Unit 1

Physical Quantities

This unit online, with its interactives:
science-playground.com/s/11f1

On 23 September 1999, a NASA spacecraft, the Mars Climate Orbiter, fired its engine to slip into orbit round Mars, and was never heard from again. It had flown far too low, and was probably torn apart in the Martian atmosphere. The cause was not a broken part. One team's software reported the push of the thrusters in pound-force seconds; the team steering the spacecraft read the numbers as newton seconds. Every push was 4.45 times bigger than they thought, and nine months of small errors added up to a lost spacecraft.

A number with the wrong unit is not a small mistake: it is a different quantity. This unit is about getting quantities right: what a physical quantity is, how it is measured, the units it is measured in, how many of its digits you can trust, and its **dimensions**, a tool so useful that it can check a formula, or even find one, without doing a single experiment.

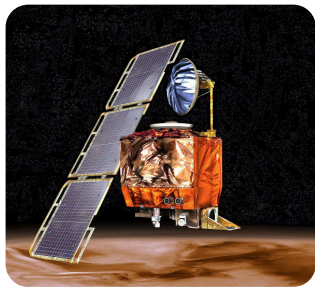


Figure 1.1 *An artist's picture of the Mars Climate Orbiter. It was lost on arrival at Mars in 1999 because two teams used different units for the same numbers.*
Photo: NASA/JPL/Corby Waste, Public domain

1.1 Introduction

Think of what happens in nature around you: clouds forming and melting away, a rainbow, rain, a pond drying up in the sun, a storm, a cold dry winter and a hot summer. These are **natural** (physical) **phenomena**. To understand them clearly we need

A STORY FROM SCIENCE

Joseph Fourier, the tailor's son

Joseph Fourier (1768–1830) was born at Auxerre in France, the son of a tailor, and was orphaned at the age of nine. The local bishop had him educated by monks, and he grew up to be one of the great mathematicians of his age. He went with Napoleon to Egypt, governed a region of France, and in his spare hours worked out how heat flows through solids.

In his book on heat, *The Analytical Theory of Heat* (1822), Fourier asked a question nobody had asked clearly before: if you change your unit of length or time, how must every quantity in an equation change with it? His answer was the idea of the **dimensions** of a physical quantity, which you will meet in Section 1.6.

The Scottish physicist **James Clerk Maxwell** built on Fourier's work from the 1860s, when he and his colleagues worked out the units of electricity, and made dimensions the everyday tool of physics, written as they still are, with [M], [L] and [T].

the laws of physics, and those laws are written in terms of **physical quantities**: length, time, mass, velocity, force, momentum, temperature and so on. So physics starts with measuring physical quantities, and that is where this book starts too.



Figure 1.2 *Joseph Fourier (1768–1830), who first wrote about the dimensions of physical quantities.*
Photo: Amédée Félix Barthélemy Geille / After Julien-Léopold Boilly, Public domain

What physics is

Physics is the most basic of the natural sciences: it tries to understand how the universe works, from its smallest pieces to its largest. A scientist who works in physics is a **physicist**.

The word comes from the Latin *physica*, "the study of nature", which comes from the Greek *phýsis* (φύσις), "nature". In Nepali it is **भौतिक विज्ञान** (Bhautik Vigyan), the sci-

13. a. The frequency f of vibration of a string depends on its length l, its tension T, and its mass per unit length m. Show by dimensions that $f \propto \frac{1}{l}\sqrt{\frac{T}{m}}$; experiment then gives $f = \frac{1}{2l}\sqrt{\frac{T}{m}}$.
    b. A force of 5 N is expressed as $5\ \text{N} = a + bt + ct^2 + dt^3$, where t is time. Find the dimensional formulas of a, b, c and d.
    c. Using the relation in a., find the dimensional formula of frequency.

14. a. How do you tell precision from accuracy? How is precision related to significant figures?
    b. Which of these measurements is the most precise, and why? i. 2 cm, ii. 2.00 cm, iii. 2.000 cm, iv. 2.0000 cm

Numerical problems

15. The velocity of a particle is $v = a + bt + ct^2$, where t is time. Find the dimensions of a, b and c, if v is in m/s and t in s.

16. A body moving through air at a high speed v feels a retarding force $F = kA\rho v^x$, where A is the body's surface area, ρ is the density of the air, and k is a dimensionless constant. Find x.

17. The velocity of a particle is $v = a + bt + \dfrac{c}{d + t}$, where t is time. Find the dimensions of a, b, c and d.

18. Find the value of 10 joules in a system whose fundamental units are 10 cm, 100 g and 30 s.

19. The length and breadth of a field are $l = (110 \pm 2)$ m and $b = (100 \pm 4)$ m. Find the area of the field with its uncertainty.

20. Convert i. 72 km/h into m/s, and ii. 20 g/cm³ into kg/m³.

21. The frequency of vibration of a stretched string depends on its tension T, its length L, and its mass per unit length m. Using dimensions, prove that $f \propto \frac{1}{L}\sqrt{\frac{T}{m}}$.

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.

ence of the material world.

Everything around you, desks, the school building, trees, buses, is made of **matter**, and everything that happens to it involves **energy**: the energy of motion, of heat, of light, of sound. So physics is the science of **matter and energy and how they interact**. It covers the very large (stars, planets and galaxies) and the very small (atoms and electrons), and everything between: motion, heat, sound, light, electricity and magnetism, radioactivity, even space and time. Its main branches are mechanics, heat, optics (light), electricity and magnetism, and modern physics.

Physics is a **quantitative** science: it deals in measurements, which come with numbers. Physicists use mathematics to build **models** that predict what nature will do, then compare the predictions with careful measurements, and improve the models where they don't agree. That cycle, predict, measure, improve, is why physics can send a spacecraft to Mars at all.

Check 1

Physics is called a quantitative science because:

a. it studies a great quantity of things
b. its ideas are tested by measurements, which come with numbers
c. it studies only quantities, never qualities like color
d. it was first written in Latin

1.2 Physical quantity

A **physical quantity** is anything in nature that can be **measured**: the mass of a person (87.5 kg), the height of a child (1.4 m), the time of a journey (2 hours). Kindness and beauty are real, but they are not physical quantities: there is no instrument to measure them.

Every measured value is written as a **number** times a **unit**:

$$\text{physical quantity} = n\,u$$

where n is the **numerical value** and u is the **unit**. "The bag of apples is 5 kg" means 5 times the unit kilogram. The same quantity can be written with different units, and then the number changes the other way: a bigger unit needs a smaller number.

$$2\ \text{h} = 120\ \text{min} = 7200\ \text{s}, \qquad n_1 u_1 = n_2 u_2$$

A number without its unit means nothing: "the road is 200 long" could be 200 m or 200 km.

Fundamental and derived quantities

Physical quantities are of two types.

1. **Fundamental (base) quantities** are a small set chosen to stand on their own: none of them is defined from the others. There are **seven**: mass, length, time, temperature, electric current, luminous intensity (how bright a light source is) and amount of substance.
2. **Derived quantities** are defined from the fundamental ones by multiplying or dividing them. Area is length × length; velocity is displacement ÷ time; momentum is mass × velocity, that is, mass × displacement ÷ time; force, pressure, energy and power are all derived the same way.

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}, \qquad \text{momentum} = \text{mass} \times \text{velocity} = \text{mass} \times \frac{\text{displaceme}}{\text{time}}$$

Two more quantities, **plane angle** and **solid angle**, are often listed beside the seven as **supplementary** quantities (Section 1.4).

Check 2

Which of these is a derived quantity?

a mass
b time
c pressure
d electric current

1.3 Measurement and its need

In Aristotle's time, more than two thousand years ago, natural science was mostly argument from what people saw and felt. Senses differ from person to person (the same water feels warm to a cold hand and cool to a hot one), so observations disagreed and people could not settle their arguments.

Lord Kelvin (1824–1907) put the modern view sharply: when you can measure what you are speaking about and express it in numbers, you know something about it; when you cannot, your knowledge is meager and unsatisfactory. A quantity enters physics only once it can be measured.

Measurement is the comparison of an unknown quantity with a known, agreed reference amount of the same kind, the **unit**. Measuring the length of a bench means finding how many meters long it is. In short, measurement assigns a number to a physical quantity by comparing it with its unit.

Why we need measurement:

Check 18

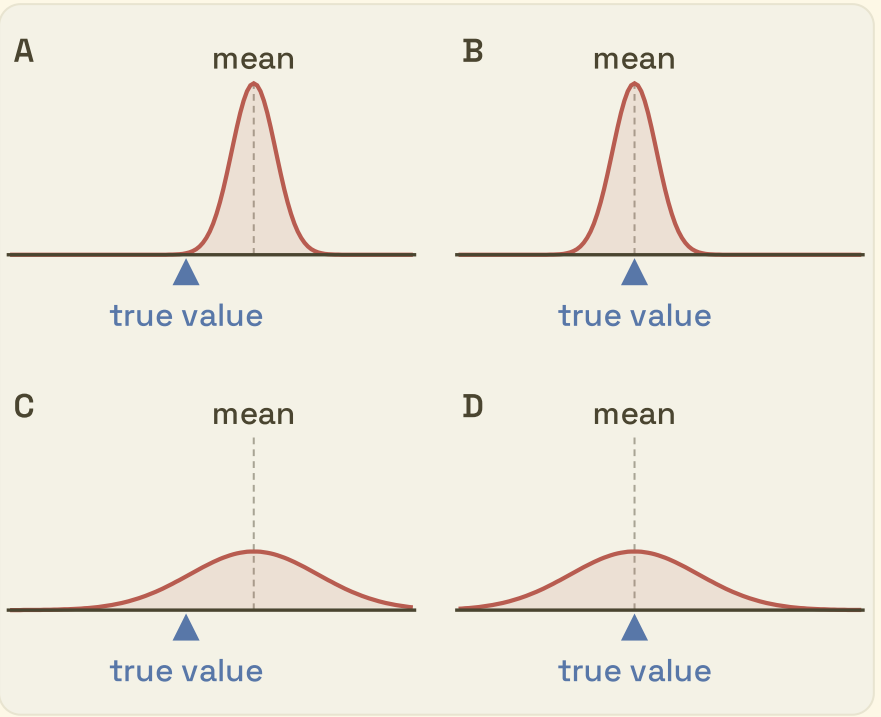


Figure 1.6 *For question 11: the spread of four sets of measurements of one quantity, each with its mean and the true value marked.*

11. In the figure above, which set of measurements is both accurate and precise?

a A
b B
c C
d D

Answer these questions

12.

a. Define the dimensional formula of a physical quantity.
b. Taking force, length and time as fundamental quantities, find the dimensional formula of energy.
c. The volume of liquid flowing per second through a horizontal pipe of length l is given by $\frac{kPa^x}{\eta l}$, where k is a dimensionless constant, P is the pressure difference between the pipe's ends, a is the radius of the pipe, and η is the coefficient of viscosity, $[\mathrm{M\,L^{-1}\,T^{-1}}]$. Find x by the method of dimensions.

Check 13

6. If force [F], length [L] and time [T] are taken as fundamental, the dimensional formula of momentum is:

a $[F\,T^{-1}]$ b $[F\,T^{2}]$
c $[F\,T]$ d $[F\,T^{-2}]$

Check 14

7. The dimensions of a light year are:

a $[T]$ b $[L\,T^{-1}]$
c $[L]$ d $[T^{-1}]$

Check 15

8. The sides of a rectangle are (0.5 ± 0.01) m and (0.2 ± 0.01) m. Its perimeter, with its uncertainty, is:

The textbook gives the options (1.4 ± 0.02) m, (1.4 ± 0.01) m, (0.1 ± 0.01) m and (0.1 ± 0.02) m, and none fits: the uncertainty of $2 \times (0.5 + 0.2)$ m is $2 \times (0.01 + 0.01) = 0.04$ m, so the book's options don't work. The options below are corrected.

a (1.4 ± 0.02) m b (1.4 ± 0.04) m
c (1.4 ± 0.01) m d (0.1 ± 0.02) m

Check 16

9. Which of these numbers has the fewest significant figures?

a 0.70860 b 0.50300
c 0.07086 d 60.5370

Check 17

10. The number of significant figures in the measurement 0.0050×10^{-12} is:

a 1 b 2
c 4 d 5

1. **To test ideas.** A hypothesis ("a longer pendulum swings more slowly") can only be tested by measuring; the things measured become the variables of the experiment.
2. **To understand phenomena.** Experiments need measurements of length, mass, time, temperature and so on.
3. **To check laws and theories.** A law is accepted only when measurements agree with it.

It also lets people trade fairly and build things that fit: a shopkeeper's kilogram must be the same as yours.

DID YOU KNOW?

Ropani, aana and the meter

Before the metric system reached them, people everywhere measured with local units, and some are still in daily use. In the hills of Nepal land is bought and sold in **ropani** and **aana**: a ropani is 74 feet × 74 feet, 5476 square feet, about 508.7 m^2, and an aana is a sixteenth of it, about 31.8 m^2. In the Tarai it is **bigha**, **kattha** and **dhur**. They work well locally, but a scientist in Japan or Brazil can't use them. That is why science needs one system of units for the whole world. In Nepal, the Nepal Bureau of Standards and Metrology keeps the national standards and checks that shops' scales and petrol pumps give true measure.

1.4 Unit of a physical quantity

A **unit** is the agreed standard amount of a quantity that others are compared with. A good unit is well defined, the same everywhere, does not change with time, and can be copied and checked easily.

Writing units correctly

The SI has rules for writing units, so that a unit means the same thing to everyone:

1. **A unit named after a person** is written in full with a small letter: newton, joule, pascal, kelvin (not Newton, Joule). Degree Celsius is the one exception.
2. **Its symbol starts with a capital letter**: N for newton, J for joule, Pa for pascal. Symbols of other units are small letters: m for meter (not M), s for second, kg for kilogram. (The liter may be written L as well as l, so that it isn't mistaken for the number 1.)

3. **Symbols are never made plural**: 5 kg, not 5 kgs; 10 m, not 10 ms (which would mean 10 milliseconds!).
4. **A product of units** is written with a space or a raised dot: N m or N·m (newton meter).
5. **A quotient of units** is written with one solidus (slash) or with a negative power: meter per second is m/s or m s^{-1}. Use only **one** slash: kg/(m s^2) or kg m^{-1} s^{-2}, never kg/m/s^2.
6. **Symbols are not abbreviations**, so they take no full stop: 50 cm, 50 mL, not 50 c.m. or 50 m.l. (except at the end of a sentence).
7. **Names of units** are made plural in the usual way (5 newtons, 3 henries), except lux, hertz and siemens, which stay the same.
8. **A prefix** is joined to the symbol with no space: GHz (gigahertz), MPa (megapascal), km (kilometer).

CAREFUL

mN, nm and N m

One letter or space changes everything. **mN** is a millinewton (10^{-3} N), a force. **nm** is a nanometer (10^{-9} m), a length. **N m** is a newton meter, the unit of work (or of torque). And **m s^{-1}** (meter per second, with a space) is not **ms^{-1}** (per millisecond).

The prefixes you will meet most in this book:

Prefix	Symbol	Factor		Prefix	Symbol	Factor
giga	G	10^9		centi	c	10^{-2}
mega	M	10^6		milli	m	10^{-3}
kilo	k	10^3		micro	μ	10^{-6}
hecto	h	10^2		nano	n	10^{-9}
deca	da	10^1		pico	p	10^{-12}

1.4.1 Fundamental and derived units

The units of the fundamental quantities are the **fundamental units** (base units): the meter, kilogram, second, kelvin, ampere, candela and mole. The units of derived quantities are **derived units**, made by multiplying and dividing fundamental units.

Exercise

Choose the correct option

Check 8

1. Which of the following is dimensionally incorrect?

a $s = ut + at^2$
b $s = ut - \frac{1}{2}at^2$
c $s = ut + 5at^2$
d $s = ut + \frac{1}{2}at$

Check 9

2. Which of the following is physically correct?

a $v = u + 2at$
b $v = u - \frac{1}{2}at$
c $v = u + \frac{1}{2}at$
d $v = at + u$

Check 10

3. The same physical quantity measured in two different systems of units has:

a the same unit but different dimensions
b the same dimensions but different units
c the same units and the same dimensions
d different units and different dimensions

Check 11

4. Which of the following physical quantities is dimensionless?

a the gravitational constant
b strain
c the universal gas constant
d Stefan's constant

Check 12

5. The velocity of a particle is $v = \frac{b}{t} + ct^2$. The dimensional formula of b is:

a [L]
b [M L]
c [T]
d [M L T^{-1}]

1.6.2 Limitations of dimensional analysis

Dimensional analysis is quick and powerful, but it can't do everything:

1. It gives **no value for a dimensionless constant**: the 2π in the pendulum's period, the ½ in $\frac{1}{2}at^2$.
2. It can't derive a relation that contains **trigonometric, exponential or logarithmic functions** ($\sin\theta$, e^{-kt}, $\log x$): their arguments are dimensionless, so they can't be found by matching powers.
3. It can't tell whether a quantity is a **scalar or a vector**: work and torque have the same dimensions.
4. It can't tell **which** quantity it has, when two share a dimensional formula; and it can't derive a relation when a quantity depends on **more than three** others in mechanics (there are only three equations, one each for M, L and T), or when a **sum** of terms is involved, as in $s = ut + \frac{1}{2}at^2$.

IN SHORT

Key ideas of Unit 1

- A **physical quantity** is anything that can be measured: quantity $= n\,u$ (number × unit). Seven are **fundamental** (mass, length, time, temperature, current, luminous intensity, amount of substance); the rest are **derived**.
- **SI** units: m, kg, s, K, A, cd, mol; since 2019 each is fixed by a constant of nature.
- **Accuracy** = closeness to the true value; **precision** = closeness of repeated readings to one another.
- **Significant figures**: the certain digits plus the first uncertain one. Leading zeros never count; trailing zeros after a decimal point do. Add/subtract: fewest decimal places. Multiply/divide: fewest significant figures.
- **Uncertainty**: add absolute uncertainties for a sum or difference, relative ones for a product or quotient.
- **Dimensions**: $[X] = [M^a L^b T^c]$. Every term of a correct equation has the same dimensions (homogeneity). Use them to check equations, derive relations, convert units and find unknown dimensions, but not to find pure numbers or functions like sin and log.

The unit of velocity is m/s; the unit of force is kg m/s², which is given its own name, the **newton** (N).

1.4.2 Systems of units

At different times and places people used different sets of units. The four systems you should know:

System	Length	Mass	Time	Where it came from
CGS	centimeter	gram	second	the metric system; used by scientists from the 1870s
MKS	meter	kilogram	second	the metric system, which began in France in the 1790s
FPS	foot	pound	second	Britain (the British system)
SI	meter	kilogram	second, + four more	agreed internationally in 1960

In **1960**, at the General Conference on Weights and Measures in Paris, the nations agreed on one system for the whole world, built on the MKS system: the **SI**, short for *Système international d'unités*, the International System of Units. Today it is used in science everywhere, and in daily life almost everywhere.

Table 1.1 The SI base quantities and units, with the two supplementary ones:

Fundamental quantity	SI unit	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
temperature	kelvin	K
electric current	ampere	A
luminous intensity	candela	cd
amount of substance	mole	mol

Supplementary quantity	SI unit	Symbol
plane angle	radian	rad
solid angle	steradian	sr

The **radian** measures a plane angle: an angle in radians is the arc it cuts off a circle divided by the radius, θ = arc ÷ radius. The **steradian** measures a solid angle, the three-dimensional "angle" of a cone: the area it cuts off a sphere divided by the radius squared.

CAREFUL

Supplementary units

Some books still call the radian and steradian **supplementary units**. Since 1995 the SI counts them as derived units with no dimensions (an arc divided by a radius is a length divided by a length). Either answer is accepted; know both names.

How the base units are defined

A unit is only as good as its definition. Since **2019**, every SI base unit is defined by fixing the exact value of a constant of nature, so that a laboratory anywhere can make the units for itself (as of 2026):

- **Second (s):** the time taken by 9 192 631 770 vibrations of the radiation given out when a **cesium-133** atom jumps between two particular energy levels (the two "hyperfine" levels of its ground state). Atomic clocks keep this time to better than a second in a hundred million years.
- **Meter (m):** the length of the path traveled by light in vacuum in $\frac{1}{299\,792\,458}$ of a second. So the speed of light is exactly 299 792 458 m/s, by definition. (1 m = 3.28 feet.)
- **Kilogram (kg):** defined by fixing the value of Planck's constant, $h = 6.626\,070\,15 \times 10^{-34}$ J s exactly; a kilogram is measured against it with an instrument called a **Kibble balance**, which weighs a mass against an electromagnetic force.
- The **kelvin**, **ampere**, **mole** and **candela** are fixed in the same way by the Boltzmann constant, the charge of the electron, the Avogadro constant and a fixed brightness of green light.

WORKED EXAMPLE

A newton in dynes

The CGS unit of force is the dyne (g cm/s²). How many dynes make one newton?

Answer

Force: $[MLT^{-2}]$, so $a = 1, b = 1, c = -2$.

$$n_2 = 1 \times \left(\frac{1\text{ kg}}{1\text{ g}}\right)^{1} \left(\frac{1\text{ m}}{1\text{ cm}}\right)^{1} \left(\frac{1\text{ s}}{1\text{ s}}\right)^{-2} = 1000 \times 100 \times 1 = 10^5$$

So **1 N = 10^5 dyne**.

4. Finding the dimensions of an unknown quantity

Every term added in an equation must have the same dimensions, so an unknown constant takes whatever dimensions make its term fit.

WORKED EXAMPLE

The constants in a velocity

The velocity of a particle is $v = A + Bt + Ct^2$, where t is time. Find the dimensional formulas of A, B and C.

Answer

By homogeneity, $[v] = [A] = [Bt] = [Ct^2] = [LT^{-1}]$. So

$$[A] = [LT^{-1}], \qquad [B] = \frac{[LT^{-1}]}{[T]} = [LT^{-2}], \qquad [C] = \frac{[LT^{-1}]}{[T^2]} = [LT^{-3}]$$

B is an acceleration; C is a rate of change of acceleration.

YOUR TURN

Inside a cosine

In the wave equation $y = a\cos(\omega t - kx)$, t is time and x is position. Find the dimensions of ω and k.

Hint

The cosine of something must be the cosine of a pure number (an angle in radians).

Answer

ωt and kx must both be dimensionless. So $[\omega] = [T^{-1}]$ and $[k] = [L^{-1}]$.

YOUR TURN

The frequency of a string

The frequency f of vibration of a stretched string depends on its tension T (a force), its length L, and its mass per unit length μ. Use dimensions to find the relation.

Answer

Let $f = k\,T^a L^b \mu^c$. With $[f] = [T^{-1}]$, $[T] = [MLT^{-2}]$ (the tension, a force), $[L] = [L]$ and $[\mu] = [ML^{-1}]$:

$$[M^0 L^0 T^{-1}] = [MLT^{-2}]^a [L]^b [ML^{-1}]^c = [M^{a+c}\, L^{a+b-c}\, T^{-2a}]$$

T: $-2a = -1 \Rightarrow a = \tfrac{1}{2}$. M: $a + c = 0 \Rightarrow c = -\tfrac{1}{2}$. L: $a + b - c = 0 \Rightarrow b = -1$.

$$f = \frac{k}{L}\sqrt{\frac{T}{\mu}}$$

Experiment gives $k = \tfrac{1}{2}$ for the lowest note.

A STORY FROM SCIENCE

How big was the bomb?

In 1945 the United States tested the first atomic bomb in New Mexico. The energy it released was secret. In 1950 the British physicist **G. I. Taylor** worked it out from a series of photographs of the fireball that had been made public, each with its time and a scale of length.

He reasoned that the fireball's radius R after a time t could depend only on the energy E released and the density ρ of the air. Only one combination of these has the dimensions of a length:

$$R = k\left(\frac{E\,t^2}{\rho}\right)^{1/5}, \qquad \text{so} \qquad E \approx \frac{\rho R^5}{t^2}$$

(check it: $[\rho R^5/t^2] = [ML^{-3}][L^5][T^{-2}] = [ML^2T^{-2}]$, an energy). From the photos his estimate came close to the real value, which was still a closely kept secret. The story is told that the U.S. government was not amused.

has dimensions $[M^a L^b T^c]$, its unit in each system is built from that system's units of mass, length and time:

$$n_1\,[M_1^a L_1^b T_1^c] = n_2\,[M_2^a L_2^b T_2^c] \quad \Rightarrow \quad n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$$

A STORY FROM SCIENCE

The kilogram that lost weight

From 1889 to 2019, the kilogram was the mass of one metal cylinder, the **International Prototype of the Kilogram**, made of 90% platinum and 10% iridium, 39 mm high and 39 mm across, and kept under three glass jars at the International Bureau of Weights and Measures at Sèvres, near Paris. Countries had official copies, and every few decades the copies were brought back to be compared with it. They had drifted apart by about 50 micrograms (the mass of a grain of sand) in a hundred years, and nobody could say whether the copies had gained or the original had lost. A standard that changes is no standard. On 20 May 2019 the cylinder was retired, and the kilogram was tied to Planck's constant, which is the same everywhere and for ever.

CAREFUL

The kilogram in older books

Some books still define the kilogram as the mass of the platinum–iridium cylinder at Sèvres. That was true until 2019. If an exam asks for it, you may give that definition, but add that the kilogram is now defined by fixing Planck's constant.

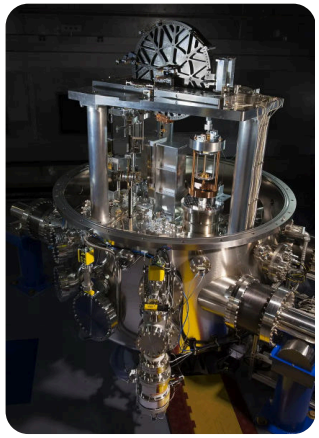


Figure 1.3 *The NIST-4 Kibble balance in the United States, one of the instruments that measure a kilogram from Planck's constant.*

Photo: Jennifer Lauren Lee, for the U.S. National Institute of Standards and Technology (NIST), Public domain

Check 3
Which is written correctly?

a 5 Kgs
b 20 Newton
c 3.0 m/s
d 50 c.m.

1.5 Precision and significant figures

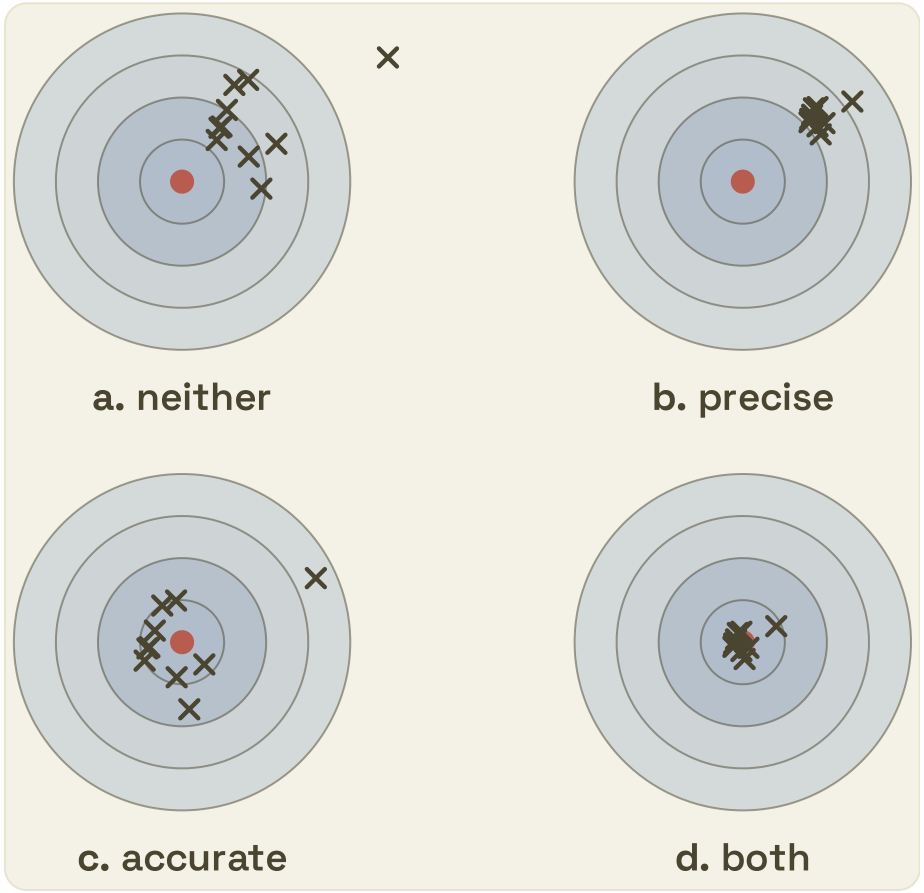


Figure 1.4 *Four sets of throws. a. Neither accurate nor precise. b. Precise but not accurate: close together, away from the center. c. Accurate but not precise: around the center on average, but scattered. d. Accurate and precise.*

These are the two qualities of a set of measurements:

- **Accuracy** is how close a measurement (or the average of several) comes to the **true value**.

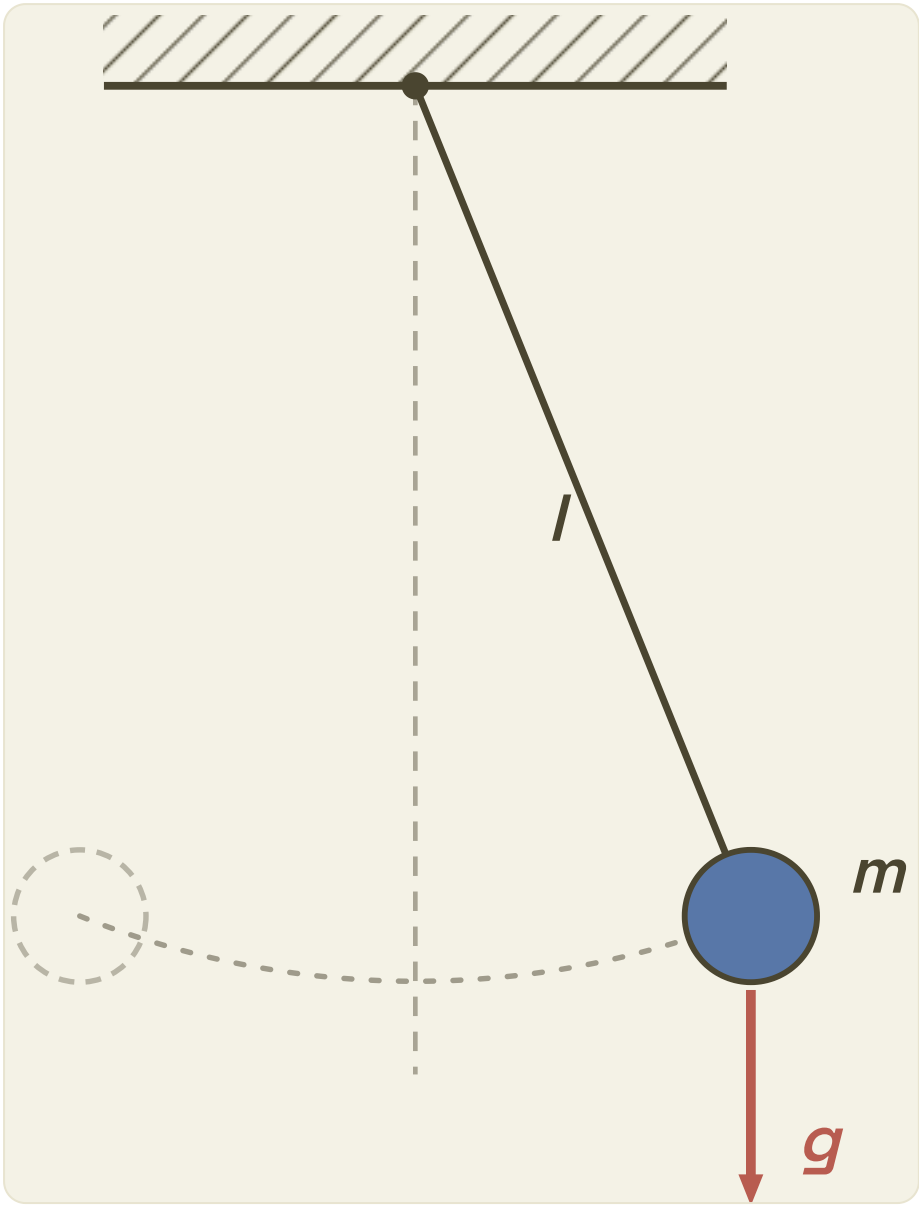


Figure 1.5 *A simple pendulum: a bob of mass m on a string of length l, swinging under gravity g. Its period T is the time of one full swing, there and back.*

3. Converting units

A quantity is the same whatever units it is measured in: $n_1 u_1 = n_2 u_2$. If the quantity

YOUR TURN

Test $v^2 = u^2 + as^2$

Is the equation $v^2 = u^2 + as^2$ dimensionally correct?

Answer

$[v^2] = [u^2] = [L^2T^{-2}]$, but $[as^2] = [LT^{-2}][L^2] = [L^3T^{-2}]$. The last term is not like the others, so the equation is **dimensionally wrong**, and so physically wrong. (The right one is $v^2 = u^2 + 2as$.)

2. Deriving a relation

If you know (or guess) which quantities something depends on, dimensions can often find how.

The **period** T of a simple pendulum (the time of one full swing, there and back) might depend on the mass m of the bob, the length l of the string, and the acceleration due to gravity g. Suppose

$$T = k\, m^a\, l^b\, g^c$$

where k is a pure number. Put in the dimensions:

$$[M^0L^0T^1] = [M]^a\,[L]^b\,[LT^{-2}]^c = [M^a\,L^{b+c}\,T^{-2c}]$$

By the principle of homogeneity, the powers of M, L and T must match on both sides:

- M: $a = 0$
- T: $-2c = 1$, so $c = -\frac{1}{2}$
- L: $b + c = 0$, so $b = \frac{1}{2}$

So

$$T = k\sqrt{\frac{l}{g}}$$

Dimensions can't find the pure number k; experiment (or a full theory, Grade 12) gives $k = 2\pi$:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The answer says something surprising, which experiment confirms: the period doesn't depend on the bob's mass ($a = 0$). A heavy bob and a light one on equal strings swing in step.

- **Precision** is how close repeated measurements come to **one another**, whatever the true value: how consistent they are, and how finely the instrument can read.

Suppose a bag of rice really has a mass of 10.0 kg.

- Readings of 9.8 kg, 10.2 kg and 10.4 kg are **accurate** (they cluster round 10 kg on average) but not very precise (they spread over 0.6 kg).
- Readings of 12.5 kg, 12.6 kg and 12.4 kg are **precise** (within 0.2 kg of each other) but not accurate: the scale has a fault that adds about 2.5 kg every time.
- Readings of 10.1 kg, 9.9 kg and 10.0 kg are both.

A faulty instrument that is wrong by the same amount every time (a scale that doesn't read zero when empty, a stretched tape measure) spoils the **accuracy** without spoiling the precision: no number of repeats will reveal it. Careless reading and wobbly hands spoil the **precision**, and averaging many readings helps.

Check 4

A student's stopwatch reads 2.31 s, 2.30 s, 2.32 s and 2.31 s for a pendulum whose true period is 2.01 s. Her readings are:

a accurate and precise
b precise but not accurate
c accurate but not precise
d neither accurate nor precise

1.5.1 Significant figures

Measure the diameter of a ball with a ruler marked in millimeters, and you might read **2.64 cm**. The 2 and the 6 are certain: you can see them on the scale. The last digit, 4, is your best estimate between two marks: it is **uncertain**, but still worth writing. The digits that are certain, plus the first uncertain one, are the **significant figures** of the measurement. The diameter 2.64 cm has **three** significant figures.

So the number of significant figures tells the reader how precisely a quantity was measured:

- A rod measured as **3.5 cm** (two significant figures) with a ruler marked in centimeters;
- as **3.56 cm** (three) with a millimeter scale;
- as **3.567 cm** (four) with a vernier caliper or micrometer screw gauge.

Each is **more precise** than the one before: it pins the length down more finely. A measurement can never be more precise than the instrument it was made with, so don't write 3.5672 cm from a ruler.

CAREFUL

More figures means more precise

Some books say that more significant figures means a more **accurate** measurement. Strictly, more significant figures means a more **precise** one. 2.0000 cm read from a micrometer that hasn't been zeroed may be very precise and still wrong. In an exam that asks which reading is "most accurate" among 2 cm, 2.00 cm, 2.000 cm and 2.0000 cm, choose the one with the most significant figures, and say that it is the most precise.

Rules for counting significant figures

The non-zero digits are easy: they always count. All the rules are about **zeros**:

Rule	Examples
1. Every **non-zero digit** is significant.	21.48 has 4
2. **Zeros between** non-zero digits are significant.	15.0041 has 6; 2001 has 4
3. **Leading zeros** (before the first non-zero digit) are not significant: they only place the decimal point.	0015 has 2; 0.006 has 1; 0.00025 has 2; 0.0080038 has 5; 0.25 has 2
4. **Trailing zeros after a decimal point** are significant: someone measured them.	5.00 has 3; 5.0 has 2; 0.200 has 3
5. **Trailing zeros in a whole number** with no decimal point are taken as not significant: they may only show the size of the number.	9800 has 2; 050600 has 3
6. In **scientific notation**, $N \times 10^a$, only the digits of N count; the power of ten never does.	1.6×10^{-19} C has 2; 6.62×10^{-34} J s has 3

Rule 5 hides a problem: is "9800 m" measured to the nearest meter or the nearest hundred meters? Scientific notation removes the doubt: 9.8×10^3 m has two significant figures, and 9.800×10^3 m has four. That is one reason scientists write numbers that way.

1.5.2 Rounding off

To **round off** a number to fewer significant figures, drop digits from the right until the number has as many figures as you want, and look at the **first digit dropped**:

1. Checking an equation

Put the dimensions of each quantity into the equation and see whether every term matches.

WORKED EXAMPLE

Is $s = ut + \frac{1}{2}at^2$ correct?

Check the equation dimensionally.

Answer

$$[L] = [LT^{-1}][T] + [LT^{-2}][T^2] \quad \Rightarrow \quad [L] = [L] + [L]$$

(The ½ is a pure number, with no dimensions, so it is left out.) Every term is a length: the equation is **dimensionally correct**.

But look at two wrong versions:

- $s = ut + at^2$ is dimensionally correct too ([L] = [L] + [L]), yet it is **physically wrong**: the ½ is missing, and dimensions can't see pure numbers.
- $s = ut^2 + \frac{1}{2}at^2$ gives $[L] = [LT] + [L]$: its first term is not a length, so it is **dimensionally wrong**, and therefore certainly physically wrong.

So:

- **An equation that is dimensionally wrong is certainly wrong.**
- **An equation that is dimensionally correct may still be wrong**, by a pure number or by a missing term.

WORKED EXAMPLE

A student's escape velocity

A student writes the escape velocity from a planet of mass M and radius R as $v = \sqrt{\frac{2GM}{R}}$. Check it.

Answer

$[G] = [M^{-1}L^3T^{-2}]$, so

$$\left[\sqrt{\frac{2GM}{R}}\right] = \sqrt{\frac{[M^{-1}L^3T^{-2}][M]}{[L]}} = \sqrt{[L^2T^{-2}]} = [LT^{-1}]$$

which is the dimension of velocity. The formula is dimensionally correct (and it is in fact the right formula, Unit 7).

WORKED EXAMPLE

Adding lengths

A student writes $2[L] + [L] = [L]$ and $[L] - [L] = [L]$. Is this right?

Answer

Yes. Dimensions don't follow ordinary arithmetic: they say what **kind** of quantity something is, not how much. Adding a length to a length gives a length (2 m + 1 m = 3 m), and taking a length from a length still gives a length (5 m − 2 m = 3 m), even when the answer is zero. So the sum or difference of two quantities of the same dimensions has those same dimensions, and numbers like the 2 have no dimensions to add.

	Has dimensions	Has no dimensions
Value can change (variable)	**dimensional variables**: velocity, work, power, pressure	**dimensionless variables**: strain, relative density, angle
Value fixed (constant)	**dimensional constants**: gravitational constant G, Planck's constant h, speed of light c	**dimensionless constants**: π, the numbers 2 and ½

A dimensionless quantity has no unit (or, like the angle, a unit that is a ratio of two equal units). Note that the coefficient of friction, which is a ratio of two forces, is dimensionless too, but it isn't a constant of nature: it depends on the two surfaces.

Check 7

Which pair has the same dimensions?

a force and energy
b work and torque
c pressure and force
d power and energy

1.6.1 Uses of dimensional analysis

Dimensional analysis has four uses:

1. to **check** whether a physical equation is correct;
2. to **derive** a relation between physical quantities;
3. to **convert** a quantity from one system of units to another;
4. to find the **dimensions of an unknown quantity** in an equation.

WORKED EXAMPLE

One significant figure

How many significant figures does 0.005 have?

Answer

The zeros before the 5 only place the decimal point (Rule 3): **one** significant figure. Written in scientific notation it is 5×10^{-3}, and the one figure is plain to see.

Check 5

How many significant figures are there in 0.0300 m?

a 1
b 2
c 3
d 5

1. **Less than 5:** the digit before it stays as it is. 123.42 ≈ 123.4 (4 significant figures) ≈ 123 (3).
2. **More than 5**, or 5 followed by any non-zero digits: the digit before it goes **up by 1**. 123.167 ≈ 123.17 (5) ≈ 123.2 (4); 2.4851 ≈ 2.49 (3).
3. **Exactly 5** (5 with nothing, or only zeros, after it): the digit before it goes **up by 1 if it is odd**, and **stays if it is even**, so that the last digit kept is even. 12.35 ≈ 12.4; 12.45 ≈ 12.4 (3 figures).

Rule 3, "round half to even", keeps a long column of rounded numbers from creeping upward on average: half the 5s round up and half round down. (In daily arithmetic, and in many calculators, a 5 always rounds up; in physics practical work, follow the rule your teacher sets.)

1.5.3 Significant figures in calculations

A calculator gives you every digit it can, but an answer can't be more precise than the measurements it came from.

Adding and subtracting: the answer keeps as many **decimal places** as the input with the **fewest decimal places**.

$$3.240\ \text{m} + 7.14\ \text{m} = 10.380\ \text{m} \ \rightarrow\ \mathbf{10.38\ m}$$

$$3.266\ \text{m} - 2.1\ \text{m} = 1.166\ \text{m} \ \rightarrow\ \mathbf{1.2\ m}$$

The second answer is 1.2 m because 2.1 m is known only to the nearest tenth of a meter, so the answer can't be known more finely.

Multiplying and dividing: the answer keeps as many **significant figures** as the input with the **fewest significant figures**.

$$118.6 \times 16.2 = 1921.32 \;\rightarrow\; \mathbf{1.92 \times 10^{3}} \text{ (3 figures, as 16.2)}$$

$$118.6 \div 16.2 = 7.3209876\ldots \;\rightarrow\; \mathbf{7.32}$$

$$125 \div 3 = 41.666\ldots \;\rightarrow\; \mathbf{4 \times 10^{1}} \text{ (1 figure, if 3 is a measured value)}$$

The last needs care. If the 3 is a measurement good to one figure, the answer has one figure: 4×10^{1} (writing "40" would hide that only the 4 counts). If the 3 is an exact count (125 g shared among exactly 3 people), it doesn't limit the answer at all: 41.7 g, to the three figures of 125.

WORKED EXAMPLE

A room's floor

A room is measured as 4.25 m long and 3.1 m wide. Find its perimeter and its area to the right number of figures.

Answer

Perimeter: $4.25 + 3.1 + 4.25 + 3.1 = 14.70$ m. The fewest decimal places is one (3.1), so the perimeter is **14.7 m**.

Area: $4.25 \times 3.1 = 13.175$ m^2. The fewest significant figures is two (3.1), so the area is **13 m^2**.

Check 6

A sample question: which of these measurements is the most precise?

a 5 kg
b 5.0 kg
c 5.00 kg
d 5.000 kg

Uncertainty in a result

A measurement is never exact, so it is written with its **uncertainty**: a length of (0.50 ± 0.01) m means the true length is probably between 0.49 m and 0.51 m. The 0.01 m is the **absolute uncertainty**; divided by the value, $0.01 \div 0.50 = 0.02$, it is the **relative** (fractional) **uncertainty**, or 2% as a **percentage uncertainty**.

When measured values are combined, their uncertainties combine too, by two rules (taking the worst case, when every error goes the same way):

- **Adding or subtracting:** add the **absolute** uncertainties. If $Z = A + B$ or $Z = A - B$, then $\Delta Z = \Delta A + \Delta B$.

dimensions help you find units, and units help you catch mistakes in an equation.

WORKED EXAMPLE

Dimensions in other fundamental quantities

Taking **force**, length and time as the fundamental quantities, find the dimensional formula of energy.

Answer

Energy = work = force × displacement, so

$$[E] = [F][L] = [F\,L\,T^{0}]$$

Any set of independent quantities can serve as fundamental ones; then the dimensional formulas change.

YOUR TURN

Power in F, L and T

Taking force, length and time as fundamental, find the dimensional formula of power.

Answer

Power = work ÷ time = force × displacement ÷ time, so $[P] = \dfrac{[F][L]}{[T]} =$ **[F L T^{-1}]**.

The principle of homogeneity

You can add a length to a length, but not a length to a time: 3 m + 2 s means nothing. This common sense becomes a powerful rule, the **principle of homogeneity of dimensions**:

In a correct physical equation, every term on both sides has the same dimensions.

Only quantities of the same kind can be added, subtracted, compared or equated. Mass can be added to mass, but not to length; force can be added to force, but not to energy. In $v = u + at$, the three terms have the same dimensions, [L T^{-1}] (since $[at] = [LT^{-2}][T] = [LT^{-1}]$), so the equation is **homogeneous**.

Kinds of physical quantity

By their dimensions, and by whether their value can change, physical quantities are of four kinds:

#	Quantity	Defining formula	Dimensional formula	SI unit
23	strain	change in length ÷ original length	[M^0 L^0 T^0]	none
24	modulus of elasticity	stress ÷ strain	[M L^{-1} T^{-2}]	Pa (N/m^2)
25	surface tension	force ÷ length	[M T^{-2}]	N/m
26	velocity gradient	dv/dx	[T^{-1}]	s^{-1}
27	coefficient of viscosity, η	force ÷ (area × velocity gradient)	[M L^{-1} T^{-1}]	N s/m^2
28	torque (moment of a force, couple)	force × perpendicular distance	[M L^2 T^{-2}]	N m
29	coefficient of linear expansion, α	change in length ÷ (original length × change in temperature)	[K^{-1}]	/K
30	specific heat capacity, s	$\frac{Q}{m\,\Delta\theta}$	[L^2 T^{-2} K^{-1}]	J/(kg K)
31	specific latent heat, L	Q/m	[L^2 T^{-2}]	J/kg
32	thermal conductivity, K	$\frac{Q\,\Delta x}{t\,A\,\Delta\theta}$	[M L T^{-3} K^{-1}]	W/(m K)
33	Stefan's constant, σ	$\frac{E}{A\,t\,T^4}$ (power per area ÷ T^4)	[M T^{-3} K^{-4}]	W/(m^2 K^4)
34	molar gas constant, R	$\frac{PV}{nT}$	[M L^2 T^{-2} K^{-1} N^{-1}]	J/(mol K)
35	electric charge	current × time, $Q = It$	[A T]	C (A s)

Two things in the table are worth noticing:

- Different quantities can share a dimensional formula: work and torque are both [M L^2 T^{-2}]; pressure, stress and modulus of elasticity are all [M L^{-1} T^{-2}]. So dimensions alone can't tell you which quantity you have.
- The SI unit of a quantity, written in base units, follows straight from its dimensional formula: [M L^2 T^{-2}] means kg m^2 s^{-2}, which is the joule. So

- **Multiplying or dividing:** add the **relative** uncertainties. If $Z = AB$ or $Z = A/B$, then $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$.
- A power counts as repeated multiplying: if $Z = A^n$, then $\frac{\Delta Z}{Z} = n\,\frac{\Delta A}{A}$.

Note that subtracting still **adds** the uncertainties: the difference of two uncertain lengths is less certain than either.

WORKED EXAMPLE

The volume of a cube

The side of a cube is measured as (2.00 ± 0.02) cm. Find its volume with its uncertainty.

Answer

$V = a^3 = 2.00^3 = 8.00$ cm^3.

Relative uncertainty in a: $\frac{0.02}{2.00} = 0.01$, that is 1%. For a cube, $\frac{\Delta V}{V} = 3 \times 0.01 = 0.03$ (3%).

$\Delta V = 0.03 \times 8.00 = 0.24$ cm^3, so $V =$ **(8.0 ± 0.2) cm^3**. A 1% error in the side becomes a 3% error in the volume.

1.6 Dimensions of a physical quantity

Every derived quantity is built from the fundamental quantities, and its **dimensions** say exactly how. In mechanics three fundamental quantities are enough, written in square brackets:

Fundamental quantity	Dimension
mass	[M]
length	[L]
time	[T]
temperature	[K] (in the SI's own notation, Θ)
electric current	[A] (or [I])
amount of substance	[N] (or [mol])
luminous intensity	[J] (or [cd])

The **dimensions** of a physical quantity are the **powers** to which the fundamental quantities must be raised to represent it. A mechanical quantity X is written

$$[X] = [M^a\, L^b\, T^c]$$

- $[M^a L^b T^c]$ is the **dimensional formula** of X;
- a, b and c are its **dimensions** in mass, length and time;
- the equation $[X] = [M^a L^b T^c]$ is its **dimensional equation**.

The square brackets are read "the dimensions of". They may be left off (M L T^{-2}), but are usually written.

To find a dimensional formula, start from the **defining equation** of the quantity, and put in the dimensions of each part.

WORKED EXAMPLE

The dimensions of velocity

Find the dimensional formula of velocity.

Answer

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}, \qquad [v] = \frac{[L]}{[T]} = [L\,T^{-1}] = [M^0\, L^1\, T^{-1}]$$

The dimensions of velocity are 0 in mass, 1 in length and −1 in time; [M^0 L T^{-1}] is its dimensional formula, and $[v] = [M^0 L T^{-1}]$ its dimensional equation. (The M^0 says, in so many words, that velocity doesn't depend on mass.)

WORKED EXAMPLE

The dimensions of power

Find the dimensional formula of power.

Answer

Power is work done per unit time, and work is force × displacement:

$$P = \frac{\text{work}}{\text{time}} = \frac{\text{force} \times \text{displacement}}{\text{time}}$$

Force = mass × acceleration, so $[F] = [M][LT^{-2}] = [MLT^{-2}]$. Then

$$[P] = \frac{[MLT^{-2}][L]}{[T]} = [M\, L^2\, T^{-3}]$$

The dimensions of the quantities in this course, worked out the same way:

#	Quantity	Defining formula	Dimensional formula	SI unit
1	area	length × breadth	[L^2]	m^2
2	volume	length × breadth × height	[L^3]	m^3
3	density	mass ÷ volume	[M L^{-3}]	kg/m^3
4	relative density (specific gravity)	density ÷ density of water	[M^0 L^0 T^0]	none
5	frequency	number of cycles ÷ time	[T^{-1}]	Hz
6	plane angle	arc ÷ radius	[M^0 L^0 T^0]	rad
7	speed	distance ÷ time	[L T^{-1}]	m/s
8	velocity	displacement ÷ time	[L T^{-1}]	m/s
9	acceleration	change in velocity ÷ time	[L T^{-2}]	m/s^2
10	linear momentum	mass × velocity, $p = mv$	[M L T^{-1}]	kg m/s
11	force	mass × acceleration, $F = ma$	[M L T^{-2}]	N
12	impulse	force × time	[M L T^{-1}]	N s
13	weight	$W = mg$	[M L T^{-2}]	N
14	work, energy	force × displacement	[M L^2 T^{-2}]	J (N m)
15	power	work ÷ time	[M L^2 T^{-3}]	W (J/s)
16	pressure	force ÷ area	[M L^{-1} T^{-2}]	Pa (N/m^2)
17	angular velocity	angle ÷ time, $\omega = \theta/t$	[T^{-1}]	rad/s
18	angular acceleration	angular velocity ÷ time, $\alpha = \omega/t$	[T^{-2}]	rad/s^2
19	gravitational constant	$G = \frac{Fr^2}{Mm}$	[M^{-1} L^3 T^{-2}]	N m^2/kg^2
20	gravitational field strength	force ÷ mass	[L T^{-2}]	N/kg
21	gravitational potential	work ÷ mass	[L^2 T^{-2}]	J/kg
22	stress	force ÷ area	[M L^{-1} T^{-2}]	Pa (N/m^2)