Unit 10 · Thermal Expansion

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Heat and Thermodynamics (Units 9–13 together): 15 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **MCQ (1 mark)**
  - density or apparent weight as temperature rises
  - equal expansions: $l_1\alpha_1 = l_2\alpha_2$
  - solid vs hollow sphere
  - which way a bimetallic strip bends
  - clock gains or loses per day
  - % change in area or volume from % in length
  - a hole in a heated disc
  - $\gamma_r = \gamma_a + \gamma_g$; mercury spilt
  - α per °F; thermal force
- **Short (5 marks, in parts)**
  - define α, β, γ; derive $\beta = 2\alpha$, $\gamma = 3\alpha$
  - real and apparent expansion from the flask figure; prove $\gamma_r = \gamma_a + \gamma_g$
  - Dulong and Petit: why, derive $\gamma = \frac{h_2 - h_1}{h_1 \Delta\theta}$, a value
  - bimetallic strip with a diagram; uses of differential expansion
  - railway gaps, coin diameter, rulers, tank, tape, wood in benzene, thermal stress

Key terms

Thermal expansion: increase in size of a body when heated (cooling: thermal contraction).

Linear expansivity α: increase in length per unit original length per degree rise.

Superficial expansivity β: increase in area per unit original area per degree rise.

Cubical expansivity γ: increase in volume per unit original volume per degree rise.

Thermal force: force set up when a body is stopped from expanding on heating or contracting on cooling; its stress is **thermal stress**.

Differential expansion: difference in the expansions of two bodies of different expansivities heated through the same range.

Real expansivity γ_r: real increase in a liquid's volume per unit original volume per degree.

Apparent expansivity γ_a: increase seen in its container per unit original volume per degree.

Formulas

$\alpha = \dfrac{l_2 - l_1}{l_1(\theta_2 - \theta_1)}$	l_1, l_2 lengths at θ_1, θ_2; unit K^{-1} (= $^\circ\mathrm{C}^{-1}$) **Use:** definition of α
$l_2 = l_1(1 + \alpha\Delta\theta)$	$\Delta\theta = \theta_2 - \theta_1$ (negative for cooling) **Use:** new length
$A_2 = A_1(1 + \beta\Delta\theta)$	A area **Use:** sheet, plate
$V_2 = V_1(1 + \gamma\Delta\theta)$	V volume **Use:** solid, liquid, inside of a vessel
$\alpha = \dfrac{\beta}{2} = \dfrac{\gamma}{3}$	isotropic solid **Use:** $\alpha : \beta : \gamma = 1 : 2 : 3$
$\text{loss} = \frac{1}{2}\alpha\Delta\theta \times t$	t time run (1 day = 86 400 s) **Use:** pendulum clock; warmer loses, colder gains
$F = YA\alpha\Delta\theta$	Y Young's modulus, A cross-section; stress $Y\alpha\Delta\theta$ **Use:** rod or wire held; not on length
$l_1\alpha = l_1'\alpha'$	lengths in inverse ratio of α **Use:** same difference in length at all temperatures

$\gamma_r = \gamma_a + \gamma_g$	glass vessel: $\gamma_g = 3\alpha_g$ **Use:** liquid in a vessel
$\rho_2 = \dfrac{\rho_1}{1+\gamma\Delta\theta}$	$\approx \rho_1(1-\gamma\Delta\theta)$ **Use:** density when heated
$\alpha = \dfrac{S_2 - S_1}{l(\theta_2 - \theta_1)}$	S_1, S_2 spherometer readings **Use:** Pullinger's apparatus
$\gamma = \dfrac{h_2 - h_1}{h_1\Delta\theta}$	h_1 cold, h_2 hot column **Use:** Dulong and Petit: **real** γ

Why things expand

- heated → atoms vibrate with larger amplitude; force between atoms **lopsided** (hard to squeeze, easier to pull apart) → average spacing ↑
- gases expand most, liquids less, solids least
- water: contracts 0 → 4 °C; densest at 4 °C
- uses: gaps or rollers on bridges and rails; cart-wheel tire fitted hot, shrinks to grip
- α depends on material only, not on length, shape or unit of length; tiny (steel 1.1×10^{-5} K^{-1})
- hollow sphere = solid sphere of same metal, same size: **expand equally**; a hole grows as if filled (hole in a heated disc gets **bigger**)

Substance	α (10^{-5} K^{-1})
fused quartz / invar	0.05 / 0.12
glass	0.3 – 0.9
steel / iron	1.1 / 1.2
copper / brass	1.7 / 1.9
aluminium	2.3

Derivations

$\beta = 2\alpha$ (square sheet, side l_1)

1. $A_2 = A_1(1+\beta\Delta\theta) \rightarrow l_2^2 = l_1^2(1+\beta\Delta\theta)$
2. $l_2 = l_1(1+\alpha\Delta\theta) \rightarrow (1+\alpha\Delta\theta)^2 = 1+\beta\Delta\theta$
3. $1 + 2\alpha\Delta\theta + \alpha^2\Delta\theta^2 = 1+\beta\Delta\theta$; α tiny → drop α^2 → $\beta = 2\alpha$

$\gamma = 3\alpha$ (cube): $(1+\alpha\Delta\theta)^3 = 1+\gamma\Delta\theta \rightarrow$ $1 + 3\alpha\Delta\theta + 3\alpha^2\Delta\theta^2 + \alpha^3\Delta\theta^3$; drop α^2, $\alpha^3 \rightarrow \gamma = 3\alpha$

- isotropic solids only (metals, glass); a crystal or wood: γ = the sum of its three α's

Pendulum clock: $T = 2\pi\sqrt{l/g}$;
$T' = T(1+\alpha\Delta\theta)^{1/2} \approx T(1+\frac{1}{2}\alpha\Delta\theta) \rightarrow$ fraction lost $\frac{1}{2}\alpha\Delta\theta$

- hot → longer rod → longer period → **loses**; cold → **gains**

Thermal stress: stopped growth $\Delta l = l\alpha\Delta\theta \rightarrow$ strain $\alpha\Delta\theta \rightarrow$ stress = $Y \times$ strain = $Y\alpha\Delta\theta$

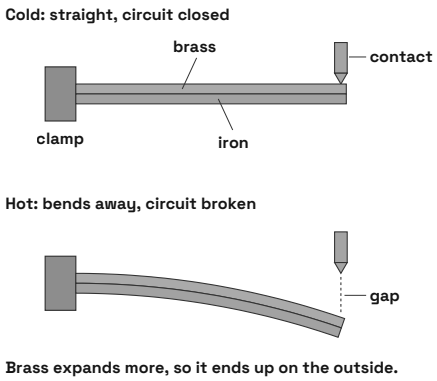
Zero differential expansion:
$d_2 - d_1 = (l_1\alpha - l_1'\alpha')\Delta\theta = 0 \rightarrow l_1\alpha = l_1'\alpha'$; metal that expands less is longer (compensated pendulum)

Expanding scale: scale expands more than the rod → reads **short** by $l(\alpha_s - \alpha_r)\Delta\theta$ (s scale, r rod) (brass scale, iron rod 50.0 cm, 30 °C → 49.988 cm)

Pullinger's apparatus

- rod length l by meter scale; rod in steam jacket, lower end on base; thermometer: θ_1
- spherometer screwed down till it touches A → galvanometer deflects → S_1; screw back up
- steam till steady θ_2; screw down to touch again → S_2
- $\Delta l = S_2 - S_1$ (spherometer reads 0.01 mm); galvanometer marks the exact moment of touch

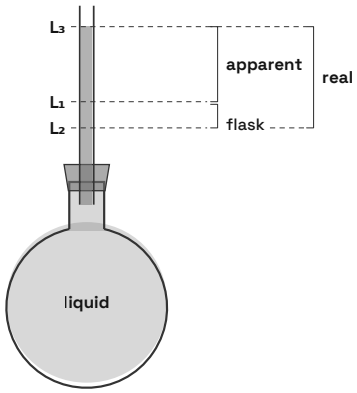
Bimetallic strip



Brass and iron joined: heated, it bends with the **more expanding** metal (brass) outside; cooled, the other way.

- **thermostat**: strip bends away past a set temperature → breaks the circuit; iron, rice cooker, water heater, oven
- differential expansion also in compensated pendulums (gridiron)

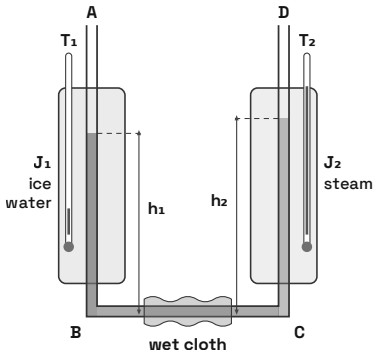
Liquids



Flask heated: level $L_1 \rightarrow$ dips to L_2 (glass expands first) $\rightarrow$ rises to L_3. L_1L_2 vessel, L_1L_3 apparent, L_2L_3 real.

- real = apparent + vessel: $L_2L_3 = L_1L_3 + L_2L_1$; $\times$ tube area $\rightarrow \Delta V_r = \Delta V_a + \Delta V_g \rightarrow \gamma_r = \gamma_a + \gamma_g$
- γ_r: liquid only; γ_a: liquid **and** vessel (mercury: smaller γ_a in glass than in quartz)
- $\gamma_g = \gamma_r \rightarrow$ level never moves; $\gamma_g > \gamma_r \rightarrow$ level falls
- heated: mass same, volume $\uparrow \rightarrow$ density $\downarrow \rightarrow$ upthrust $\downarrow \rightarrow$ metal's apparent weight in the liquid $\uparrow$; 5 L of benzene weighs more in winter
- wood just sinks when its density = liquid's: $\rho_0/(1+\gamma\theta)$ for each, equal

Dulong and Petit



U-tube: limb AB in ice water (θ_1, height h_1), CD in steam (θ_2, h_2); BC kept cool.

- B and C at one level, liquid at rest $\rightarrow \rho_1 g h_1 = \rho_2 g h_2 \rightarrow h_1/h_2 = \rho_2/\rho_1 = 1/(1+\gamma\Delta\theta) \rightarrow \gamma = \frac{h_2 - h_1}{h_1 \Delta\theta}$
- pressure doesn't depend on width $\rightarrow$ glass's expansion doesn't enter $\rightarrow$ **real** expansivity directly

COMMON MISTAKES

✗ hole in a heated plate shrinks
✓ it grows, as if filled with the metal.

✗ hollow sphere expands more (or less)
✓ **equally**, same metal and size.

✗ $\gamma = 2\alpha$ or $\beta = 3\alpha$
✓ $\beta = 2\alpha, \gamma = 3\alpha$.

✗ pendulum clock gains in summer
✓ longer rod, longer period: **loses**.

✗ longer rail, bigger thermal force
✓ $F = YA\alpha\Delta\theta$: **no length** in it.

✗ strip bends with iron outside on heating
✓ **brass** (more α) outside.

✗ level seen = real expansion
✓ seen = **apparent**; real = apparent + vessel.

✗ $\Delta\theta$ in °F used with α in $^\circ\text{C}^{-1}$
✓ convert: $\Delta C = \frac{5}{9}\Delta F$.