Unit 13 · Ideal Gas

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Heat and Thermodynamics (Units 9–13 together): 15 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** a 5-mark short question from this unit: three assumptions of the kinetic theory (3); what to do since no gas is ideal (2)
- **MCQ (1 mark)**
  - $c_{rms} \propto \sqrt{T}$; $P \propto c_{rms}^2$
  - c_{rms} ratio $O_2 : H_2$
  - KE per volume $= \frac{3}{2}P$
  - KE at 0 °C vs 273 °C
  - same for all gases at one T: KE
  - T for double c_{rms}; N_2 vs O_2
  - KE of a molecule from k_B
  - molecules per m^3 at new P, T
  - gas laws: volume, density
  - $C_p - C_v$ universal; $N \to 2N$
- **Short (5 marks, in parts)**
  - why a gas presses on walls; load on a rigid box
  - volume of a mole at STP
  - CO_2 stays, Moon has no air; tire pressure after a drive
  - equal volumes, equal molecules; Boyle's law
  - not all molecules at one speed; heating a sealed gas
  - molecules in 1 cm^3; pumping out gas and temperature
  - c_{rms} of helium; combined gas law; air per m^3; balloon; methane's density; P from c_{rms}
- **Derivation (a short question's part):** $P = \frac{1}{3}\rho c_{rms}^2$, then c_{rms}, KE and temperature

Key terms

Volume coefficient γ_P: increase in volume per unit **volume at 0 °C** per degree, at constant pressure; $\frac{1}{273.15}$ K^{-1} for every gas (Charles's law).

Pressure coefficient γ_V: increase in pressure per unit **pressure at 0 °C** per degree, at constant volume; $\frac{1}{273.15}$ K^{-1} (pressure law).

Absolute zero: −273.15 °C, where an ideal gas's pressure (constant V) or volume (constant P) would become zero.

Ideal gas: no forces between molecules except in collisions, no volume of their own; obeys Boyle's and Charles's laws exactly.

Mole: amount with exactly $6.022\,140\,76 \times 10^{23}$ particles (N_A).

Molar mass M: mass of one mole (in g, = relative molecular mass).

Avogadro's law: equal volumes of all gases at the same T and P contain equal numbers of molecules.

R: work done by one mole of ideal gas heated through 1 K at constant pressure; 8.31 J/(mol K).

rms speed: square root of the mean of the squares of the molecules' speeds.

Boltzmann's constant: $k_B = R/N_A = 1.38 \times 10^{-23}$ J/K, gas constant per molecule.

Formulas

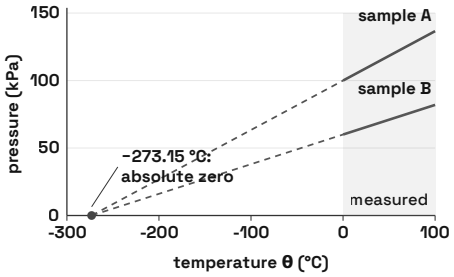
$\gamma_P = \frac{V_\theta - V_0}{V_0\,\theta}$, $\gamma_V = \frac{P_\theta - P_0}{P_0\,\theta}$	V_0, P_0 at 0 °C **Use:** both $\frac{1}{273.15}$ K^{-1}
$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$	T in K **Use:** combined gas equation
$PV = nRT$	P Pa, V m^3, n mol, T K **Use:** ideal gas equation
$PV = mrT$, $r = R/M$	r gas constant per unit mass (H_2 4.16, O_2 0.26 J/(g K)) **Use:** in terms of mass
$n = \frac{m}{M}$, $N = nN_A$	22.4 L per mole at NTP **Use:** moles, molecules
$P = \frac{1}{3}\rho\, c_{rms}^2$	ρ density; also $\frac{1}{3}mn_V\, c_{rms}^2$ **Use:** pressure from kinetic theory
$c_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}}$	M in **kg/mol** **Use:** rms speed
$E = \frac{3}{2}RT$	per mole; n moles: $\frac{3}{2}nRT$ **Use:** KE of a mole
$\frac{1}{2}mc_{rms}^2 = \frac{3}{2}k_BT$	m one molecule **Use:** KE of a molecule
$E' = \frac{3}{2}P$	KE per unit volume **Use:** pressure and KE
$PV = Nk_BT$	N molecules **Use:** counting molecules

- $c_{rms} \propto \sqrt{T}$ (one gas); $\propto 1/\sqrt{M}$ (same T): $\frac{c_1}{c_2} = \sqrt{\frac{M_2}{M_1}} \to O_2 : H_2 = 1 : 4$
- $R = \frac{1.013\times10^5 \times 22.4\times10^{-3}}{1\times273} = 8.31$ J/(mol K); dimensions $[ML^2T^{-2}K^{-1}mol^{-1}]$
- one O_2 molecule: $M/N_A = 5.32 \times 10^{-26}$ kg

- for one mole of ideal gas $C_p - C_v = R$ (the extra work $P\Delta V$): universal

Gas laws and absolute zero

- P–θ (constant V) and V–θ (constant P) lines of every gas meet the axis at **−273.15 °C**
- $P_\theta = \frac{P_0}{273.15}(273.15 + \theta) \to P \propto T$, $T = \theta + 273.15$
- coefficients per unit value **at 0 °C**, not at the start
- at 0 K the random KE is the **least possible** (classical: zero); no Kelvin temperature is negative
- real gases ≈ ideal at **low pressure, high temperature**; liquefy before 0 K



P–θ (or V–θ) lines of any gas, carried on, reach zero at −273.15 °C.

$PV = nRT$ **from Boyle and Charles** (one mole, $P_1V_1T_1 \to P_2V_2T_2$)

1. constant T_1, $P_1 \to P_2$: $P_1V_1 = P_2V \to V = P_1V_1/P_2$
2. constant P_2, $T_1 \to T_2$: $V/T_1 = V_2/T_2 \to V = V_2T_1/T_2$
3. equal: $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \to PV/T = R$ for one mole, same for every gas; n moles: $PV = nRT$

- **R's meaning**: $P\Delta V = R\Delta T$ = work by a mole heated 1 K at constant P
- NTP/STP here: 0 °C, 1 atm (1.013×10^5 Pa = 760 mm Hg), 22.4 L/mol
- N_A is the number in a mole of **anything**; only the gas's volume depends on conditions

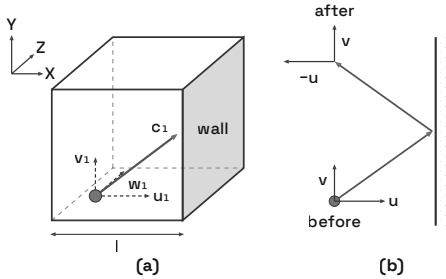
Kinetic theory

- macroscopic (P, V, T) vs microscopic (each molecule's m, v, KE)
- intermolecular forces: attract a little apart, repel when pushed; gas molecules ≈ 10 × their width apart at NTP
- physical change: molecules rearranged (boiling water ≈ 41 kJ/mol); chemical: atoms inside rearranged (splitting water ≈ 286 kJ/mol)

Postulates

1. very many identical, tiny, perfectly elastic spheres
2. their own volume negligible
3. no forces except in collisions; straight lines between
4. random motion, all directions, a range of speeds
5. collisions perfectly elastic
6. collision time negligible
7. Newton's laws apply
8. uniform density

Pressure: derivation (cube side l, N molecules of mass m)



Molecule with velocity c_1 (components u_1, v_1, w_1) bouncing off the shaded wall.

1. one bounce: momentum $mu_1 \to -mu_1$; wall gets $2mu_1$
2. hits again after $2l/u_1$
3. force $F_1 = \frac{2mu_1}{2l/u_1} = \frac{mu_1^2}{l}$
4. all: $P_x = \frac{F_x}{l^2} = \frac{m}{V}(u_1^2 + \cdots + u_N^2)$
5. random: $P_x = P_y = P_z \to P = \frac{m}{3V}(c_1^2 + \cdots + c_N^2)$, as $c^2 = u^2 + v^2 + w^2$
6. $c_{rms}^2 = \frac{c_1^2 + \cdots + c_N^2}{N} \to P = \frac{mN}{3V}c_{rms}^2$
7. $mN/V = \rho \to P = \frac{1}{3}\rho c_{rms}^2$

- $P \propto \rho$: half the volume, twice the hits (Boyle's law)
- $P \propto c_{rms}^2$: faster molecules hit more often **and** harder
- $\frac{1}{3}$: each direction gets a third of c^2 on average
- a load on a rigid box doesn't change the gas's pressure (same ρ, same c_{rms})

rms speed, KE, temperature

- one mole: $\rho = M/V \to PV = \frac{1}{3}Mc_{rms}^2 = RT \to c_{rms} = \sqrt{3RT/M}$
- $\times \frac{3}{2}$: $\frac{1}{2}Mc_{rms}^2 = \frac{3}{2}RT$; $\div N_A$: $\frac{1}{2}mc_{rms}^2 = \frac{3}{2}k_BT$
- $E' = \frac{1}{2}\rho c_{rms}^2 = \frac{3}{2} \times \frac{1}{3}\rho c_{rms}^2 = \frac{3}{2}P$
- speeds 1, 2, 3, 4, 5 m/s: average 3, rms $\sqrt{55/5} = 3.32$ m/s
- molecules have a spread of speeds (Maxwell): peak just below c_{rms}, long fast tail; heating widens and shifts it
- 300 K: $N_2 \approx 517$ m/s, $H_2 \approx 1930$ m/s
- **temperature** = measure of the average KE of the molecules' **random** motion; same $T \to$ same average KE, any gas
- heated sealed gas: KE ↑ ∝ T, speeds spread to higher values, pressure ↑
- light molecules (H_2, He) fast enough to escape more easily than heavy CO_2; the Moon's weak gravity lets its gas escape

COMMON MISTAKES

✗	T in °C in $PV = nRT$
✓	**kelvin**: 27 °C = 300 K.
✗	M in g/mol in $\sqrt{3RT/M}$
✓	**kg/mol**: O_2 32×10^{-3}.
✗	c_{rms} doubles when T doubles
✓	$\times\sqrt{2}$; double c_{rms} needs **4 ×** T (0 °C → 819 °C).
✗	heavier gas, more KE at one T
✓	same KE; heavier gas is **slower**.
✗	γ_P per unit volume at the start
✓	per unit volume **at 0 °C**.
✗	every molecule moves at c_{rms}
✓	a spread of speeds.
✗	molecules completely at rest at 0 K
✓	least possible KE (zero-point motion).

Science Playground, Katunje, Bhaktapur
https://science-playground.com/learn/grade-11-physics/ideal-gas