Unit 23 · DC Circuits

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Electricity and Magnetism (Units 19–23 together): 16 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **MCQ (1 mark)**
  - a wire folded in two (CDC's model paper) or stretched: new R
  - ρ from V, I, l, A; number of electrons from I, t
  - terminal p.d. $V = E - Ir$; larger r: both meters fall
  - current ratios in a network; same lamps rearranged; efficiency of lamps or a motor
- **Short (5 marks, in parts)**
  - a 2 kW heater (CDC's model paper): what it means, R at 220 V, how to get more heat
  - derive $I = nAvq$; drift velocities; why a lamp lights at once
  - Ohm's law; I–V of resistor, lamp, diode; thermistor, LDR, strain gauge
  - derive series and parallel rules; potential divider and sensors
  - e.m.f. vs p.d.; internal resistance from a V–I graph; power $VI = I^2R = V^2/R$

Key terms

Current (I): rate of flow of charge; ampere (A), an SI base unit.

Coulomb: charge passing in 1 s at 1 A; 1 C = 1 A s.

Conventional current: from + to − outside the source; opposite to electron flow.

Drift velocity: average speed of carriers along a conductor.

Number density (n): free carriers per m^3.

Potential difference: electrical energy turned into other forms per coulomb through a component; volt = J/C.

E.m.f.: other energy turned into electrical energy per coulomb by a source; volt.

Resistance (R): opposition to current; V/I; ohm = V/A.

Ohmic conductor: obeys Ohm's law (straight I–V through the origin).

I–V characteristic: graph of current against p.d. for a component.

Thermistor (NTC): resistance falls a lot as temperature rises.

LDR: resistance falls as light gets brighter.

Strain gauge: fine zigzag wire whose R rises when stretched.

Resistivity (ρ): resistance of a 1 m cube of the material; Ω m.

Superconductor: zero resistance below its critical temperature.

Potential divider: resistors in series sharing a p.d.

Internal resistance (r): resistance inside a source.

Terminal p.d.: p.d. across its terminals.

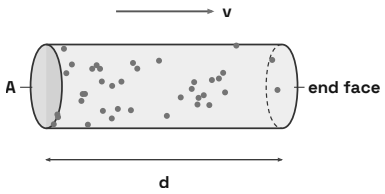
Power: rate of energy transfer; watt = J/s.

Formulas

$I = \frac{\Delta Q}{\Delta t}$	1 A = 1 C/s **Use:** current
$I = nAvq$	n per m^3, v drift **Use:** drift
$V = \frac{W}{Q}, E = \frac{W}{Q}$	p.d. in a component, e.m.f. in a source **Use:** energy per coulomb
$R = \frac{V}{I}$	any component **Use:** resistance
$R = \frac{\rho l}{A}$	l along the current **Use:** resistivity
$R = R_1 + R_2; \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$	series; parallel **Use:** combinations
$V_1 = \frac{R_1}{R_1 + R_2} V_{in}, \frac{V_1}{V_2} = \frac{R_1}{R_2}$	series pair **Use:** potential divider
$E = V + Ir, I = \frac{E}{R + r}$	Ir "lost volts" **Use:** internal resistance
$P = VI = I^2R = \frac{V^2}{R}, W = VIt$	watts, joules **Use:** power, energy

Current and drift

- e.g. 250 mA for 1 h → 900 C = 5.6×10^{21} electrons; 2.0 mA for 5.0 min → 0.60 C
- lightning: electrons = It/e
- carriers: electrons (metals); positive ions with conventional current, negative ions against (solutions, gases)
- free electrons: random ≈ 10^6 m/s, no net movement; field adds a slow drift towards +



Length d, area A, n carriers per m^3 of charge q drifting at v.

1. carriers in the length: nAd; charge $Q = nAdq$
2. all pass the end in $t = d/v$
3. $I = Q/t = nAvq$

- $v = I/nAq$: thinner wire or fewer carriers → faster
- e.g. 160 mA: tungsten filament 0.025 mm → 51 mm/s; copper leads 0.72 mm → 0.031 mm/s
- e.g. 8.2 A, 2.5 mm^2 copper → 0.24 mm/s; 0.25 A, 0.65 mm → 5.5×10^{-5} m/s; tungsten 0.046 mm, 0.50 A → 15 mm/s
- e.g. Cu^{2+} ($q = 2e$), 30 mA, 100 cm^2 → 1.6×10^{-8} m/s; semiconductor strip → 0.30 m/s
- lamp lights at once: wire already full of electrons; the **field** travels at nearly c

p.d. and e.m.f.

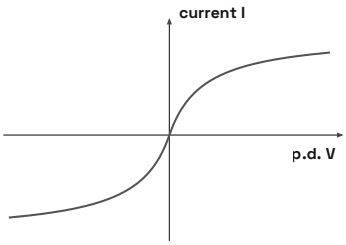
- water picture: height difference ↔ p.d.; flow ↔ current; paddle wheel ↔ lamp
- kettle 12.5 A, 540 kJ in 3 min → 2250 C, 240 V
- voltmeter **in parallel** (across); ammeter **in series**
- **e.m.f.**: work done **on** each coulomb in a source; **p.d.**: work done **by** each coulomb in a component
- round a loop: Σ e.m.f. = Σ p.d. (6 V → 4 V + 2 V)
- 1.5 V, 20 C → 30 J

Resistance, Ohm's law, characteristics

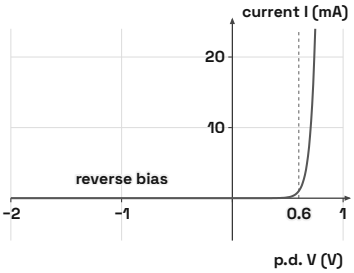
- drifting electrons collide with vibrating ions → energy to the lattice → heat
- hotter metal: ions vibrate more → R **rises**

Ohm's law: current through a metal conductor ∝ p.d., at constant temperature and conditions

- $R = V/I$ holds for every component, at each point; Ohm's law only for some
- V–I graph: gradient = R (steeper → larger); I–V: gradient = $1/R$



Filament lamp: straight at first, then flattens as it heats (R rises); same in both directions.



Silicon diode: no current in reverse; forward, nearly none until ≈ 0.6 V, then rises steeply.

- **diode**: one-way valve; conducts in **forward bias** (the way the symbol's triangle points; band at the bar end)
- **NTC thermistor** (metal oxides): more carriers freed as it warms → R falls, non-linear (≈ 3800 Ω at 0 °C → 100 Ω at 100 °C); cut-outs, heaters, rice cookers, fridges, fever thermometers, car engines
- thermistor at constant temperature: straight I–V; self-heating curves it **upward**
- **LDR** (cadmium sulphide): light frees electrons → R falls; dark ≈ MΩ; non-linear
- **strain gauge**: stretched → longer and thinner → R up; $\Delta R = \rho \Delta L/A \propto$ strain (bridges, aircraft wings)

Resistivity

1. $R \propto l$ (more ions in the way)
2. $R \propto 1/A$ (more paths)
3. $R = \rho l/A$, $\rho = RA/l$

- ρ depends on the material: carrier density and mobility; not on size or shape
- silver 1.6×10^{-8}, copper 1.68×10^{-8}, aluminium 2.7×10^{-8}, tungsten 5.6×10^{-8}, nichrome ≈ 1.1×10^{-6}, graphite ≈ 3×10^{-5}, silicon ≈ 2×10^{3}, ABS ≈ 10^{15} Ω m
- temperature: metals ↑; semiconductors ↓; superconductor drops to 0 at its critical temperature
- carbon chip $10 \times 5 \times 1$ mm, 3.0×10^{-5} Ω m: along 10 mm → 0.060 Ω; through 1 mm → 0.60 mΩ
- nichrome 1 m, 1 mm^2, 2 V, 4 A → 5×10^{-7} Ω m
- **stretched** to 2× length (same volume, area halves) → **4R**; **folded** in two (half length, double area) → **R/4**

Series and parallel

Series (no junction, same I)

1. $V = V_1 + V_2$ (energy)
2. $V_1 = IR_1$, $V_2 = IR_2$; $V = IR$
3. $IR = IR_1 + IR_2 \rightarrow R = R_1 + R_2$

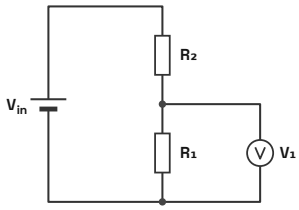
Parallel (same V)

1. $I = I_1 + I_2$ (charge conserved at a junction)
2. $I_1 = V/R_1$, $I_2 = V/R_2$; $I = V/R$
3. $\div V$: $1/R = 1/R_1 + 1/R_2$; two: $R_1R_2/(R_1 + R_2)$

- series: larger than the largest; parallel: **smaller than the smallest**

- 6 Ω ∥ 3 Ω → 2 Ω; four 1.5 Ω, two pairs in parallel, pairs in series → 1.5 Ω
- 1.0 Ω across the battery ∥ (2 + 4 + 3) Ω: $I_0/I_1 = 10$
- removing a parallel branch → total R up, current down

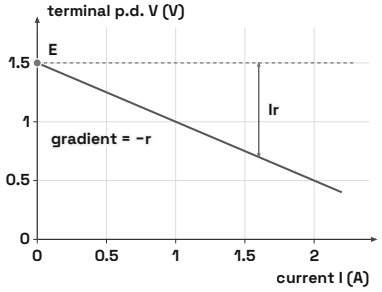
Potential divider



R_1, R_2 in series across V_{in}; voltmeter across R_1.

- same $I \rightarrow V_1/V_2 = R_1/R_2$: larger resistance, larger share
- e.g. 3.0 Ω and 6.0 Ω on 18 V → 6 V, 12 V; 2 A in both
- **sensor**: thermistor or LDR as R_2
  - warmer (thermistor R falls) or brighter (LDR) → V_1 across the fixed resistor **rises**
  - output across the thermistor → **falls** when warmed
  - street light: output across the LDR → rises when **darker**
- e.g. 6.0 V, 1800 Ω, dark: 1.2 V across the resistor → LDR 7200 Ω; light 2400 Ω → 3.4 V across the LDR
- garden lights at 200 lux (LDR ≈ 350 kΩ), 5.0 V: LDR on top, ≈ 350 kΩ fixed below, output across it → 2.5 V at 200 lux, falls when darker

Internal resistance



Terminal p.d. against current: intercept E, gradient $-r$; lost volts Ir.

1. $E = V + Ir$ (energy round the loop)
2. $V = -rI + E$: straight line, negative slope
3. $V = IR \rightarrow I = \dfrac{E}{R + r}$

- open circuit ($I = 0$): terminal p.d. = E
- AA cell $r \approx 0.2$ Ω; matters when the load is small
- e.g. torch 6.4 Ω, 4.5 V, 0.4 Ω → 0.66 A; 2.0 V, 0.50 Ω, 1.0 A → 1.5 V
- same E, larger r: current and terminal p.d. both **fall**
- measure: vary the load, read V and I, plot

Power

1. $W = QV$, $Q = It \rightarrow W = VIt$
2. $P = W/t = VI$
3. $V = IR \rightarrow P = I^2R = V^2/R$

- I^2R for series (same I: larger R, more power); V^2/R for parallel (same V: smaller R, more power)
- 250 mW max, 47 kΩ → $I = \sqrt{P/R} = 2.3$ mA
- 1000 W heater on 230 V → 4.35 A, 52.9 Ω hot
- **2 kW heater**: uses 2000 J each second at its rated p.d.; at 220 V, $R = V^2/P = 24.2$ Ω; more heat: **smaller** R (V^2/R) at the same V
- same brightness: least power = most efficient (LED 6 W)
- motor lifting m by h in t: efficiency = $\dfrac{mgh}{VIt}$

COMMON MISTAKES

✗ conventional current = electron flow
✓ **opposite** directions.

✗ electrons rush round at near light speed
✓ drift ≈ mm/s; the field travels fast.

✗ e.m.f. is a force
✓ energy per coulomb, in volts.

✗ voltmeter in series
✓ **across** the component.

✗ Ohm's law is $R = V/I$
✓ that defines R; Ohm's law is $I \propto V$ at constant temperature.

✗ R of a stretched wire doubles
✓ length × 2, area ÷ 2 → **4R**.

✗ parallel total between the two values
✓ **smaller than the smallest**.

✗ ρ depends on length
✓ R does; ρ is the material's.

✗ higher-power heater has more resistance
✓ at one V, **less** ($P = V^2/R$).

✗ terminal p.d. = e.m.f. always
✓ only on open circuit.