Unit 1

Answers: Physical Quantities

Quick checks

1 b A quantitative science expresses its ideas as numbers and equations and checks them against measured numbers. Physics does study color, too: as the wavelength of light, a quantity.

2 c Pressure is force ÷ area, and force is mass × acceleration: it is built from mass, length and time. Mass, time and electric current are three of the seven fundamental quantities.

3 c The kilogram's symbol is kg (small k, never plural), a unit named after a person is written newton with a small n (or N as a symbol), and a symbol takes no full stops: 50 cm. 3.0 m/s follows every rule.

4 b They agree with each other to within 0.02 s, so they are precise, but every one is about 0.3 s too long: not accurate. A steady error like this often comes from the same reaction delay each time, or a stopwatch that runs fast.

5 c The first two zeros only place the decimal point; the 3 counts; the two zeros after it, written after the decimal point, are measured digits and count too: 3.00×10^{-2} m, three figures.

6 d 5.000 kg is known to the nearest gram, 5 kg only to the nearest kilogram. Whether it is also the most **accurate** depends on whether the scale reads true, which the number alone can't tell you.

7 b Work is force × displacement and torque is force × perpendicular distance: both [M L^2 T^{-2}]. Force [M L T^{-2}], energy [M L^2 T^{-2}], pressure [M L^{-1} T^{-2}] and power [M L^2 T^{-3}] all differ.

8 d $[at] = [LT^{-2}][T] = [LT^{-1}]$, a velocity, not a length. In the other three every term is [L]; the pure numbers 1, ½ and 5 don't change the dimensions (only one of them, ½, is physically right).

9 d All four are dimensionally correct ([L T^{-1}] in every term). Only $v = u + at$, written here as $at + u$, is physically right: dimensions can't catch a wrong number.

10 b Dimensions describe the kind of quantity, which doesn't change; units are its size in a particular system. A force is [M L T^{-2}] whether it is measured in newtons or dynes.

11 b Strain is a change in length divided by a length: [M^0 L^0 T^0]. G is [M^{-1} L^3 T^{-2}], R is [M L^2 T^{-2} K^{-1} N^{-1}] and σ is [M T^{-3} K^{-4}].

12 a $[b/t] = [v] = [LT^{-1}]$, so $[b] = [LT^{-1}][T] = [L]$.

13 c Momentum = force × time (impulse equals change in momentum, $F = \Delta p/\Delta t$), so $[p] = [FT]$.

14 **c** A light year is a **distance**, the distance light travels in one year (about 9.46×10^{15} m), not a time.

15 **b** Perimeter $= 2(l + b) = 2(0.5 + 0.2) = 1.4$ m. In a sum the absolute uncertainties add, and there are four sides: $0.01 + 0.01 + 0.01 + 0.01 = 0.04$ m. So (1.4 ± 0.04) m. 0.1 m is no measure of this rectangle at all, and $\pm$ 0.01 m or $\pm$ 0.02 m forgets that each of the four sides adds its own uncertainty.

16 **c** Counting: 0.70860 has 5 (the last zero, after the decimal point, counts), 0.50300 has 5, 0.07086 has 4 (the leading zeros don't count), 60.5370 has 6.

17 **b** The power of ten doesn't count, nor do the leading zeros; the 5 and the zero after it do: 5.0×10^{-15}, two figures.

18 **b** A narrow peak means the readings agree closely (precise); a mean on the true value means they are right on average (accurate). B has both. A is precise but its mean is off the true value; D is accurate but spread out; C is neither.

12.

a. The **dimensional formula** of a physical quantity is the expression that shows which fundamental quantities, raised to which powers, make it up: for a mechanical quantity, $[M^a L^b T^c]$. For example, force is [M L T^{-2}].

b. Energy = force × displacement, so $[E] = [F][L] =$ **[F L T^0]**.

c. $[\,] = [L^3 T^{-1}]$.

$$\left[\frac{kPa^x}{\eta l}\right] = \frac{[ML^{-1}T^{-2}][L^x]}{[ML^{-1}T^{-1}][L]} = [M^0\, L^{x-1}\, T^{-1}]$$

Matching the powers of L: $x - 1 = 3$, so $x = 4$. (The flow goes as the fourth power of the radius, Poiseuille's law: half as wide a pipe carries a sixteenth as much.)

13.

a. Let $f = k\, l^x\, T^y\, m^z$, with k a pure number.

$$[T^{-1}] = [L]^x [MLT^{-2}]^y [ML^{-1}]^z = [M^{y+z}\, L^{x+y-z}\, T^{-2y}]$$

T: $-2y = -1 \Rightarrow y = \frac{1}{2}$. M: $y + z = 0 \Rightarrow z = -\frac{1}{2}$. L: $x + y - z = 0 \Rightarrow x = -1$.

So $f = \dfrac{k}{l}\sqrt{\dfrac{T}{m}} \propto \dfrac{1}{l}\sqrt{\dfrac{T}{m}}$. Dimensions can't give the number k; experiment (and the theory of waves on a string) gives $k = \frac{1}{2}$.

b. Every term is a force, [M L T^{-2}]:

$[a] =$ **[M L T^{-2}]**; $[b] = [MLT^{-2}]/[T] =$ **[M L T^{-3}]**; $[c] = [MLT^{-2}]/[T^2] =$ **[M L T^{-4}]**; $[d] = [MLT^{-2}]/[T^3] =$ **[M L T^{-5}]**.

c.

$$[f] = \frac{1}{[L]}\sqrt{\frac{[MLT^{-2}]}{[ML^{-1}]}} = \frac{1}{[L]}\sqrt{[L^2T^{-2}]} = \frac{[LT^{-1}]}{[L]} = [M^0 L^0 T^{-1}]$$

as a frequency should be.

14. a. **Accuracy** is how close a measurement is to the true value. **Precision** is how close repeated measurements are to each other, and how finely a value is read. Readings can be precise but inaccurate (a faulty instrument that is wrong the same way every time) or accurate on average but imprecise (scattered round the true value).
The number of significant figures shows the precision: a measurement with more significant figures was read to a finer division, so it is more precise.
b. **iv. 2.0000 cm**: it has five significant figures, so it was read to 0.0001 cm, while 2 cm was read only to the nearest centimeter. (Whether it is also the most **accurate** depends on whether the instrument reads true; the digits alone can't say.)

15. Each term must be a velocity, [L T^{-1}]:
$[a] = \mathbf{[L\ T^{-1}]}$; $[b] = [LT^{-1}]/[T] = \mathbf{[L\ T^{-2}]}$; $[c] = [LT^{-1}]/[T^2] = \mathbf{[L\ T^{-3}]}$.

16. $$[MLT^{-2}] = [L^2][ML^{-3}][LT^{-1}]^x = [M\,L^{x-1}\,T^{-x}]$$
T: $-x = -2$, so $x = 2$ (and L checks: $x - 1 = 1$). Air drag at high speed grows as the **square** of the speed: drive twice as fast and the drag is four times as large.

17. d is added to t, so $[d] = \mathbf{[T]}$. Then every term of v is [L T^{-1}]:
$[a] = \mathbf{[L\ T^{-1}]}$; $[b] = \mathbf{[L\ T^{-2}]}$; $\left[\dfrac{c}{d+t}\right] = [LT^{-1}] \Rightarrow [c] = [LT^{-1}][T] = \mathbf{[L]}$.

18. Energy: $[ML^2T^{-2}]$, so $a = 1$, $b = 2$, $c = -2$. With $n_1 = 10$ in the SI (kg, m, s):
$$n_2 = n_1\left(\frac{M_1}{M_2}\right)\left(\frac{L_1}{L_2}\right)^2\left(\frac{T_1}{T_2}\right)^{-2} = 10 \times \frac{1}{0.1} \times \left(\frac{1}{0.1}\right)^2 \times \left(\frac{1}{30}\right)^{-2}$$
$$n_2 = 10 \times 10 \times 100 \times 900 = 9 \times 10^6$$
10 J = **9×10^6 new units**. (The new unit of energy is tiny: 0.1 kg × (0.1 m)^2 ÷ (30 s)^2 ≈ 1.1×10^{-6} J.)

19. $A = lb = 110 \times 100 = 11\,000$ m^2.
For a product, the relative uncertainties add:
$$\frac{\Delta A}{A} = \frac{2}{110} + \frac{4}{100} = 0.018 + 0.040 = 0.058$$
$\Delta A = 0.058 \times 11\,000 \approx 640$ m^2, about 0.06×10^4 m^2.
Area = **$(1.10 \pm 0.06) \times 10^4$ m^2**.

20. i. $72 = 72 \times \dfrac{1000}{3600} = \mathbf{20\ m/s}$. (To change km/h to m/s, divide by 3.6.)

ii. $20\ \frac{}{3} = 20 \times \dfrac{10^{-3}}{10^{-6}\ 3} = 20 \times 10^3 = \mathbf{2.0 \times 10^4\ kg/m^3}$.

21. Let $f = kT^{x}L^{y}m^{z}$. Put in the dimensions, $[f] = [T^{-1}]$, tension $[MLT^{-2}]$, $[L]$, $[m] = [ML^{-1}]$:

$$[M^{0}L^{0}T^{-1}] = [M^{x+z}\,L^{x+y-z}\,T^{-2x}]$$

Time: $-2x = -1 \Rightarrow x = \tfrac{1}{2}$. Mass: $x + z = 0 \Rightarrow z = -\tfrac{1}{2}$. Length: $x + y - z = 0 \Rightarrow y = -\tfrac{1}{2} - \tfrac{1}{2} = -1$.

$$f = kT^{1/2}L^{-1}m^{-1/2} = \frac{k}{L}\sqrt{\frac{T}{m}}, \qquad\qquad f \propto \frac{1}{L}\sqrt{\frac{T}{m}}$$

(This is question 13a again. Tighten a guitar string and it plays higher; press it shorter on the fret and it plays higher; a thick, heavy string plays lower: all three are in the formula.)