Unit 17 · Lenses

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Wave and Optics (Units 14–18 together): 9 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **MCQ (1 mark)**
  - unit of power (diopter); powers in contact
  - object at F → parallel rays
  - least object–image distance, $4f$
  - lens in a liquid of equal or greater μ
  - silvered lens; area magnification m^2
- **Short (5 marks, in parts)**
  - define convex lens + one daily use; why "converging" (CDC's model paper)
  - μ of a lens's glass from the angles of a ray (Snell, Unit 15)
  - derive the lens formula; lens maker's formula; $1/F = 1/f_1 + 1/f_2$
  - same-size real image, with a ray diagram
  - lens in water, bubble, drop; diverging pair; zero-power sunglasses
  - u, v, f, P numericals; displacement method

Key terms

Lens: uniform transparent material bounded by two refracting surfaces, at least one curved.

Convex (converging): thicker in the middle. **Concave (diverging)**: thinner in the middle.

Meniscus: one face convex, one concave (spectacle lenses).

Centers of curvature (C_1, C_2): centers of the spheres the faces belong to; **radii** R_1, R_2.

Principal axis: line through C_1 and C_2.

Optical center (O): middle of a thin lens on the axis; a ray through it goes on undeviated.

Principal focus (F): where rays parallel to the axis meet (convex) or seem to come from (concave); two, F_1 and F_2.

Focal length (f): OF; + convex, − concave.

Focal plane: plane through F, ⊥ axis; oblique parallel rays meet in it.

Aperture: effective diameter through which light passes.

Power (P): ability to converge or diverge light; $1/f$, f in m; unit **diopter** (D).

Visual angle: angle an object subtends at the eye's nodal point.

Angular magnification (M): β/α, image's and object's visual angles, both at D.

Least distance of distinct vision (D): ≈ 25 cm, nearest the eye sees clearly.

Linear magnification (m): image size ÷ object size.

Formulas

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$	real +, virtual −; f convex +, concave − **Use:** lens formula
$m = \frac{I}{O} = \frac{v}{u}$	+ real inverted, − virtual upright **Use:** linear magnification
$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$	R: bulging +, curved in −, flat ∞; μ relative to the surroundings **Use:** lens maker's formula
$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$	thin lenses in contact **Use:** combination
$P = \frac{1}{f}, P = P_1 + P_2 + \cdots$	f in **m**; powers with signs **Use:** power
$h' = \theta \times a$	$a \approx 17$ mm, nodal point to retina **Use:** retinal image ∝ visual angle
$M = \frac{\beta}{\alpha}$	visual angles **Use:** angular magnification

Lenses

Convex (thicker in middle)	Concave (thinner in middle)
biconvex (equiconvex if $R_1 = R_2$)	biconcave
plano-convex	plano-concave
concavo-convex: convex face curved **more**	convexo-concave: concave face curved **more**

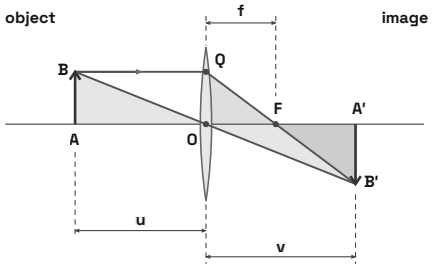
- lens = stack of thin prisms, bases towards the thicker part → convex bends rays **towards** the axis (converging), concave **away** (diverging)
- shorter f → stronger; wider aperture → brighter (and more edge blur)
- uses: glasses, contact lenses, phone cameras, microscopes, binoculars, telescopes, projectors, magnifiers; the eye's cornea and lens

Sign convention (real is positive)

1. thin lens, small aperture, rays near the axis
2. distances from the **optical center**
3. real objects and images +; virtual −
4. convex: real or virtual images; concave: only virtual (real object)
5. f, R: convex +, concave −

- "Cartesian" books: $1/f = 1/v - 1/u$; same images; never mix

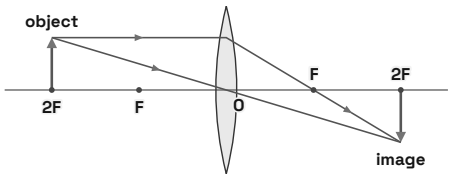
Lens formula



Object AB, real image A′B′. Ray from B ∥ axis → Q → through F; ray through O straight on.

1. $\triangle$ABO ~ $\triangle$A′B′O: $\frac{A'B'}{AB} = \frac{A'O}{AO}$
2. $\triangle$OQF ~ $\triangle$A′B′F, OQ = AB: $\frac{A'B'}{AB} = \frac{A'F}{OF}$
3. A′F = A′O − OF: $\frac{v}{u} = \frac{v-f}{f} \rightarrow uv = vf + uf$
4. $\div uvf$: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

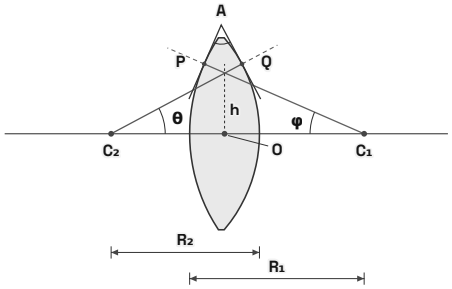
- step 1 gives $m = v/u$; area magnification m^2
- virtual image or concave lens: same formula with signs



Object at 2F → real, inverted, same size image at 2F.

- same size, real: $v = u = 2f$; object–image distance $4f$, the **least** for a real image
- object at F → $1/v = 0$ → rays leave **parallel**
- e.g. $u = 20$ cm, $f = 10$ cm → $v = +20$ cm (2F)
- e.g. 5 D, real image ×3: $f = 20$ cm, $v = 3u \rightarrow u = 26.7$ cm, $v = 80$ cm
- e.g. concave, $f = -20$ cm, object at its focus → $v = -10$ cm, $m = -0.5$ (5 cm → 2.5 cm, upright)
- e.g. convex $f = 16$ cm, virtual ×2 → $u = 8$ cm
- smaller **virtual** image → **concave** lens ($f = 12$ cm, ⅓ size → $u = 24$ cm, $v = -8$ cm; 20 cm, ⅔ → $v = -13.3$ cm, $f = -40$ cm)
- **displacement method**: object and screen fixed, two lens positions swap u, v; ×2.5, moved 30 cm → $u = 20$, $v = 50$, $f = 14.3$ cm

Lens maker's formula



Near P and Q the lens is a thin prism; its angle $A = \theta + \varphi$, the angles the radii make with the axis.

1. ray at height h ∥ axis → through F: $\delta = h/f$ (small $\tan\delta \approx \delta$)
2. lens near P, Q = thin prism: $\delta = A(\mu - 1) \rightarrow 1/f = (A/h)(\mu - 1)$
3. radii ⊥ faces → $A = \theta + \varphi$; $\theta = h/R_2$, $\varphi = h/R_1$
4. $A/h = 1/R_1 + 1/R_2 \rightarrow \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

- h cancels → every parallel ray meets at one F
- concave: both R negative → f negative

- assumes thin lens, rays near the axis, one color

μ relative to the medium around

- **in water**: ${}_w\mu_g = 1.5/1.33 = 1.13 \rightarrow$ power **falls** ($\approx$ ¼); f 24 cm $\rightarrow$ 93.9 cm; +5 D $\rightarrow$ +1.28 D
- liquid **denser** than the glass: $\mu < 1 \rightarrow$ converging lens **diverges**
- liquid of **equal** μ: $f = \infty$, lens invisible
- **air bubble in water**: $\mu = 1/1.33 = 0.75 \rightarrow$ **diverging**, though convex in shape (shines by TIR)
- **water drop in air**: converging
- **sunglasses**: both faces curve the same way, R and $-R \rightarrow 1/f = 0$, zero power
- equiconvex glass (1.5): $f = R$; 4 D $\rightarrow R = 25$ cm
- plano-convex: $R_2 = \infty$; $R = 12$ cm, $1.5 \rightarrow f = 24$ cm
- concavo-convex, $R_1 = +15$, $R_2 = -30$ cm, $1.6 \rightarrow f = +50$ cm
- lens on a mirror with a liquid between: liquid = plano-concave lens; pin at the combination's focus coincides with its image (24 cm faces, pin 40 cm $\rightarrow$ $\mu_l = 1.4$)
- water under a lens on a mirror $\rightarrow$ power **decreases**
- convex lens with its second face silvered $\rightarrow$ **concave mirror**

Lenses in contact, power

1. L_1 alone: $1/f_1 = 1/u + 1/v'$
2. I' a virtual object for L_2: $1/f_2 = 1/(-v') + 1/v$
3. add: $1/f_1 + 1/f_2 = 1/u + 1/v = 1/F$

- powers add, **with signs** (why opticians use powers)
- 1 m $\rightarrow$ 1 D; 25 cm $\rightarrow$ 4 D; convex +, concave −
- +12 D and −2 D $\rightarrow$ 10 D, 10 cm; +10 cm and −20 cm $\rightarrow$ +5 D
- +6 D, −4 D $\rightarrow$ +2 D, converging, 50 cm; −4 D, +2.5 D $\rightarrow$ −66.7 cm, diverging
- 5 cm convex + 10 cm concave $\rightarrow F = +10$ cm, converging
- convex + concave of **equal** $f \rightarrow P = 0$, a glass plate
- pair **diverges** when the concave lens is stronger; converges when $f_1 < f_2$ (convex shorter)
- more power: combine lenses of the same sign
- power $= 1/f$: shorter f bends more

Visual angle, magnification

- apparent size set by the **visual angle** at the nodal point ($\approx$ 7 mm behind the cornea), not real size
- retinal image $h' = \theta a \propto \theta$: 1.6 m friend at 160 m = 1 cm coin at 1 m (0.01 rad)
- magnifier: object inside F, upright virtual image at D, larger angle β than α (object at D)
- simple magnifier, image at D: $M = 1 + D/f$

COMMON MISTAKES

✗ $P = 1/f$ with f in cm
✓ **meters** first.

✗ add focal lengths in contact
✓ add **powers** ($1/F = 1/f_1 + 1/f_2$).

✗ a convex shape always converges
✓ depends on μ **relative to the surroundings** (bubble in water diverges).

✗ μ of glass = 1.5 in water
✓ use 1.5/1.33 = 1.13.

✗ concavo-convex = diverging
✓ thicker in the middle: **converging**.

✗ smaller virtual image from a convex lens
✓ only a **concave** lens.

✗ $f_1 f_2/(f_1 + f_2)$ as the power
✓ that is F; power is $(f_1 + f_2)/f_1 f_2$.