Unit 21 · Potential, Potential Difference and Potential Energy

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Electricity and Magnetism (Units 19–23 together): 16 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **MCQ (1 mark)**
  - $V = W/Q$; $V = Ed$ (cm → m)
  - test charge doubled: V same, E_p doubles
  - which statement about $V = 0$ and $E = 0$ is valid
  - midpoint of two like charges; force on a proton and an electron in one field
  - a uniform field could be any kind of field
- **Short (5 marks, in parts)**
  - define potential; derive $V = kQ/r$; derive $E = -\Delta V/\Delta x$
  - potential vs field; potential vs potential energy; sign of E_p for like and unlike charges
  - where $V = 0$ on the line of two charges; where $E = 0$
  - equipotentials ⊥ field lines; V–x, E–x graphs; areas and slopes
  - electron gun $\frac{1}{2}mv^2 = eV$; stopping p.d.; Bohr atom's energies; electric vs gravitational fields

Key terms

Electric potential (V): work done per unit positive charge in bringing it from infinity to the point; scalar; volt (J/C).

Volt: 1 J of work to bring 1 C there (or between two points).

Electric potential energy: qV, belongs to a charge at a point; joules.

Potential difference: work per unit positive charge from A to B, $V_B - V_A$.

Equipotential surface: all points at the same potential.

Potential gradient: change of potential per unit distance, $\Delta V/\Delta x$.

Electron volt (eV): energy a charge e gains through 1 V; 1.60×10^{-19} J.

Electron gun: hot filament gives electrons, a + anode accelerates them in a vacuum.

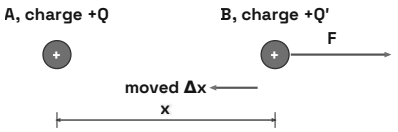
Formulas

$V = \dfrac{W}{Q}$	W from infinity **Use:** potential
$V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r} = \dfrac{kQ}{r}$	signed Q; zero at ∞ **Use:** point charge
$E_p = qV = \dfrac{kQq}{r}$	+ like, − unlike **Use:** potential energy
$E = -\dfrac{\Delta V}{\Delta x}$; uniform $E = \dfrac{V}{d}$	V/m = N/C **Use:** field from potential
$W = q\,\Delta V$, field's work $-q\,\Delta V$	outside agent, no speeding up **Use:** moving a charge
$\frac{1}{2}mv^2 = eV$, $v = \sqrt{\dfrac{2eV}{m}}$	from rest **Use:** electron gun
$1\text{ eV} = 1.60 \times 10^{-19}\text{ J}$	energy, not potential **Use:** unit

Electric potential

- like gravity's potential: work per unit from infinity; infinity = far enough that the force is nil
- V belongs to a **point**; $E_p = QV$ to a **charge** there (2 C at 6 V → 12 J)
- e.g. 8.0×10^{-4} J for +2.0 μC → $V = 400$ V
- **unlike** charges: E_p **negative**; work needed to separate (bound, e.g. atom)
- **like** charges: E_p **positive**; work stored, released as they fly apart

$V = kQ/r$



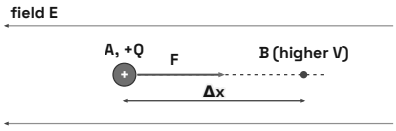
+Q at A pushes +Q′ at B (distance x) with F; an agent moves B a step Δx towards A.

1. $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{QQ'}{x^2}$
2. step towards A, against F (Δx negative): $\Delta W = -F\,\Delta x$
3. $W = -\dfrac{QQ'}{4\pi\varepsilon_0}\displaystyle\int_{\infty}^{r}\frac{dx}{x^2} = \frac{QQ'}{4\pi\varepsilon_0 r}$
4. $Q' = 1\text{ C} \to V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r}$

- **scalar**: several charges → algebraic sum, signs, no angles
- sign follows the charge; points at equal r → equipotential spheres
- e.g. 1.0 nC at 10 cm → 90 V
- e.g. two +2.0 nC, 20 cm apart, midpoint: $V = 360$ V, $E = 0$; work for +1.0 nC = 3.6×10^{-7} J
- ±4 nC, midpoint: $V = 0$, $E \neq 0$
- Q from V: 165 V at 15 cm → 2.8 nC

- $V/E = r$: 120 V, 600 V/m → 0.20 m

E and V



+Q moved from A to B, distance Δx, against E: B at higher potential.

1. $\Delta W = F\Delta x$
2. against the field: $F = -EQ \rightarrow \Delta W = -EQ\Delta x$
3. by definition $\Delta W = Q\Delta V$
4. equate → $E = -\dfrac{\Delta V}{\Delta x}$

- **minus**: E points the way V **falls**
- 20 V → 50 V over 0.15 m: 200 V/m towards the 20 V end
- V constant in a region → $E = 0$ there (inside a charged metal sphere)
- $V = 0$ doesn't mean $E = 0$, nor the reverse

Where $V = 0$ ($+3 \times 10^{-8}$ C at O, -2×10^{-8} C at A, 15 cm)

- between: $3/x = 2/(15 - x)$ → **9 cm** from O
- beyond A: $3/x = 2/(x - 15)$ → **45 cm** from O
- beyond O: never (bigger + charge nearer)

Where $E = 0$ (Q and $2Q$, ℓ apart): nearer the smaller

- opposite signs: outside, $\approx 2.4\ell$ beyond Q
- same signs: between, $\approx 0.41\ell$ from Q
- two equal + charges: $E = 0$ at the midpoint, $V = 2kQ/r$, never 0

Potential difference, electron volt

- 1 V: 1 J per C between two points; a 9 V battery's + terminal 9 V above its −
- a + charge to higher V: gains E_p, must be pushed
- **electron** to higher V: $\Delta E_p = (-e)\Delta V < 0$ → speeds up; − charges "fall uphill"
- released at rest: + charges to **lower** V, − to **higher**; E_p falls in both
- field's work on -7.7 μC raised 65 V: $-q\Delta V = +5.0 \times 10^{-4}$ J
- in a uniform field proton and electron: forces equal, opposite

eV: visible photon 2–3 eV; hydrogen 13.6 eV; X-ray 60–70 keV; uranium fission 200 MeV

- proton, alpha through 500 V → 500 eV, 1000 eV (charge counts, not mass)

Electron gun: $v \propto \sqrt{V}$: 4 × voltage → 2 × speed

- 100 V → 100 eV = 1.60×10^{-17} J, 5.93×10^6 m/s
- 1200 V → 2.1×10^7 m/s
- 9.0 V across 4.5 mm: $E = 2.0 \times 10^3$ V/m; electron 1.8×10^6 m/s
- stop an electron at 6.0×10^5 m/s: $\frac{1}{2}mv^2/e = 1.0$ V (towards lower potential)

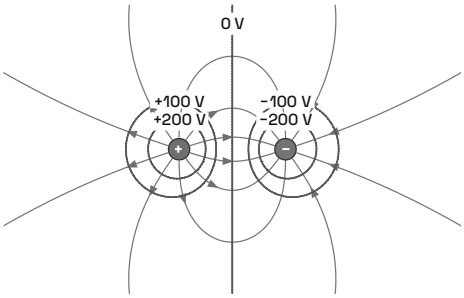
Bohr hydrogen ($r = 0.53 \times 10^{-10}$ m)

- $V = ke/r = 27$ V
- $ke^2/r^2 = mv^2/r \rightarrow E_k = ke^2/2r = 13.6$ eV
- $E_p = -27.1$ eV; total -13.6 eV; ionization 13.6 eV

Proton to touch a silicon nucleus (+14e): $r = 1.2 + 3.6 = 4.8 \times 10^{-15}$ m → $V = k(14e)/r = 4.2$ MV

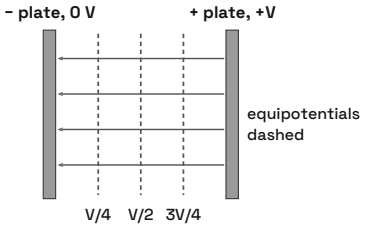
Equipotentials

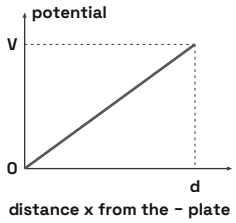
- along an equipotential: no work → field ⊥ motion → **field lines ⊥ equipotentials**
- closer equipotentials → steeper gradient → stronger field
- point charge: spheres; 2.0 nC at 3, 6, 9, 12 cm → 600, 300, 200, 150 V (wider gaps outward)



+1.0 nC and −1.0 nC: equipotentials round each; 0 V is the straight line midway; field lines cross them at 90°.

- two equal +: own equipotentials close in; farther out one surrounds both

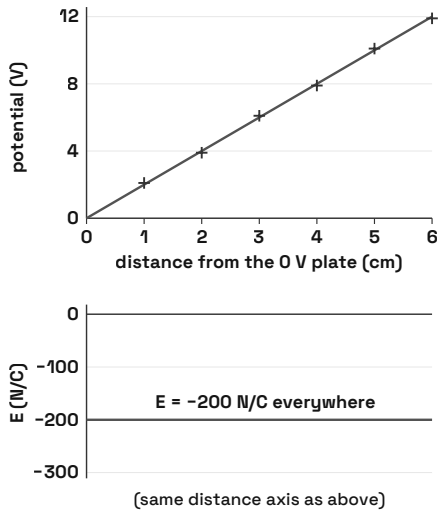




Plates: equipotentials at $V/4$, $V/2$, $3V/4$ equally spaced; $V \propto$ distance from the − plate.

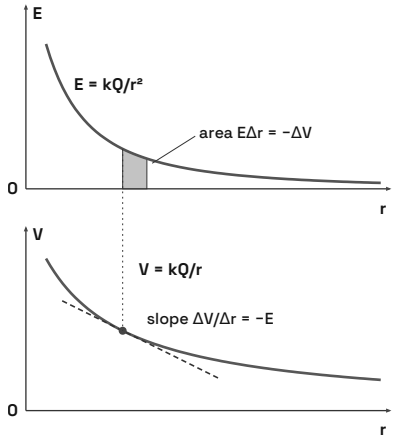
- uniform: $E = V/d$; 240 V, 2.0 cm, 0.50 cm from − → 60 V, 12,000 V/m
- **water-tray activity**: 9 V across salty water, meter's COM on −; 2, 4, 6, 8 V lines straight and even with two strips, crowded round a coin

Graphs



Graphs: (b) V against x between plates 6.0 cm apart, 0–12 V: straight line. (c) $E = -200$ N/C everywhere.

- slope of V–x = 12/0.060 = 200 V/m = size of E; E negative: points against x
- upstream (against E) → higher V; downstream → lower
- electron from 0 V to 12 V: E_p −12 eV, E_k +12 eV (1.9×10^{-18} J)



Positive point charge: area under E–r = fall in V; slope of V–r = $-E$.

- **area under** E–r = change in potential; from r to $\infty = V$ at r
- $dV = -E\,dr \rightarrow V_r = -\int_{\infty}^{r} \frac{kQ}{r^2} dr = \frac{kQ}{r}$
- V–r: + charge positive, − charge negative, both → 0 far away; E falls faster ($1/r^2$)
- e.g. 2.0 nC: $V_{10} - V_{20} = 90$ V; electron outwards: work $+1.4 \times 10^{-17}$ J (90 eV)

Electric vs gravitational fields

	Gravitation	Electric
acts on	mass (all bodies)	charge (+ or −)
force	attractive only	attractive or repulsive
strength	1	$\approx 10^{36}$ (two protons)
field	$g = F/m$, N/kg	$E = F/Q$, N/C
radial V	$-GM/r$, always −	kQ/r, sign of Q
shielding	none	metal box

- both: non-contact, infinite range, inverse square, field = −potential gradient, $V = 0$ at ∞
- a uniform field's lines could be gravitational, magnetic or electric

COMMON MISTAKES

✗ potential = potential energy
✓ V per coulomb at a point; $E_p = qV$ of a charge.

✗ potentials added as vectors
✓ **scalars**: add with signs.

✗ $V = kQ/r^2$
✓ $V = kQ/r$; $E = kQ/r^2$.

✗ $V = 0 \rightarrow E = 0$
✓ E depends on how V **changes**.

✗ E points to higher potential
✓ to **lower**.

✗ electrons move to lower potential
✓ **higher**; their E_p still falls.

✗ eV a unit of potential
✓ a unit of **energy**.

✗ speed ∝ voltage
✓ $\propto \sqrt{V}$.

✗ larger test charge, larger V
✓ V unchanged; E_p grows.