Unit 16 · Refraction through Prisms

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Wave and Optics (Units 14–18 together): 9 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **MCQ (1 mark)**
  - what equals e at minimum deviation; which statement is false
  - $A > 2C$: no emergent ray
  - A for minimum deviation at a given i
  - $\mu = \cot(A/2) \leftrightarrow \delta_m = 180° - 2A$
  - TIR in a prism (CDC's model paper: $C = 45°$, equilateral prism)
- **Short (5 marks, in parts)**
  - show $r_1 + r_2 = A$, $\delta = i + e - A$
  - derive $\mu = \sin\frac{A+\delta_m}{2} / \sin\frac{A}{2}$; show $i = e$ at minimum
  - thin prism $\delta = (\mu - 1)A$
  - factors of deviation; no lateral shift for a prism; prism in water
  - e, δ from i; δ_m, μ; silvered face; ray just fails to emerge

Key terms

Prism: transparent medium bounded by two plane faces meeting along a straight edge.

Refracting faces: the two faces light passes through.
Refracting edge: where they meet.

Angle of the prism (A): angle between the refracting faces.

Principal section: cross-section at right angles to the refracting edge.

Angle of emergence (e): between the emergent ray and the normal at the second face.

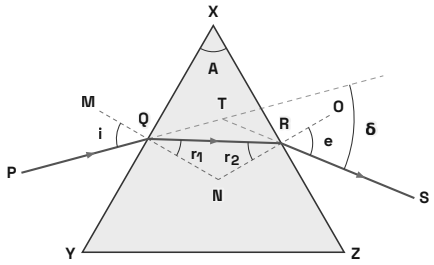
Angle of deviation (δ): between the directions of the incident and emergent rays.

Angle of minimum deviation (δ_m): the least deviation, as i changes.

Formulas

$r_1 + r_2 = A$	r_1, r_2 angles inside the glass **Use:** any ray
$\delta = i + e - A$	i, e incidence, emergence **Use:** any ray
$\sin i = \mu \sin r_1$, $\sin e = \mu \sin r_2$	Snell at each face **Use:** follow a ray
$\mu = \dfrac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$	at minimum deviation **Use:** measure μ
$\delta = (\mu - 1)A$	small A, small i **Use:** thin prism

A ray through a prism



$A = 60°$, $\mu = 1.5$, $i = 45°$: $r_1 = 28.1°$, $r_2 = 31.9°$, $e = 52.4°$, $\delta = 37.4°$. Incident and emergent rays produced meet at T.

$r_1 + r_2 = A$

1. XQNR: angles at Q, R $= 90° \rightarrow A + \angle QNR = 180°$
2. $\triangle$QNR: $r_1 + r_2 + \angle QNR = 180°$
3. compare $\rightarrow r_1 + r_2 = A$

$\delta = i + e - A$: turned $i - r_1$ at Q and $e - r_2$ at R, same way $\rightarrow \delta = (i + e) - (r_1 + r_2)$ (or exterior angle of $\triangle$QTR)

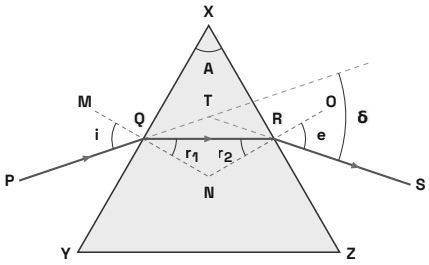
- glass prism in air: always turned **towards the base**
- slab: faces parallel $\rightarrow$ only a sideways shift; prism: faces lean $\rightarrow$ turned, so **no lateral shift** for a prism (rays not parallel)
- one color at a time; white light $\rightarrow$ dispersion (Unit 18)
- deviation depends on
  - angle of incidence
  - angle of the prism (larger $\rightarrow$ more)
  - μ relative to the surroundings (color: violet more)

- grazing ($i = 90°$), $A = 60°$, $\mu = 1.5$: $r_1 = C = 41.8°$, $r_2 = 18.2°$, $e = 27.9°$, $\delta = 57.9°$
- normal incidence: $r_1 = 0 \to r_2 = A$; just fails to emerge $\to A = C$ (41.8° for glass)
- leaves grazing after normal incidence $\to A = C$
- $A > 2C$: $r_2 \geq A - C > C$ for every ray $\to$ no emergent ray
- one face silvered, ray returns on itself: meets the silver along the normal $\to r_1 = A$, $\sin i = \mu \sin A$ ($\sqrt{2}$, $30° \to 45°$)
- emergent ray with the face $= 90° - e$

Activity 16.1 (pins): outline, normal, $i = 30°$; pins P_1 P_2, line up P_3 P_4 through the prism; join, measure i, r_1, r_2, e, δ; repeat 35°–60°; plot δ against i

Minimum deviation

- $i \uparrow \to \delta$ **falls**, reaches δ_m, **rises**: a U-shaped δ–i graph with one lowest point
- 60° glass: $\approx 47°$ at 30°, 37.2° at 48.6°, $\approx 39°$ at 60°



Minimum deviation, 60° glass prism: $i = e = 48.6°$, $r_1 = r_2 = 30°$, ray inside $\parallel$ base, $\delta_m = 37.2°$.

At minimum deviation

- $i = e$
- $r_1 = r_2 = r$
- ray inside **parallel to the base** (equal sides)

Why $i = e$: reversibility: a ray in at i, out at e, reversed goes in at e, out at i, same deviation $\to$ if i and e differed, two angles with the least deviation; only one minimum $\to i = e$

Derivation

1. $r_1 = r_2 = r$, $r_1 + r_2 = A \to r = A/2$
2. $i = e$, $\delta = \delta_m \to \delta_m + A = 2i \to i = \dfrac{A + \delta_m}{2}$
3. $\mu = \sin i / \sin r \to \mu = \sin \frac{A+\delta_m}{2} \div \sin \frac{A}{2}$

- most accurate way to measure μ: spectrometer; rock the prism until the image stops and turns back
- with calculus: $d\delta/di = 0 \to de/di = -1 \to i = e$, $r_1 = r_2$
- e.g. $A = 60°$, $\mu = 1.5 \to i = 48.6°$, $\delta_m = 37.2°$
- e.g. $A = 60°$, $\delta_m = 34° \to i = 47°$, $\mu = 1.46$; $\delta_m = 50° \to \mu = 1.638$
- e.g. $\mu = 1.5$, $i = 50°$ at minimum $\to r = 30.71°$, $A = 61.42°$
- equilateral, $\mu = \sqrt{2} \to i = 45°$, $\delta_m = 30°$
- $\delta_m = 180° - 2A \Leftrightarrow \mu = \cot(A/2)$
- **in water**: relative $\mu = 1.5/1.33 = 1.13 \to \delta_m$ **smaller**
- right-angled isosceles prism totally reflecting at the long face (45°): $\mu \geq \sqrt{2}$

Thin (small-angled) prism

1. small angles: $\sin\theta \approx \theta \to i = \mu r_1$, $e = \mu r_2$
2. $\delta = \mu(r_1 + r_2) - A = \mu A - A$
3. $\delta = (\mu - 1)A$

- same deviation for every ray falling **nearly normally**; depends only on μ, A
- glass (1.5): turns light through **half its angle**; e.g. $4° \to 2°$; $3°$ needs $6°$
- needs small A **and** small i (4° prism: 2.0° at $i = 0$, $\approx$ 2.9° at 40°)

Uses

- prism glasses (an eye turned in or out; **prism diopter**: 1 cm at 1 m)
- steering a laser beam (Risley pair)
- shaping a beam (anamorphic pair)
- alignment
- achromatic prism: crown + opposite flint, colors cancel, deviation stays

COMMON MISTAKES

✗ $\mu = \sin(A + \delta_m)/\sin A$
✓ **halve** first: $\sin \frac{A+\delta_m}{2} / \sin \frac{A}{2}$.

✗ at minimum deviation $i = r_1$
✓ $i = e$ and $r_1 = r_2$; $r_1 < i$.

✗ δ keeps falling as i grows
✓ falls to δ_m, then **rises**.

✗ angle with the face as i
✓ $i = 90° -$ angle with the face.

✗ $\delta = (\mu - 1)A$ for any prism
✓ only **thin** prism, small i.

✗ a ray can pass a glass prism in air undeviated
✓ never: $\delta \geq \delta_m > 0$.