Unit 15 · Refraction at Plane Surfaces

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Wave and Optics (Units 14–18 together): 9 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **MCQ (1 mark)**
  - what stays the same on refraction (frequency)
  - v in glass; time through a slab
  - critical angle from μ, or μ from it
  - color with the smallest critical angle
  - bubble in a slab seen from two sides
  - optical fiber's principle
- **Short (5 marks, in parts)**
  - why light refracts; lateral shift: define, derive, how it changes
  - real and apparent depth: derive $\mu = R/a$; a pool looks shallow
  - conditions for TIR; why "total"; TIR from the rarer side? with a diagram
  - mirage, crack in glass, diamond, Sun oval at the horizon, shimmering
  - critical angle at glass–water; microscope and turpentine; layers of glass and water

Key terms

Refraction: bending of light passing from one medium into another, because its speed changes.

Interface: boundary between two media.

Normal: line at right angles to the surface at the point of incidence; i, r measured from it.

Optically denser: light slower, larger μ. **Rarer**: light faster, smaller μ. Not mass density (kerosene floats on water, yet $1.44 > 1.33$).

Absolute refractive index: $c \div$ speed in the medium; no unit.

Relative refractive index ${}_a\mu_b$: of b with respect to a.

Principle of reversibility: a reversed ray retraces its path through any reflections and refractions.

Lateral shift: perpendicular distance between the incident ray (produced) and the emergent ray of a slab.

Real / apparent depth: actual depth / depth at which it seems to be (a virtual image).

Apparent (normal) shift: how far the object seems raised.

Critical angle C: angle of incidence in the denser medium for which the angle of refraction is 90°.

Total internal reflection (TIR): all light reflected back into the denser medium at more than C.

Mirage: image of the sky or a distant object seen below it, as if in water.

Formulas

$\frac{\sin i}{\sin r} = {}_1\mu_2$	i, r from the normal **Use:** Snell's law
$\mu = \frac{c}{v}$	$c = 3 \times 10^8$ m/s **Use:** absolute index
$\mu_a \sin i = \mu_b \sin r$	absolute indices **Use:** any two media
${}_a\mu_b = \frac{1}{{}_b\mu_a}$	reversibility **Use:** swap the media
${}_w\mu_g = \frac{{}_a\mu_g}{{}_a\mu_w}$	from ${}_a\mu_w \cdot {}_w\mu_g \cdot {}_g\mu_a = 1$ **Use:** glass w.r.t. water
$L = \frac{t \sin(i - r)}{\cos r}$	t slab thickness **Use:** lateral shift
$\mu = \frac{R}{a}$	R real, a apparent depth **Use:** looking straight down
$d = R\left(1 - \frac{1}{\mu}\right)$	d apparent shift **Use:** how far raised
$a = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} + \cdots$	layers **Use:** apparent depth of stacked layers
$\sin C = \frac{1}{\mu}$	μ denser w.r.t. rarer **Use:** critical angle

Refraction and Snell's law

Laws of refraction

1. incident ray, refracted ray, normal at the point: one plane
2. $\sin i / \sin r = \text{constant}$, for a given pair of media and color (**Snell's law**)

- rarer → denser: $r < i$, bends **towards** the normal; denser → rarer: away
- speeds: vacuum 3×10^8, water $\approx 2.25 \times 10^8$, glass $\approx 2 \times 10^8$ m/s
- on refraction **frequency stays**; speed and wavelength ($\lambda = v/f$) change
- along the normal: changes speed, no bending; $\sin i / \sin r = 0/0 \to \mu$ not found from angles there
- absolute $\mu < 1$ impossible for visible light (faster than light in vacuum)
- red fastest in glass, violet slowest

Medium	μ
air	1.0003
water	1.33
kerosene	1.44
crown / flint glass	1.52 / 1.62
diamond	2.42

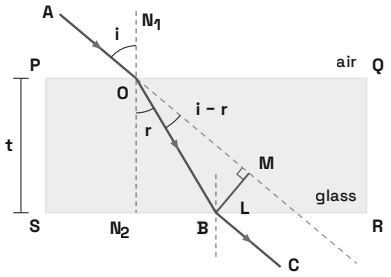
μ depends on

- the material
- color: larger for violet than red
- temperature: hotter → smaller
- pressure (gases); impurities

Relations

- reversibility: ${}_a\mu_b = \sin i / \sin r$, reversed ${}_b\mu_a = \sin r / \sin i \to$ product 1
- air → water → glass → air (parallel faces): emergent ray ∥ incident ray → ${}_a\mu_w \times {}_w\mu_g \times {}_g\mu_a = 1$
- e.g. ${}_w\mu_g = 1.5/1.33 = 1.13$; ${}_g\mu_w = 1/1.13 = 0.885$
- e.g. water → glass at 30°: $\sin r = 1.33 \times 0.5/1.5 \to r = 26.3°$
- light 500 m in water: $v = 3 \times 10^8/1.33 = 2.26 \times 10^8$ m/s → 2.2×10^{-6} s

Lateral shift through a slab

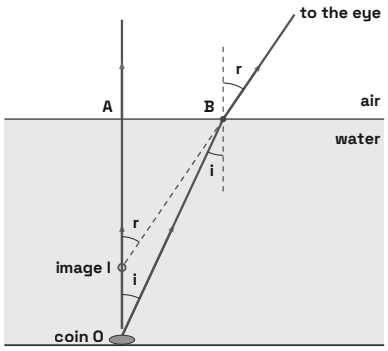


AO refracted along OB, leaves along BC ∥ AO; BM ⊥ AO produced = lateral shift L.

1. ray turns through $\angle BOM = i - r$: $BM = OB \sin(i - r)$
2. $\triangle ON_2B$: $\cos r = t/OB \to OB = t/\cos r$
3. $L = \dfrac{t \sin(i - r)}{\cos r}$

- depends on t, i, μ (through r)
- $i = 0$: $L = 0$; small i: $L \approx t\,i(1 - 1/\mu) \propto i$
- **grazing** ($i \to 90°$): $L \to t$, the largest
- e.g. $i = 45°$, $t = \sqrt{2}$ cm, $\mu = 2 \to r = 20.7°$, $L = 0.62$ cm

Real and apparent depth



Coin O at real depth AO, seen at I: apparent depth AI. OA leaves undeviated; OB bends away from the normal.

1. ${}_w\mu_a = \sin i / \sin r$; $\sin i = AB/OB$, $\sin r = AB/IB$
2. looking straight down: B near A → $IB \approx IA$, $OB \approx OA$
3. ${}_w\mu_a = a/R \to {}_a\mu_w = \dfrac{R}{a}$

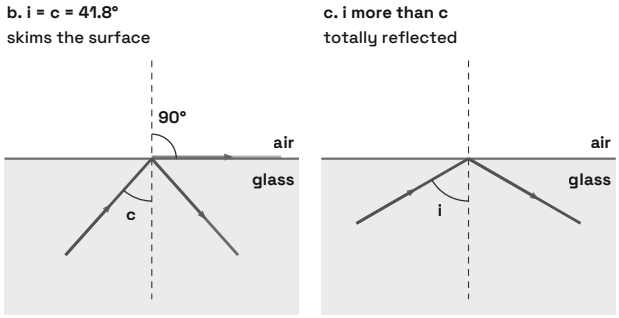
- object in rarer, eye in denser (fish sees a bird): μ = apparent height ÷ real height → looks **farther**
- from a slant: looks shallower still
- glass slab: shift ≈ ⅓ of thickness

- **e.g.** tank 40 cm, $\mu = 4/3 \rightarrow$ looks 30 cm deep
- **e.g.** microscope raised 1.25 cm for 4 cm turpentine $\rightarrow a = 2.75$, $\mu = 1.45$
- **e.g.** 6 cm glass + 4 cm water $\rightarrow a = 4 + 3.0 = 7.0$ cm
- bubble seen 6 cm from one face, 4 cm from the other (μ 1.5) $\rightarrow 9 + 6 = 15$ cm thick

Everyday

- clear pool looks shallow: test with a stick
- stick bent at the surface
- tree taller to a swimmer
- violet letters under a slab raised most (μ largest)
- shimmering at midday: hot air pockets, moving, smaller μ
- Sun oval at the horizon: denser air lower down lifts the lower edge more

Critical angle, total internal reflection



Glass ($\mu = 1.5$). At $C = 41.8°$: refracted ray skims the surface. At $60° > C$: totally reflected. (Below C: refracts out, a little reflected.)

- at C: $\mu \sin C = \sin 90° \rightarrow \sin C = 1/\mu$
- water 48.8°; glass 41.8°; diamond 24.4°; glass–water: $\sin C = 1.33/1.5$, $C = 62.5°$
- closer indices $\rightarrow$ larger C; violet $\rightarrow$ smallest C
- speed doubles across the boundary $\rightarrow C = 30°$

Conditions for TIR

1. light from **denser towards rarer**
2. angle of incidence $> C$

- "total": all reflected, none refracted, almost none absorbed (a silvered mirror absorbs ~ a tenth)
- from the rarer side: always $\sin r = \sin i/\mu < 1/\mu \rightarrow$ always refracted; **no TIR**
- circle of light in a 2.0 cm slab, radius 3.2 cm: $3.2 = 2 \times 2.0 \tan C \rightarrow \tan C = 0.8$, $\mu = 1.6$

Mirage: hot road heats the air above $\rightarrow \mu$ falls towards the road $\rightarrow$ light from the sky bends away from the normal layer by layer, finally TIR, turns up into the eye $\rightarrow$ seen below the road, inverted, like water. Cold sea: light bent down $\rightarrow$ ships in the sky (superior mirage).

Uses

- **optical fiber**: core in cladding of slightly lower μ, plastic coat; ~10^{-4} m thick; light reflects totally again and again; no loss
  - telecommunications (internet, phone, TV)
  - medicine: endoscope
  - data centers
- **totally reflecting prism**: 90°, 45°, 45°, crown glass ($C \approx 42°$); light meets the face at $45° > C$; turns through 90° (periscope) or 180°; binoculars, periscopes, cameras; better than mirrors: all light, one image, no silvering to tarnish
- **diamond**: $C = 24.4°$; cut faces $\rightarrow$ many TIRs $\rightarrow$ sparkle
- **crack in glass**: air film; light meets it at $> C \rightarrow$ shines

COMMON MISTAKES

✗ light bends towards the normal going into air
✓ into a **rarer** medium it bends **away**.

✗ frequency changes on refraction
✓ frequency stays; speed and wavelength change.

✗ $\sin C = \mu$
✓ $\sin C = 1/\mu$ (denser w.r.t. rarer).

✗ glass–water C with $\mu = 1.5$
✓ use 1.33/1.5.

✗ TIR when going into glass
✓ only from **denser to rarer**.

✗ $\mu = R/a$ at any angle
✓ only looking **straight down**.

✗ lateral shift largest at normal incidence
✓ it is **zero** there; largest at grazing.