Unit 22 · Capacitors

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Electricity and Magnetism (Units 19–23 together): 16 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **MCQ (1 mark)**
  - $Q = CV$ (μF × V)
  - ratios of charge and energy at the same V
  - C and energy from a V–Q graph
  - C when A, d or the dielectric change; series of equal capacitors
- **Short (5 marks, in parts)**
  - define capacitance; capacitance of a parallel-plate capacitor + one use (CDC's model paper)
  - three equal capacitors (CDC's model paper: 1000 μF) in series, parallel, mixed
  - derive $C = \varepsilon_0 A/d$; series and parallel formulas; $W = \frac{1}{2}QV$
  - what a dielectric does and why; dielectric strength; disconnected capacitor with the slab removed
  - charge and p.d. on each capacitor of a network; energy stored; W–Q graph

Key terms

Capacitor: two conductors separated by an insulator; stores charge (and energy) for a while.

Dielectric: the insulator between the plates; polarized in a field.

Capacitance (C): charge on either plate per unit p.d., Q/V.

Farad (F): 1 C/V; = $A^2\,s^4\,kg^{-1}\,m^{-2}$; usually μF, nF, pF.

Permittivity (ε): how a material weakens a field; $\varepsilon = \varepsilon_r\varepsilon_0$; F/m.

Relative permittivity (dielectric constant) ε_r, K: $\varepsilon/\varepsilon_0$.

Polarization: molecules stretched or turned by a field, + ends one way.

Dielectric strength: greatest field an insulator takes without breaking down.

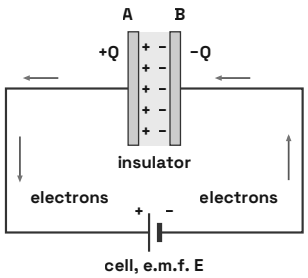
Voltage rating: highest safe p.d., printed beside C.

Electrolytic capacitor: oxide film on a foil as dielectric; must go the right way round.

Formulas

$C = \frac{Q}{V}, Q = CV$	Q on one plate **Use:** definition
$C = \frac{\varepsilon_0 A}{d}, C = \frac{\varepsilon_r\varepsilon_0 A}{d}$	$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m **Use:** parallel plates
$C = C_1 + C_2 + \cdots$	same V, charges add **Use:** parallel
$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$; two: $\frac{C_1C_2}{C_1 + C_2}$	same Q, p.d.s add **Use:** series
$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$	area under V–Q **Use:** energy stored

Storing charge



Cell pulls electrons off plate A, pushes as many onto B: A +Q, B −Q.

1. the + terminal pulls electrons off A; − pushes as many onto B; none crosses the gap

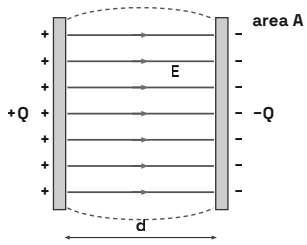
2. A +Q, B −Q: **equal**, charge conserved round the loop
3. growing p.d. pushes back; current falls
4. p.d. = e.m.f. → current stops: fully charged

- total charge **zero**; "stores 20 μC" = on each plate
- disconnected: charges stay; through a lamp: brief glow
- current only a moment: gap → no complete loop

Capacitance

- $Q \propto V$ for one capacitor; C = slope of Q–V (of V–Q: $1/C$)
- C set by **construction**, not by Q or V
- uses: fan (2–4 μF), camera flash (hundreds of μF), radio tuning (pF), power supply smoothing (mF)
- e.g. 2.5 μC at 15 V → 170 nF; 2.5 μF at 300 V → 0.75 mC; 220 μF at 6 V → 1.32 mC
- 60 μC at 12 V → 5 μF; at 18 V → 90 μC
- graph: 660 μC at 3 V → 220 μF

Parallel-plate capacitor



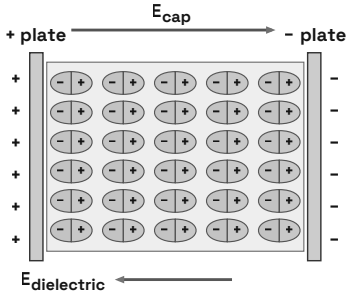
Plates of area A, gap d: uniform field between, bulging at the edges.

1. Gauss: $E = \sigma/\varepsilon_0$, $\sigma = Q/A \rightarrow E = \dfrac{Q}{\varepsilon_0 A}$
2. uniform: $V = Ed = \dfrac{Qd}{\varepsilon_0 A}$
3. $C = Q/V \rightarrow Q$ cancels $\rightarrow C = \dfrac{\varepsilon_0 A}{d}$
4. dielectric: $\varepsilon_0 \rightarrow \varepsilon_r \varepsilon_0$

- depends on: area (larger → more), gap (smaller → more), dielectric ($\times \varepsilon_r$)
- ε_r: air 1.0006, Teflon 2.1, polystyrene ≈ 2.6, paper 2–4, mica 5–7, glass 5–10, water ≈ 80, barium titanate ≥ 1000
- e.g. 0.015 m^2, 2.0 mm, air → 66 pF
- e.g. 100 cm^2 ($\rightarrow 1.0 \times 10^{-2}$ m^2), 1.5 cm → 5.9 pF; polystyrene (3) → 18 pF
- double A and d → unchanged; 50 pF with mica (6) → 300 pF
- foil + newspaper: a few nF; press (smaller d) → more; five sheets → less

Dielectrics

- **jobs**: keep the plates apart at a small fixed gap; raise C by ε_r
- **breakdown**: field tears electrons loose → spark; air ≈ 3 kV/mm, paper ≈ 15, Teflon ≈ 60, mica ≥ 100 kV/mm
- lightning: air breaking down on a huge scale
- thinnest Teflon for 500 V: $d = V/E_{max} \approx 8$ μm; use thicker (margin)



Molecules line up + ends along E_{cap}; − face next to the + plate, + face next to the − plate.

Why C rises (charged, disconnected: Q fixed)

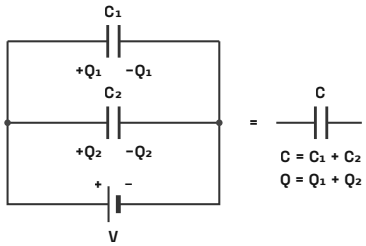
1. surface charges set up $E_{dielectric}$ against $E_{cap} \rightarrow E_{net} = E_{cap}/\varepsilon_r$
2. $V = E_{net} d$ falls
3. $C = Q/V$ rises by ε_r

- still connected: V fixed, battery pushes **more** Q on
- disconnected, slab in: energy $Q^2/2C$ **falls** (field pulls the slab in)
- 137 pF, 2.47 nC, ε_r 3.1 removed: V 18.0 → 55.9 V (rises ≈ 38 V)

Practical capacitors: large A, tiny d, high ε_r

- ceramic (pF–μF); film, rolled foils (fan capacitor); electrolytic (μF–F, oxide on + foil, paper soaked in electrolyte; **polarity**: backwards → oxide breaks, can burst)

Combinations



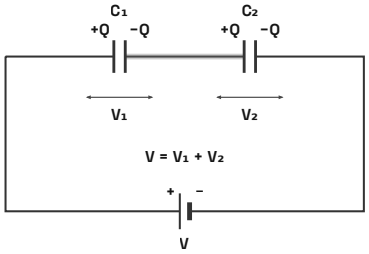
Parallel: each has the supply's V; $Q_1 + Q_2 = Q$.

Parallel

1. same V across each
2. $Q = Q_1 + Q_2$, $Q_1 = C_1 V$, $Q_2 = C_2 V$

3. $CV = C_1V + C_2V \rightarrow C = C_1 + C_2$

- larger than any one (areas added); 22 + 47 μF at 12 V → 264, 564 μC; 69 μF



Series: outer plates +Q, −Q from the supply; inner pair −Q, +Q by induction.

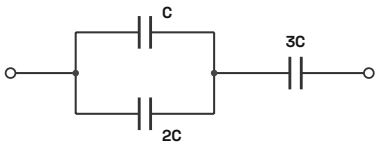
Series

1. same Q on each (inner plates joined, isolated, charged by induction)
2. $V = V_1 + V_2$, $V_1 = Q/C_1$, $V_2 = Q/C_2$
3. $Q/C = Q/C_1 + Q/C_2 \rightarrow 1/C = 1/C_1 + 1/C_2$

- **smaller than the smallest** (gaps added); p.d. shared → higher total voltage rating
- smaller capacitor takes the **larger** p.d.: 2 and 4 μF, 12 V → 16 μC, 8 V and 4 V
- 20 and 10 μF, 12 V → 6.67 μF, 80 μC, 4 V and 8 V
- n equal: series C/n, parallel nC; three 6 μF → 2 μF or 18 μF
- three equal C: two in series + one in parallel → $1.5C$; two parallel + one series → $\frac{2}{3}C$
- opposite of resistors

Networks

1. combine clear series or parallel groups; redraw; repeat
2. total $Q = CV$
3. back: series parts same Q; parallel parts same V

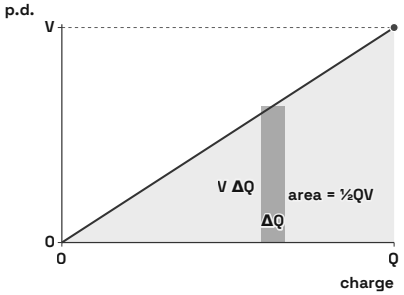


C and 2C in parallel, in series with 3C.

- $C + 2C = 3C$; with 3C in series → **1.5C**
- C = 2 μF, 12 V: 3 μF, 36 μC; 3C: 6 V; pair 6 V; C 12 μC, 2C 24 μC

Energy stored

- electrons pushed onto the − plate against repulsion, pulled off the + plate against attraction → work stored in the field, from the battery



V against Q: strip $V\,\Delta Q$ = work for ΔQ; triangle = total.

1. small ΔQ at p.d. V: $\Delta W = V\,\Delta Q$ (a strip)
2. all strips = area under V–Q
3. triangle: $W = \frac{1}{2}QV$
4. $Q = CV \rightarrow \frac{1}{2}CV^2$; $V = Q/C \rightarrow Q^2/2C$

- calculus: $W = \int_0^Q \frac{Q}{C}\,dQ = \frac{Q^2}{2C}$
- the half: each charge crosses on average $V/2$
- battery gives QV; other $\frac{1}{2}QV$ → heat in wires and battery
- $W \propto V^2$: 3 V → 9 V gives 9 ×; at one V, Q and $W \propto C$ (4200 vs 420 μF: both × 10)
- W–Q: parabola through the origin ($W \propto Q^2$)
- e.g. 5000 μF, 25 V → 1.6 J; 8.0 μF, 12 V → 0.58 mJ
- e.g. flash 200 μF, 300 V → 9.0 J; out in 1 ms → 9000 W
- graph: 30 mC at 12 V → 2.5 mF, 180 mJ
- 47 μF: 100, 200, 500 μC → 0.11, 0.43, 2.7 mJ

COMMON MISTAKES

✗ capacitor's total charge is Q
✓ **zero**: +Q and −Q.

✗ C rises with V
✓ C fixed by build; Q rises.

✗ series: add capacitances
✓ add **reciprocals** (the reverse of resistors).

✗ series: equal p.d.s
✓ equal **charge**; smaller C gets more p.d.

✗ $W = QV$
✓ $\frac{1}{2}QV$.

✗ $W = \frac{1}{2}Q^2C$
✓ $Q^2/2C$ (check units: $C^2/F = J$).

✗ area in cm^2, d in mm
✓ m^2, m.

✗ dielectric lowers C
✓ raises it by ε_r.

✗ electrolytic either way round
✓ + to the higher potential.