Unit 20 · Electric Field

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Electricity and Magnetism (Units 19–23 together): 16 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **MCQ (1 mark)**
  - path of an electron between plates
  - field pattern of a point charge
  - definition of E; $V = Ed$ (cm → m); field direction between plates
  - flux through a tilted sheet; Gauss: only the charge inside
- **Short (5 marks, in parts)**
  - sketch the field of two equal charges (CDC's model paper: two −Q) and find E at a point
  - rules of field lines; a student's wrong drawing
  - derive $E = kQ/r^2$, $E = V/d$, the parabola $y = qEx^2/2mv^2$
  - flux, Gauss's law; E of a sphere (inside, outside), a line, a sheet
  - oil drop held still: q, number of electrons; E–$1/r^2$ graph

Key terms

Field: region where an object feels a force without being touched.

Electric field: region round charges where another charge feels a force.

Test charge: small positive charge that doesn't disturb the field it explores.

Field line: line whose direction at each point is that of the force on a + charge.

Uniform field: same size and direction everywhere (between parallel plates).

Radial field: lines straight out from a point charge or sphere.

Neutral point: where fields cancel, $E = 0$.

Dipole: equal and opposite charges a small distance apart.

Electric field strength (E): force per unit positive charge; vector; N/C = V/m.

Electric flux (Φ): $E \times$ area at right angles to it; number of lines through a surface; N m^2/C.

Gaussian surface: imaginary closed surface chosen for Gauss's law.

Charge densities: ρ per volume, λ per length, σ per area.

Formulas

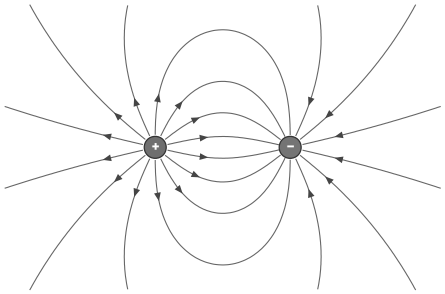
$E = \frac{F}{q}, F = qE$	q test charge **Use:** definition
$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$	away from +, towards − **Use:** point charge, sphere outside
$E = \frac{V}{d}$	V p.d., d plate gap **Use:** uniform field
$\frac{1}{2}mv^2 = qV$	from rest **Use:** accelerated along the field
$y = \frac{qE}{2mv^2}x^2, y_L = \frac{qV_P L^2}{2mdv^2}$	v along the plates, length L **Use:** deflection, a parabola
$\Phi = EA\cos\theta$	θ field to the normal **Use:** flux
$\Phi = \frac{q}{\varepsilon_0}$	net q inside a closed surface **Use:** Gauss's law
$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{a^3}$	$r < a$, insulating sphere **Use:** inside
$E = \frac{\lambda}{2\pi\varepsilon_0 r}$	long line **Use:** $\propto 1/r$
$E = \frac{\sigma}{2\varepsilon_0}$; plates $E = \frac{\sigma}{\varepsilon_0}$	large sheet **Use:** uniform

Field lines

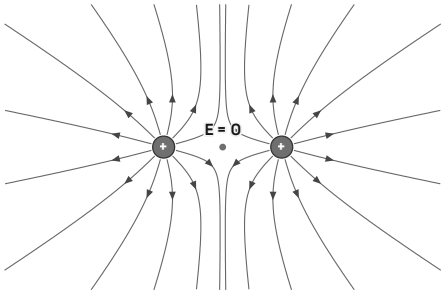
Rules (Faraday)

1. start on + charges, end on − (or at infinity)
2. arrow: direction of force on a + charge
3. close together where strong
4. never cross (one direction at a point)
5. meet a conductor at 90° (else free charges would move)

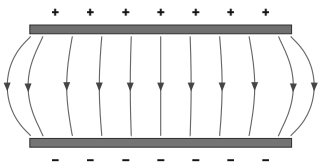
- **not** the path of a charge: they give the force (acceleration), not the velocity
- isolated −: straight radial lines, arrows **towards** it; never circles



Dipole: every line from + ends on −.



Two equal + charges: lines bend apart; neutral point midway. Two −Q: the same shape, arrows reversed (inwards).



Parallel plates: straight, parallel, evenly spaced between them (uniform); bowing at the ends.

- a + charge near a metal sheet: lines curve to meet it at 90°, ending on induced − charges
- student's drawing errors: arrows from − to +; lines not at 90° to the plates

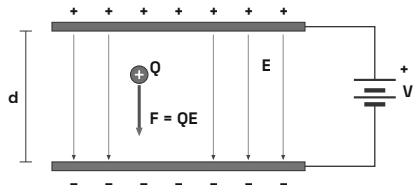
Field strength

E of a point charge

1. Coulomb: $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$
2. $E = F/q \to q$ cancels $\to E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

- independent of the test charge; $1/r^2$: twice as far, a quarter
- e.g. proton at 0.53×10^{-10} m $\to 5.1 \times 10^{11}$ N/C; air sparks at $\approx 3 \times 10^6$ N/C
- e.g. +2.0 μC feels 0.040 N east $\to E = 2.0 \times 10^4$ N/C east; on −4.0 μC: 0.080 N **west**
- E–$1/r^2$ graph: straight through the origin, gradient kQ; $1.25 \times 10^{-7} \to Q = 1.4 \times 10^{-17}$ C $\approx$ 87e; after alpha decay (−2e) less steep (85/87)

Uniform field between plates



Plates d apart, p.d. V: same force $F = QE$ anywhere between them.

$E = V/d$

1. force $F = QE$, same everywhere
2. from the + plate to − plate: work $W = Fd = QEd$
3. $V = W/Q = Ed \to E = V/d$

- 1 V/m = 1 J/(C m) = 1 N/C
- e.g. 12 V, 4.0 mm $\to 3.0 \times 10^3$ V/m; on an electron 4.8×10^{-16} N towards **+**
- e.g. 650 N/C, 12 cm $\to V = 78$ V
- field from higher to lower potential: +10 V above 0 V, 5 cm $\to$ 200 N/C **down**
- along the field from rest: $a = qE/m$, $\frac{1}{2}mv^2 = qV$ (electron gun, X-ray tube)

Path across the field: a parabola

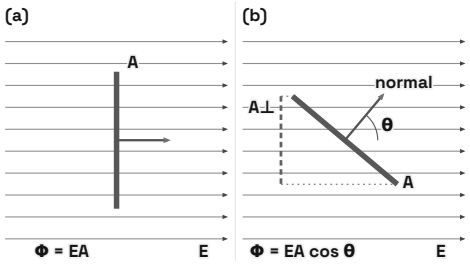
1. across: $a = \frac{qE}{m} = \frac{qV_P}{md}$, constant
2. along: no force, $x = vt$
3. across from rest: $y = \frac{1}{2}at^2$
4. $t = x/v \to y = \frac{qE}{2mv^2}x^2$: parabola
5. leaving: $t = L/v$, $y = \frac{qV_PL^2}{2mdv^2} \propto V_P$ (oscilloscope)

- after the plates: straight on along the tangent
- electron bends towards **+** plate; field down $\to$ force up (path A)
- e.g. 2.0×10^7 m/s, plates 5.0 cm long, 2.0 cm apart, 100 V $\to$ 2.7 mm
- weight $\approx 10^{-14} \times$ electric force $\to$ ignore gravity
- proton vs electron, same speed: deflected the other way, $\approx$ 1/1840 as much

Oil drop held still: $qE = mg$, $E = V/d$, $m = \rho \cdot \frac{4}{3}\pi r^3$

- e.g. 1.50 kV, 2.10 cm, $r = 1.27$ μm, 950 kg/m^3 $\to mg = 7.99 \times 10^{-14}$ N, $q = 1.12 \times 10^{-18}$ C = **7 e** (Millikan: charge quantized)

Flux



Flux: (a) surface ⊥ field: $\Phi = EA$. (b) normal at θ: lines through $A_\perp = A\cos\theta$.

- greatest EA at $\theta = 0°$; **zero** when the surface lies along the field (90°)

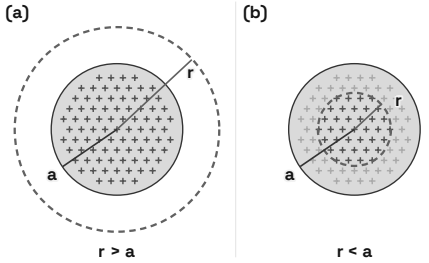
- closed surface: normal outwards; out +, in −
- e.g. 0.20 m^2, 500 N/C, 60° → 50 N m^2/C
- cube in a uniform field: faces along it 0; face square on s^2E; turned 45° → $s^2E/\sqrt{2}$ each; whole cube 0 net

Gauss's law

- $\Phi = q/\varepsilon_0$: flux out of any closed surface = net charge inside ÷ ε_0
- empty box: what flows in flows out → 0; + inside: out; − inside: in
- charges **outside** add nothing; only **net** charge counts; shape and size don't matter
- one of Maxwell's four equations
- e.g. +5, −2, +1 nC inside, +10 nC outside → 4 nC/ε_0

Point charge: sphere radius r, E same and ⊥ all over → $\Phi = \frac{kq}{r^2} \times 4\pi r^2 = q/\varepsilon_0$, independent of r; backwards it gives Coulomb's law

Uniformly charged sphere (insulating, radius a)



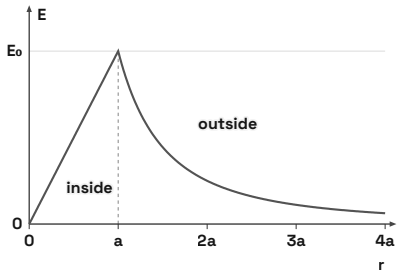
Gaussian spheres of radius r: a. outside, encloses all of Q. b. inside, encloses part.

Outside ($r > a$): encloses $Q \rightarrow E \cdot 4\pi r^2 = Q/\varepsilon_0 \rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$, as a point charge (gravity too)

Inside ($r < a$)

1. $q_{in} = \rho \cdot \frac{4}{3}\pi r^3$
2. $E \cdot 4\pi r^2 = \frac{\rho\frac{4}{3}\pi r^3}{\varepsilon_0} \rightarrow E = \frac{\rho r}{3\varepsilon_0}$
3. $\rho = \frac{Q}{\frac{4}{3}\pi a^3} \rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{a^3}$

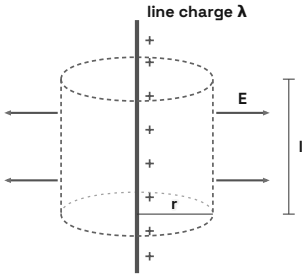
- zero at the center, $\propto r$ inside, greatest at the surface kQ/a^2, then $1/r^2$; no jump



E against r: straight line up to E_0 at the surface, then falling as $1/r^2$.

- e.g. 2.0 μC, a = 10 cm: 9.0×10^5 (5 cm), 1.8×10^6 (10 cm), 4.5×10^5 N/C (20 cm)

Line and sheet of charge



Cylinder of radius r, length l round the line: flux through the curved side only.

1. symmetry: E radial, same at the same r
2. ends: field along them, no flux; curved side: $E \cdot 2\pi rl$
3. enclosed $\lambda l \rightarrow E \cdot 2\pi rl = \lambda l/\varepsilon_0 \rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$

- $\propto 1/r$: 3 × as far → ⅓ ($6.0 \times 10^4 \rightarrow 2.0 \times 10^4$ N/C)

Sheet: pillbox through it; curved side no flux; two ends $2EA = \sigma A/\varepsilon_0 \rightarrow E = \sigma/2\varepsilon_0$, independent of distance

- two plates $\pm\sigma$: add between (σ/ε_0), cancel outside → why the field between plates is uniform

COMMON MISTAKES

✗ field line = path of a charge
✓ direction of the **force**, not the motion.

✗ field lines can cross
✓ never: one direction at a point.

✗ arrows towards +
✓ **away** from +, towards −.

✗ E depends on the test charge
✓ q cancels.

✗ $V = Ed$ with d in cm
✓ meters.

✗ electron moves along E
✓ **against** E, towards the + plate.

✗ $\Phi = EA$ for a tilted surface
✓ $EA\cos\theta$, θ from the **normal**.

✗ charges outside change the total flux
✓ only charge **inside** counts.

✗ field inside a charged insulating sphere is zero
✓ $\propto r$; zero only at the center (zero throughout inside a **conductor**).

✗ line of charge $E \propto 1/r^2$
✓ $1/r$; sheet: no r at all.