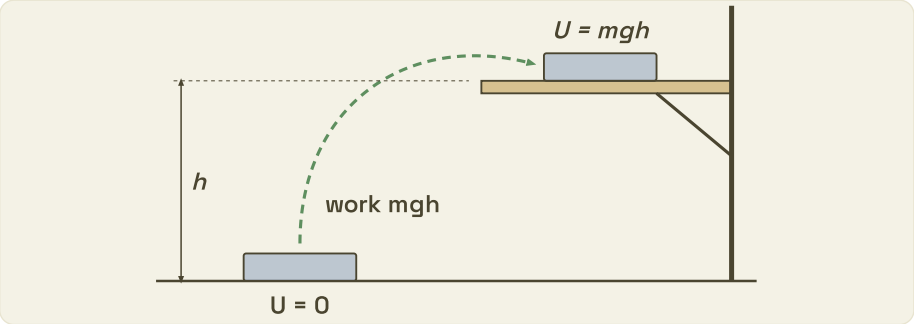


Figure 5.5 *Lifting a book of mass m from the floor onto a shelf a height h above it takes work mgh. The work is stored as the book's potential energy, and comes back as kinetic energy if it falls.*

Kinetic and potential energy together

A body can have both kinds at once. Their sum is its **total mechanical energy**:

$$E = K + U$$

- A stone resting on the ground has no kinetic energy; its potential energy depends on the level we call zero.

IN SHORT

Key ideas of Unit 5

- **Work** $W = Fd\cos\theta = \vec{F}\cdot\vec{d}$, a scalar; zero if there is no displacement or the force is at 90° to it; negative if the force opposes the motion. 1 J = 1 N m; $[\mathrm{ML^2T^{-2}}]$.
- A **variable force**: $W = \int F\,dx$, the area under the force–displacement graph.
- **Power** $P = W/t$, instantaneous $P = \vec{F}\cdot\vec{v}$; 1 W = 1 J/s, 1 hp = 746 W; $[\mathrm{ML^2T^{-3}}]$. 1 kWh = 3.6×10^6 J is energy.
- **Kinetic energy** $K = \frac{1}{2}mv^2 = p^2/2m$, never negative. **Work–energy theorem:** $W_{\text{net}} = \Delta K$.
- An explosion from rest shares kinetic energy inversely as the masses: $K_1/K_2 = m_2/m_1$.
- **Potential energy** is stored energy of position or shape; near the Earth $U = mgh$; ΔU = work done against the force.
- **Conservation of energy:** energy is never created or destroyed, only changed in form. Mechanical energy $K + U$ is conserved when only conservative forces act; with friction, $E_f = E_i + W_f$.
- **Conservative forces** (gravity, springs, electric): work independent of path, zero round a closed path, recoverable. **Non-conservative** (friction, drag): path-dependent, not recoverable.
- **Collisions:** momentum and total energy always conserved. **Elastic:** KE conserved, $e = 1$. **Inelastic:** KE lost; **perfectly inelastic:** they stick, $e = 0$. Coefficient of restitution $e = \frac{v_2 - v_1}{u_1 - u_2}$, $0 \le e \le 1$.
- 1D elastic: $u_1 - u_2 = v_2 - v_1$; equal masses exchange velocities; light on heavy bounces back; heavy on light sends it off at $2u_1$.

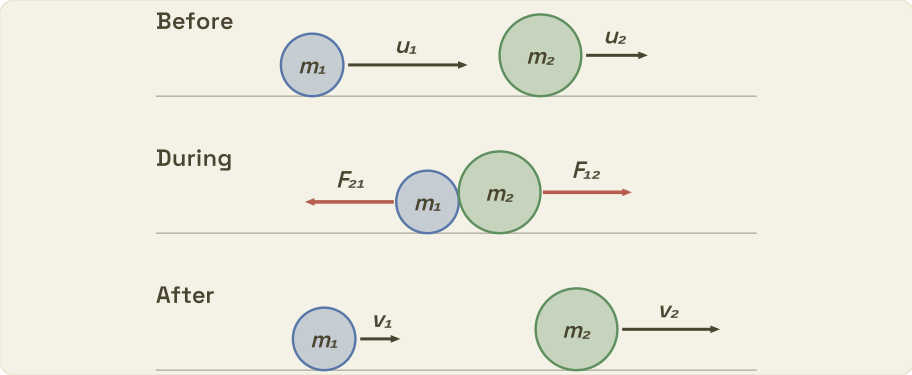


Figure 5.14 *A head-on collision of two bodies along one line. Before: body 1 (mass m_1) moving at u_1 catches up with body 2 (mass m_2) moving at u_2, with $u_1 > u_2$. During: each pushes on the other. After: they separate at v_1 and v_2, with $v_2 > v_1$.*

In a one-dimensional elastic collision, the relative velocity of approach before the collision equals the relative velocity of separation after it (that is, $e = 1$).

4. **Find v_1.** From (5.3), $v_2 = u_1 - u_2 + v_1$. Put this into (5.1): $m_1u_1 - m_1v_1 = m_2(u_1 - u_2 + v_1 - u_2)$, which rearranges to $(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$:

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2 \tag{5.4}$$

5. **Find v_2** in the same way (or by swapping the labels 1 and 2):

$$v_2 = \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2 \tag{5.5}$$

Equations (5.4) and (5.5) give both velocities after the collision from the masses and the velocities before.

Special cases

a. Equal masses ($m_1 = m_2$). Then (5.4) and (5.5) give $v_1 = u_2$ and $v_2 = u_1$: **the two bodies simply exchange velocities.** If the second was at rest ($u_2 = 0$), the first stops dead and the second moves off with the first one's velocity, as in a Newton's cradle or a straight snooker shot. A neutron hitting a proton (nearly the same mass) head-on stops and sends the proton on, which is why water and paraffin wax, rich in hydrogen, slow down neutrons so well.

b. A light body hits a very heavy one at rest ($m_1 \ll m_2$, $u_2 = 0$). Putting $m_1 \approx 0$ gives $v_1 \approx -u_1$ and $v_2 \approx 0$: **the light body bounces straight back with the same speed, and the heavy body hardly moves.** A ball bouncing off a wall.

c. A very heavy body hits a light one at rest ($m_1 \gg m_2$, $u_2 = 0$). Putting $m_2 \approx 0$ gives $v_1 \approx u_1$ and $v_2 \approx 2u_1$: **the heavy body carries on almost unchanged, and the light body shoots off at twice its speed.** A fast alpha particle passes through a cloud of electrons in an atom almost undisturbed, and a cricket bat swung hard sends a light ball off faster than the bat.

Check 4

A ball of mass 2 kg moving at 3 m/s hits a ball of mass 2 kg at rest, head-on and elastically. Afterwards:

a both move at 1.5 m/s
b the first stops and the second moves at 3 m/s
c the first bounces back at 3 m/s
d both stop

- **Water in a reservoir** high in the hills has potential energy. As it is piped down to a town, or through a turbine, its potential energy turns into kinetic energy.
- The energy stored in a stretched catapult's band becomes the kinetic energy of the stone; a bent bow's energy becomes the arrow's.
- When a tire bursts, the energy stored in the compressed air turns into the kinetic energy of the rushing air, and into sound and heat.
- A body can have potential energy and no momentum (a book on a shelf), since momentum depends only on motion; and a body with no kinetic energy has no momentum.

When something swings or bounces back and forth, its energy changes from one kind to the other. For a mass on a spring (Figure 5.6), at the ends of its swing ($x = \pm A$) it stops: all its energy is potential, stored in the spring. As it passes the middle ($x = 0$) it moves fastest: all its energy is kinetic. In between it has some of each, and the sum stays the same, as long as nothing (friction, air) takes energy away.

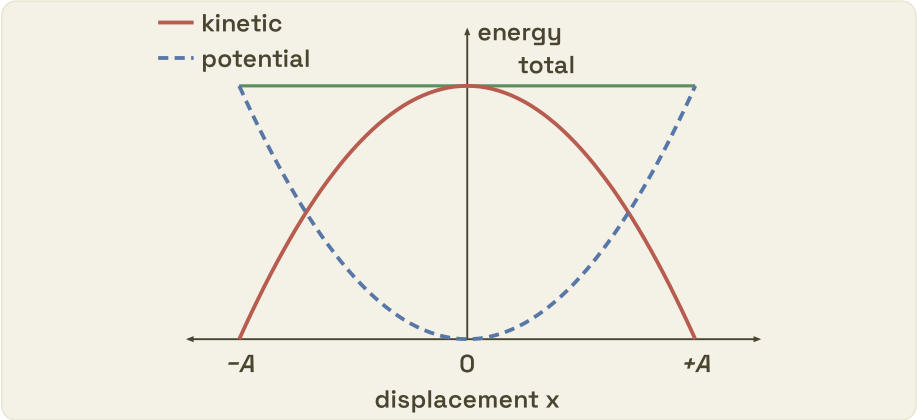


Figure 5.6 *The energy of a mass bobbing on a spring between $x = -A$ and $x = +A$. The potential energy stored in the spring (dashed) is largest at the ends and zero in the middle; the kinetic energy (solid) is largest in the middle and zero at the ends. At every point they add up to the same total energy.*

Change in potential energy and work

When a body is raised through h, gravity pulls it down while it moves up, so gravity does **negative** work, $-mgh$, and the potential energy **increases** by mgh. When the body falls, gravity does positive work and the potential energy decreases. In general:

$$\Delta U = -W_{\text{gravity}} = W_{\text{against gravity}}$$

The change in potential energy is the work done against the force (or minus the work done by it).

WORKED EXAMPLE

Up a slope

A 10 kg block is pushed 10 m up the slope of a wedge inclined at 30° to the ground. Find the potential energy it gains.

Answer

Only the height gained counts: $h = L\sin 30^\circ = 10 \times 0.5 = 5$ m.
$U = mgh = 10 \times 9.8 \times 5 =$ **490 J** (500 J with g = 10 m/s²). The block gains the same potential energy if lifted straight up 5 m: the slope only lets you use a smaller force over a longer distance.

YOUR TURN

Pumping water up

A pump lifts water vertically through 100 m, and the work done is 294 kJ. What volume of water was pumped? (Density of water = 1000 kg/m³.)

Answer

$mgh = 294\,000$ J, so $m = \dfrac{294\,000}{9.8 \times 100} = 300$ kg. Volume $= \dfrac{300}{1000} =$ **0.30 m³** (300 liters).



Figure 5.7 *The Kulekhani reservoir in Makwanpur, held back by its dam high in the hills. Its water runs down through a tunnel and pipes to power stations far below: gravitational potential energy becomes kinetic energy, and then electrical energy.*
Photo: Lakhindra, CC BY-SA 4.0

5.6 Principle of conservation of energy

You know many forms of energy: mechanical, heat, sound, light, chemical, electrical, nuclear. Energy keeps changing from one form to another, and whenever one form disappears, another appears in exactly the same amount.

- A radio turns the chemical energy of its battery (or the electrical energy from the mains) into sound, and some heat.
- A fan turns electrical energy into the kinetic energy of its blades and of the air, and some heat and sound.

In every case the energy lost in one form equals the energy gained in the others. This is the **principle of conservation of energy**:

Energy can neither be created nor destroyed; it can only be changed from one form to another. The total energy of an isolated system stays constant.

- It is an **experimental** law. It can't be proved by mathematics alone; it comes from countless measurements, starting with Joule's, who showed in the 1840s that a fixed amount of work always makes the same amount of heat.
- It is believed to hold **everywhere**: no experiment or observation has ever found it broken.
- It links mechanics with heat, light, electricity and every other part of physics, which is why energy is one of the great **unifying** ideas of science.
- **Mechanical** energy alone stays constant only when the forces at work are **conservative** (Section 5.7), like gravity. Friction turns mechanical energy into heat, which is no longer available to do useful work. The total energy is still conserved; the mechanical energy is not.

A swinging pendulum shows the exchange well (Figure 5.8). At the end of its swing the bob stops for an instant, at its highest: all its energy is potential. At the bottom it moves fastest: all its energy is kinetic. Without air resistance it would swing up to the same height on the other side for ever.

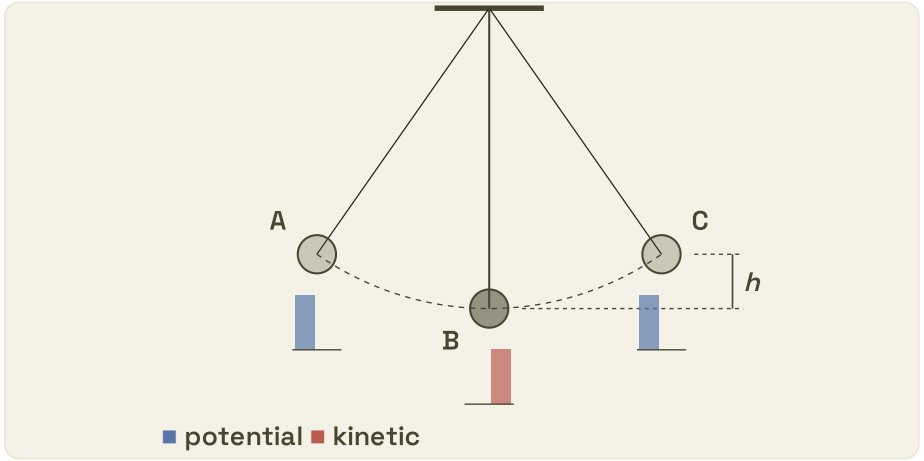


Figure 5.8 *A pendulum swinging between A and C through its lowest point B. At A and C the bob stops for an instant: its energy is all potential (blue bar). At B, lowest and fastest, it is all kinetic (red bar). Between, it has some of each, and the total stays the same.*

5.8.1 Types of collision

Elastic collision

In an **elastic** collision both **momentum** and **kinetic energy** are conserved:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

No kinetic energy is turned into heat, sound or other forms; the forces between the bodies during the collision are conservative. Collisions between atoms, molecules of a gas, and nuclear particles are often perfectly elastic. Collisions of billiard balls, or the steel balls of a Newton's cradle, are **nearly** elastic: a little energy goes into the click you hear and a little warming.

In an elastic collision:

1. momentum is conserved;
2. kinetic energy is conserved;
3. total energy is conserved;
4. no mechanical energy is changed into other forms; the forces involved are conservative.

Inelastic collision

In an **inelastic** collision **momentum is conserved** but **kinetic energy is not**:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2, \qquad \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 > \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Some kinetic energy is lost, turned into heat, sound and the work of bending and denting:

$$\Delta K = \left(\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2\right) - \left(\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2\right)$$

The forces involved are non-conservative. Cars that crash and crumple, a lump of clay dropped on the floor, a tackle in football in which two players fall together: all are inelastic. Almost every collision of everyday objects is inelastic to some degree.

In an inelastic collision:

1. momentum is conserved;
2. kinetic energy is **not** conserved;
3. total energy is conserved;
4. part of the mechanical energy is changed into other forms; the forces involved are non-conservative.

Perfectly inelastic collision. When the bodies stick together and move on with a common velocity v, the loss of kinetic energy is as large as it can be. Momentum gives

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v \quad \Rightarrow \quad v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

> CAREFUL
> **Momentum is always conserved; kinetic energy only sometimes**
>
> A common slip is to say that a collision is inelastic because momentum is lost. In any collision with no outside force, momentum is conserved, elastic or not. Only the kinetic energy tells the two kinds apart.

One-dimensional elastic collision

Two bodies of masses m_1 and m_2 move along one line with velocities u_1 and u_2, with $u_1 > u_2$, so the first catches up with the second (Figure 5.14). They collide elastically for a short time and then move on along the same line with velocities v_1 and v_2, with $v_2 > v_1$.

1. **Momentum is conserved:**
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \Rightarrow \quad m_1(u_1 - v_1) = m_2(v_2 - u_2) \tag{5.1}$$
2. **Kinetic energy is conserved:**
$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \Rightarrow \quad m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \tag{5.2}$$
3. **Divide (5.2) by (5.1).** Since $u_1^2 - v_1^2 = (u_1 - v_1)(u_1 + v_1)$, and the same on the right:
$$u_1 + v_1 = v_2 + u_2 \quad \Rightarrow \quad u_1 - u_2 = v_2 - v_1 \tag{5.3}$$

WORKED EXAMPLE

A pull against friction

A 2 kg body at rest on a table is pulled by a horizontal force of 7 N. The coefficient of kinetic friction is 0.1. Find, for the first 10 s: a. the work done by the pull, b. the work done by friction, c. the work done by the net force, d. the change in its kinetic energy.

Answer

Friction: $f = \mu mg = 0.1 \times 2 \times 9.8 = 1.96$ N. Net force $7 - 1.96 = 5.04$ N, so $a = 5.04/2 = 2.52$ m/s².
In 10 s, from rest: $s = \frac{1}{2}at^2 = \frac{1}{2} \times 2.52 \times 100 = 126$ m, and $v = at = 25.2$ m/s.

a. $W_{\text{pull}} = 7 \times 126 = \mathbf{882\ J}$.
b. $W_f = -1.96 \times 126 = \mathbf{-247\ J}$.
c. $W_{\text{net}} = 5.04 \times 126 = \mathbf{635\ J}$ (= 882 − 247).
d. $\Delta K = \frac{1}{2} \times 2 \times 25.2^2 - 0 = \mathbf{635\ J}$.

The work done by the **net** force equals the change in kinetic energy, as the work–energy theorem says. The 247 J done against friction became heat.

YOUR TURN

A curved potential energy

A conservative force acts on a 4 kg particle along the x-axis. Its potential energy is $U(x) = 40 + (x - 6)^2$ J (x in meters). At x = 8 m its kinetic energy is 30 J. Find its greatest kinetic energy.

Answer

At x = 8 m, $U = 40 + 4 = 44$ J, so the total energy is $44 + 30 = 74$ J, and it stays 74 J.
The kinetic energy is greatest where the potential energy is least: $dU/dx = 2(x - 6) = 0$ at x = 6 m, where $U_{\min} = 40$ J. So $K_{\max} = 74 - 40 = \mathbf{34\ J}$.

Check 3

A sledge slides down a snowy slope from the same height by two routes: a short steep one and a long gentle one. Friction is the same on both. It reaches the bottom:

a at the same speed by both routes, because gravity is conservative
b faster by the short route
c faster by the long route
d at a speed that doesn't depend on friction

5.8 Collisions: elastic and inelastic

In physics, a **collision** is an interaction in which two or more bodies exert strong forces on each other for a short time, so that their motion changes suddenly. Usually they touch: two billiard balls, a bat and a cricket ball, two cars. But they needn't: an alpha particle fired at a gold nucleus is turned back by electric repulsion without touching it, and a spacecraft swinging past a planet "collides" with it through gravity.

In every collision of bodies on their own (no outside force), **momentum is conserved** (Unit 4), and the **total energy is conserved**. What may change is the **kinetic** energy.

Coefficient of restitution

How bouncy is a collision? For two bodies colliding head-on, the **coefficient of restitution** e is the ratio of their **relative velocity of separation** after the collision to their **relative velocity of approach** before it:

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

where u_1, u_2 are the velocities before and v_1, v_2 after. Its value lies between 0 and 1, $0 \le e \le 1$:

- $e = 1$: **perfectly elastic** collision; the bodies separate as fast as they approached;
- $e = 0$: **perfectly inelastic** collision; they don't separate at all, but move on together;
- $0 < e < 1$: an ordinary **inelastic** collision.

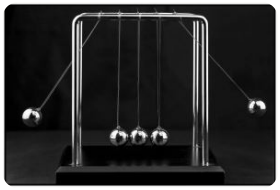


Figure 5.13 *A Newton's cradle, its two end balls caught at the top of their swings. Lift one ball and let it go: it stops dead and the ball at the far end flies out at the same speed. Steel balls collide almost elastically.*

Photo: Sheila Sund from Salem, United States, CC BY 2.0

5.6.1 Conservation of mechanical energy in a falling body

A body falling freely, with air resistance too small to matter, feels only gravity, and its mechanical energy is conserved: **the total mechanical energy of an isolated system, acted on only by conservative forces, stays constant.**

Proof. A body of mass m is let go from rest at A, a height h above the ground (Figure 5.9). It passes B, a distance x below A, with velocity v_B, and reaches the ground at C ($BC = h - x$) with velocity v_C.

At A ($u_A = 0$, height h):

$$K_A = 0, \quad U_A = mgh, \quad E_A = 0 + mgh = mgh$$

At B (fallen x, height $h - x$): from $v_B^2 = u_A^2 + 2gx = 2gx$,

$$K_B = \frac{1}{2}m(2gx) = mgx, \quad U_B = mg(h - x), \quad E_B = mgx + mgh - mgx = mgh$$

At C (fallen h, height 0): from $v_C^2 = 2gh$,

$$K_C = \frac{1}{2}m(2gh) = mgh, \quad U_C = 0, \quad E_C = mgh + 0 = mgh$$

At A, B and C, and in fact at every point, the total mechanical energy is mgh. As the body falls, potential energy changes into kinetic energy, and the total doesn't change.

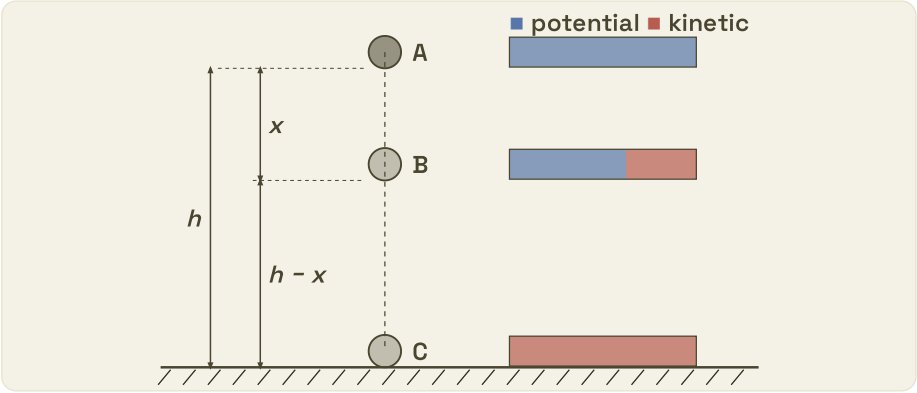


Figure 5.9 *A body let go from rest at A, a height h above the ground, passes B after falling x and reaches the ground at C. The bars show its energy at each point: all potential at A, part potential and part kinetic at B, all kinetic at C. The total is mgh throughout.*

DID YOU KNOW?

Did you know?

- A raindrop or a parachutist falling through air is **not** in free fall: air resistance, a non-conservative force, takes away mechanical energy as heat, so it reaches a steady terminal speed instead of speeding up for ever.
- A bus climbing a hill at a steady speed keeps its kinetic energy while its potential energy grows. That energy comes from its fuel: the driver must press the accelerator harder on the way up, even though the bus isn't speeding up.

WORKED EXAMPLE

How high will it roll?

A steel ball of mass 0.125 kg with 0.25 J of kinetic energy rolls along a horizontal track and on up an inclined track (Figure 5.10). How high up the slope does it rise, if friction is negligible?

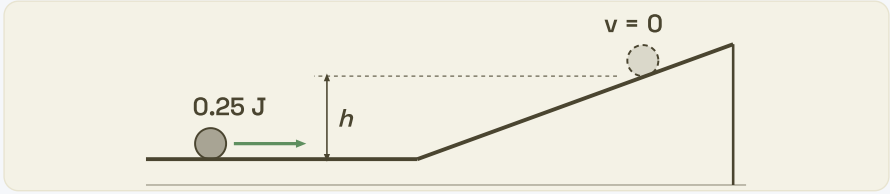


Figure 5.10 *A ball rolling along a level track and up a slope. It rises until all its kinetic energy has turned into potential energy, and stops at a height h.*

Answer

At the bottom its energy is all kinetic, 0.25 J. At its highest point it stops (v = 0), so its energy is all potential:

$$mgh = 0.25 \quad \Rightarrow \quad h = \frac{0.25}{0.125 \times 9.8} = \mathbf{0.20}\ \text{m}$$

YOUR TURN

A roller coaster car

A roller coaster car of mass 200 kg starts at A, 2 m above B, and rolls down the track to B, where its speed is 12 m/s (Figure 5.11). Assuming no friction between A and B, find the speed with which it started at A.



Figure 5.11 *A roller coaster car starts at A, 2 m above the lowest point B, and rolls down the track to B.*

Answer

Energy at A = energy at B: $\frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2$, so (the mass cancels)

$$v_A^2 = v_B^2 - 2gh = 144 - 2 \times 9.8 \times 2 = 104.8 \quad \Rightarrow \quad v_A = \mathbf{10.2}\ \text{m/s}$$

5.7 Conservative and non-conservative forces

5.7.1 Conservative forces

A force is **conservative** if the total mechanical energy stays constant when only it does work. Gravity, the force of a spring (elastic force) and the electrostatic force are conservative. A conservative force:

1. **does work that depends only on the start and end points, not on the path.** Lifting a box onto a shelf 2 m high, gravity does −mg × 2 m of work, whether you lift it straight up, carry it up a ramp, or take it the long way round by the stairs (Figure 5.12).
2. **does zero work round any closed path.** Carry the box up and back down to where it started: gravity does −mgh on the way up and +mgh on the way down, zero in all.
3. **gives back all the work done against it.** The work you do lifting the box is stored as potential energy, and all of it comes back as the box falls. This is what lets us define a **potential energy** for it.
4. **keeps mechanical energy conserved:**

$$E_f = E_i, \quad \text{that is} \quad K_f + U_f = K_i + U_i$$

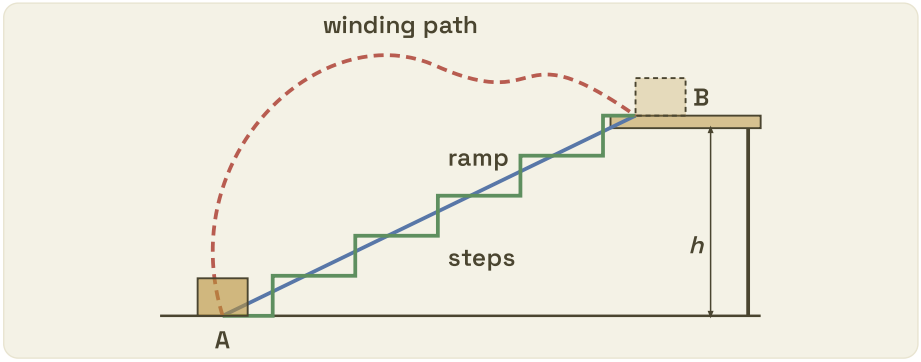


Figure 5.12 *Three ways of taking a box from the floor at A to a shelf at B, h higher: up a ramp, up a flight of steps, and along a long winding path. Gravity does the same work, −mgh, on every path. Friction does more work on the longer paths.*

5.7.2 Non-conservative forces

A force is **non-conservative** if the mechanical energy is not conserved when it acts: part of the energy is turned into heat, sound or other forms. Friction, air resistance and the drag of liquids (viscous forces) are non-conservative; so are the pushes and pulls of people, engines and muscles, which add energy from fuel or food. A non-conservative force:

1. **does work that depends on the path.** The longer the path, the more work friction does: walking from home to college by a long road, you do more work against friction than by a short one.
2. **does work that isn't zero round a closed path.** Friction opposes you both ways, so on the way there and back it does negative work each time; the total is not zero.
3. **does work that can't be recovered.** The energy friction turns into heat doesn't come back as motion.
4. **breaks the conservation of mechanical energy:** with friction, $E_f < E_i$.

We can still use energy, by **counting the work** of the non-conservative forces:

$$E_f = E_i + W_{nc} \quad \text{or} \quad K_f + U_f = K_i + U_i + W_f$$

where W_f is the work done by friction, which is **negative**. The mechanical energy lost is the heat made by friction.

WORKED EXAMPLE

Friction on a roller coaster

A roller coaster of mass 2500 kg moves at 10 m/s over the top of a hill 13 m high. When it reaches the top of the next hill, 8 m high, it is moving at 13 m/s. How much work did friction do on it?

Answer

$$\tfrac{1}{2}mv_f^2 + mgh_f = \tfrac{1}{2}mv_i^2 + mgh_i + W_f$$

$$\tfrac{1}{2}(2500)(13^2) + 2500(9.8)(8) = \tfrac{1}{2}(2500)(10^2) + 2500(9.8)(13) + W_f$$

$$211\,250 + 196\,000 = 125\,000 + 318\,500 + W_f$$

$$W_f = 407\,250 - 443\,500 = -\mathbf{36\,250}\ \text{J}$$

Friction did about −36 kJ of work: negative, because it acts against the motion. 36 kJ of mechanical energy became heat in the wheels, the track and the air.