Unit 18 · Dispersion

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Wave and Optics (Units 14–18 together): 9 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **MCQ (1 mark)**
  - what fixes a color (frequency)
  - image by a lens with chromatic aberration (colored)
  - achromatic condition, $f_r - f_v = 0$
  - rainbow: dispersion + one reflection inside the drop (CDC's model paper's option: "dispersion and total internal reflection"; the reflection is really partial)
  - prism angle for no dispersion; ω from f_r, f_v
- **Short (5 marks, in parts)**
  - cause of dispersion; none for one color; none in a slab; a mirror has no chromatic aberration
  - angular dispersion, dispersive power: define, relate, what they depend on
  - derive $f_r - f_v = \omega f$; derive $\omega/f + \omega'/f' = 0$
  - doublet's focal lengths; two prisms: deviation without dispersion and the reverse
  - pure and impure spectrum; spherical aberration and its cures

Key terms

Dispersion: splitting of white light into its colors (VIBGYOR).

Spectrum: band of colors from light split by wavelength.

Pure spectrum: colors don't overlap. **Impure**: they overlap.

Angular dispersion (θ): $\delta_v - \delta_r$, angle between the extreme colors.

Mean color: yellow, deviated by about the average; δ, μ, f without suffix are for it.

Dispersive power (ω): angular dispersion ÷ mean deviation; no unit.

Aberration: difference of a real image from the lens formula's.

Chromatic aberration: a lens can't focus all colors at one point; measured as $f_r - f_v$.

Spherical aberration: paraxial and marginal rays focus at different points, even for one color.

Paraxial / marginal rays: rays near the axis / through the rim.

Circle of least confusion: smallest disc of light between the two foci.

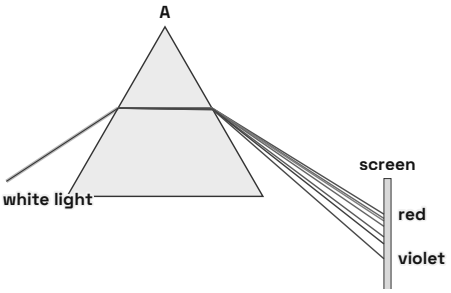
Achromatic doublet: two lenses in contact free from chromatic aberration; **achromatism**.

Direct-vision prism: dispersion without deviation.

Formulas

$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$	A, B, C constants of the material **Use:** Cauchy: μ falls as λ grows
$\theta = \delta_v - \delta_r = (\mu_v - \mu_r)A$	thin prism, angle A **Use:** angular dispersion
$\omega = \frac{\delta_v - \delta_r}{\delta} = \frac{\mu_v - \mu_r}{\mu - 1}$	μ mean (yellow) **Use:** dispersive power
$f_r - f_v = \omega f$	f mean focal length **Use:** chromatic aberration
$\frac{\omega}{f} + \frac{\omega'}{f'} = 0$	thin lenses in contact **Use:** achromatic doublet
$\omega(\mu - 1)A = \omega'(\mu' - 1)A'$	prisms opposite ways **Use:** no dispersion
$(\mu - 1)A = (\mu' - 1)A'$	prisms opposite ways **Use:** no deviation
$I \propto \frac{1}{\lambda^4}$	particles $\ll \lambda$ **Use:** Rayleigh scattering

Dispersion and its cause



White light through a glass prism: red bent least, violet most.

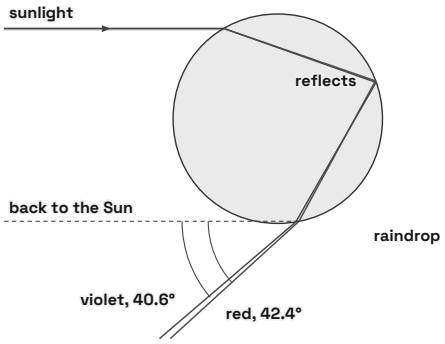
- μ depends on wavelength, larger for shorter λ: $\mu_v > \cdots > \mu_r$

- $\delta = (\mu - 1)A \to \delta_v > \cdots > \delta_r$: colors leave in different directions
- red fastest in glass ($v = c/\mu$); in vacuum all colors at c
- **color is fixed by frequency**: speed and wavelength change on entering glass, frequency doesn't
- prism apex up → violet lowest (most deviated, towards the base), red at the top
- **monochromatic** light: one λ, one μ → deviated, not dispersed
- **slab**: colors split at the first face, parallel second face turns each back → all leave parallel to the incident beam, no color seen
- deeper reason: glass's electrons have natural frequencies in the ultraviolet; violet is nearer them → slowed more

Results of dispersion

Rainbow: Sun behind you, drops in front; each drop refracts (and disperses), reflects once off the back, refracts out

- red at ≈ 42°, violet ≈ 40° from the point opposite the Sun → red outside
- secondary bow ≈ 51°, two reflections, colors reversed; darker sky between
- no rainbow on the Moon: no air, no drops
- not a pure spectrum: the Sun is ½° wide, colors overlap

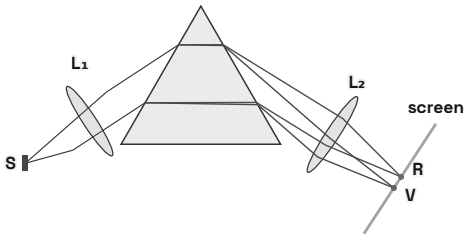


Ray in a raindrop: refracted in, reflected at the back, refracted out; red 42.4°, violet 40.6° to the line back to the Sun.

Blue sky, red sunset: scattering, not dispersion

- air molecules scatter $I \propto 1/\lambda^4$: blue (450 nm) ≈ 6 × red (700 nm) → sky blue
- not violet: less violet in sunlight, eyes less sensitive to it
- Sun near the horizon: ≈ 38 × more air, blue scattered out → red, orange
- "Some books say … due to dispersion": no prism-like splitting happens there

Spectrum



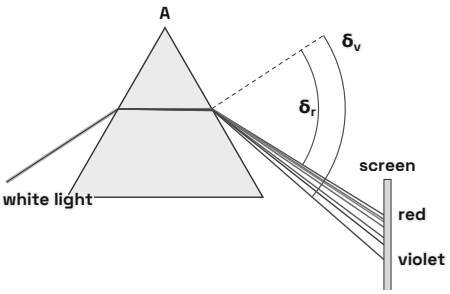
Pure spectrum: slit S at the focus of L_1 → parallel beam; each color leaves parallel; L_2 focuses red at R, violet at V.

Conditions for a pure spectrum

1. narrow slit
2. parallel beam on the prism (slit at the focus of the first lens)
3. prism at minimum deviation
4. emerging beams focused by an **achromatic** lens

- **impure**: light spreading from a source or wide slit straight on the prism → color patches overlap

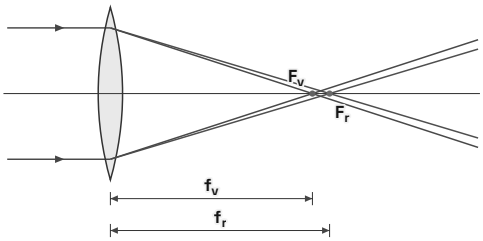
Angular dispersion, dispersive power



δ_v, δ_r from the incoming direction; angle between the emerging rays = $\delta_v - \delta_r$.

- $\theta = (\mu_v - 1)A - (\mu_r - 1)A = (\mu_v - \mu_r)A$
- $\omega = \dfrac{(\mu_v - \mu_r)A}{(\mu - 1)A}$ → A cancels
- angular dispersion: material **and** prism angle; dispersive power: **material only** (and colors compared)
- 8° prism, same glass as a 4° one → double θ, same ω
- e.g. 8° crown, 1.530/1.514 → $\delta_v = 4.24°$, $\delta_r = 4.11°$, $\theta = 0.13°$
- e.g. $A = 5°$, 1.66/1.54 → 0.6°
- e.g. crown 1.523/1.513 → $\omega = 0.010/0.518 = 0.0193$; flint 1.773/1.743 → 0.0396
- mean μ not given → average of μ_v and μ_r

Chromatic aberration



Convex lens: violet focuses at F_v, nearer than red's F_r.

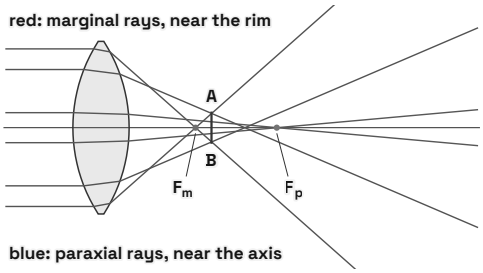
- lens = stack of prisms → disperses; violet bent more → $f_v < f_r$
- image of a white object: **colored** edges
- concave lens: f negative, $f_r - f_v$ negative; violet focus again nearer
- **mirror**: reflection the same for every λ → **no** chromatic aberration (why big telescopes use mirrors)

Derivation

1. mean: $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{f(\mu - 1)}$
2. red: $\frac{1}{f_r} = \frac{\mu_r - 1}{f(\mu - 1)}$; violet: $\frac{1}{f_v} = \frac{\mu_v - 1}{f(\mu - 1)}$
3. subtract: $\frac{f_r - f_v}{f_r f_v} = \frac{\mu_v - \mu_r}{f(\mu - 1)} = \frac{\omega}{f}$
4. $f_r f_v \approx f^2 \to f_r - f_v = \omega f$

- e.g. $f_r = 26, f_v = 25$ cm → $f = 25.5$, aberration 1 cm, $\omega = 0.039$
- e.g. $\omega = 0.026$, aberration 0.351 cm → $f = 13.5$ cm
- e.g. concave crown, R 25 and 30 cm, 1.523/1.5204 → $f_v = -26.07, f_r = -26.20$, size 0.13 cm

Spherical aberration



Marginal rays (red) cross at F_m, nearer than paraxial (blue) at F_p; AB the circle of least confusion.

- spherical faces bend marginal rays too much; happens with one color
- convex: positive; concave: negative (marginal seem to come from nearer)

Reducing it (never removed with spherical faces)

1. **stop**: block the rim → sharper, dimmer (smaller camera aperture)
2. **share the bending**: plano-convex with the curved face towards parallel light
3. **two lenses**: convex + concave, aberrations opposite; or two plano-convex lenses apart by $f_1 - f_2$
4. **aspheric** faces: flatter towards the rim (phone cameras)

Achromatic combination

1. red: $\frac{1}{F_r} = \frac{\mu_r - 1}{f(\mu - 1)} + \frac{\mu_r' - 1}{f'(\mu' - 1)}$
2. violet: same with μ_v, μ_v'
3. $F_r = F_v \to \frac{\mu_v - \mu_r}{f(\mu - 1)} + \frac{\mu_v' - \mu_r'}{f'(\mu' - 1)} = 0$
4. $\frac{\omega}{f} + \frac{\omega'}{f'} = 0$

- f, f' **opposite signs**: one convex, one concave
- **different materials**: $\omega = \omega' \to f = -f'$, power 0
- $f'/f = -\omega'/\omega$: more dispersive glass has the longer f → converging doublet = convex **crown** (small ω) + concave **flint**
- uses: cameras, binoculars, telescopes, microscopes, spectrometers, fiber couplers, projectors

Finding the two lenses: $f_1/f_2 = -\omega_1/\omega_2$, then $1/F = 1/f_1 + 1/f_2$

- 0.018, 0.027, $F = +30$ cm → crown +10, flint −15 cm
- 0.015, 0.030, $F = 100$ cm → 50 and −100 cm; 80 cm → 40, −80; +5 D → +10, −20 cm
- ratio 1 : 3, $F = 40$ cm → $26\frac{2}{3}$ and −80 cm
- only the **ratio** of the ωs matters (0.013, 0.026 → same as 0.015, 0.030)

Two prisms, apexes opposite

- **deviation without dispersion** (achromatic prism): $\omega(\mu - 1)A = \omega'(\mu' - 1)A'$; net deviation $(\mu - 1)A - (\mu' - 1)A'$
- e.g. 0.03, 1.53 with 5°, 0.045, 1.66 → $A = 0.1485/0.0159 = 9.34°$; light still turned 1.65°
- **dispersion without deviation** (direct-vision prism, pocket spectroscope): $(\mu - 1)A = (\mu' - 1)A'$; net dispersion $(\mu_v - \mu_r)A - (\mu_v' - \mu_r')A'$

COMMON MISTAKES

✗ color fixed by wavelength
✓ by **frequency** (it doesn't change in glass).

✗ blue sky by dispersion
✓ by **scattering**, $I \propto 1/\lambda^4$.

✗ ω depends on the prism angle
✓ A cancels; only the material.

✗ red deviated most
✓ **violet** (largest μ).

✗ $f_v > f_r$ for a convex lens
✓ violet focuses **nearer**: $f_v < f_r$.

✗ doublet of one glass
✓ needs **two glasses**; one convex, one concave.

✗ convex lens of flint in a converging doublet
✓ convex is **crown** (smaller ω).

✗ mirrors have chromatic aberration
✓ reflection is the same for every color.

✗ negative dispersive power
✓ always positive.