Unit 8 · Elasticity

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Mechanics (Units 1–8 together): 26 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** an MCQ (the stiffest of several materials from their stress–strain graphs: the steepest line); a 5-mark short question as the alternative to Unit 4's: state Hooke's law, a tire's rubber warming over a load–unload cycle
- **MCQ (1 mark)**
  - mass released suddenly: extension $2mg/k$
  - Poisson's ratio's expression
  - rod heated between supports: compressive stress
  - most elastic: steel; Y falls as temperature rises
  - elastomers; shearing changes shape
  - wire under its own weight: $mgL/2AY$
  - bulk modulus from a volume change
  - elastic after-effect
- **Short (5 marks, in parts)**
  - Hooke's law; does it hold for all materials?
  - greatest extension within the elastic limit
  - three moduli; water's elasticity (bulk only)
  - guitar string's stretch; Poisson's ratio; after-effect
  - elastic PE, derived; energy in a compressed bar
  - % change in volume from σ; decrease in a wire's area
  - force to double a length; steel and copper wires joined
  - elastic limit, breaking stress, hysteresis

Key terms

Deforming force: a force that changes a body's shape or size; **restoring force**: the inside force that resists it.

Elasticity: a body regains its shape and size when the deforming force is removed.

Plasticity: it keeps the new shape (putty, clay).

Elastic limit: the greatest stress from which a body still fully recovers.

Hooke's law: within the elastic limit, stress ∝ strain ($F = -kx$).

Stress: restoring force per unit area, F/A; Pa (N/m^2); normal (tensile, compressive) or tangential (shearing).

Strain: change ÷ original size; no unit; longitudinal, volumetric, shear.

Young's modulus Y: tensile stress ÷ longitudinal strain (length).

Bulk modulus B: pressure ÷ volumetric strain (volume); **compressibility** = 1/B.

Modulus of rigidity η: shear stress ÷ shear strain φ (shape).

Poisson's ratio σ: lateral strain ÷ longitudinal strain.

Breaking stress: the stress at which a body breaks; **ultimate stress**: the greatest on the curve.

Elastic after-effect: delay in regaining the original shape after the force is removed.

Elastic hysteresis: unloading curve below the loading curve; the loop's area is heat each cycle.

Elastic fatigue: breaking under many repeated loads below the breaking stress.

Elastomer: a material that stretches far, elastically, without obeying Hooke's law (rubber).

Formulas

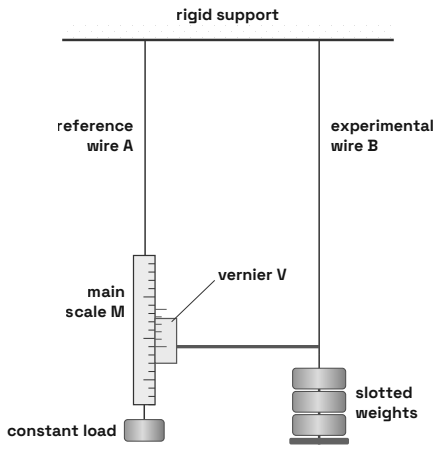
$F = -kx$	k force constant, N/m **Use:** Hooke's law, spring
$Y = \frac{F/A}{x/L} = \frac{FL}{Ax}$	$A = \pi d^2/4$ **Use:** Young's modulus
$Y = \text{slope} \times \frac{L}{A}$	slope of F against x **Use:** from the experiment
$B = \frac{P}{-\Delta V/V}$	**Use:** bulk modulus
$\eta = \frac{F/A}{\varphi}, \varphi \approx \frac{x}{L}$	φ angle of shear **Use:** rigidity
$W = \frac{1}{2}Fx = \frac{1}{2}kx^2 = \frac{1}{2}\frac{YA}{L}x^2$	**Use:** elastic PE in a wire
$u = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2}Y\varepsilon^2$	per unit volume **Use:** energy density
$\sigma = \frac{\Delta D/D}{x/L}$	0 to 0.5 **Use:** Poisson's ratio
$\frac{\Delta V}{V} = \varepsilon(1 - 2\sigma)$	ε longitudinal strain **Use:** volume change
$F = YA\alpha\Delta\theta$	Unit 10 **Use:** heated rod held fixed

Elasticity

- why: atoms held by spring-like forces; pulled apart → attract, pushed → repel
- "more elastic" = **harder to deform**: steel more elastic than rubber
- uses: crane ropes sized to stay within the elastic limit; bridges designed not to sag or break; stranded cables bend easily

- solids have all three moduli; liquids and gases only **bulk**; Y falls as temperature rises

Hooke's law: verifying it



Reference wire A with the main scale and a small load; test wire B with the vernier and hanger, from one support.

1. small starting load on B; note the vernier
2. add equal loads one at a time, wait, read; then remove them, read again (same → no permanent set)
3. extension = reading − start; mean of loading and unloading
4. plot load against extension: a straight line through the origin → Hooke's law
5. L by meter rule, d by screw gauge in several places → Y = slope × L/A

- reference wire: cancels the support's sag and the temperature's expansion
- Hooke's law fails for rubber and beyond the proportional limit

Elastic potential energy

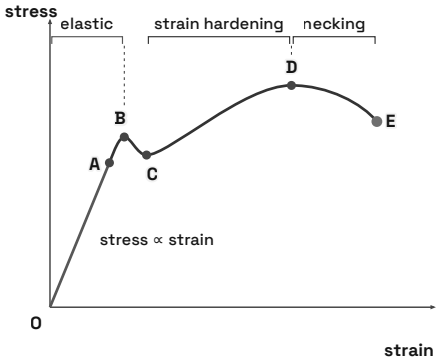
1. at extension x': $F' = \frac{YA}{L}x'$
2. work for dx': $dW = \frac{YA}{L}x'\,dx'$
3. $W = \int_0^x \frac{YA}{L}x'\,dx' = \frac{YA}{2L}x^2 = \frac{1}{2}Fx$
4. graph: area of the triangle under F–x = $\frac{1}{2}Fx$

- ÷ volume AL: $\frac{1}{2}$ stress × strain
- mass hung and released suddenly: $mgx = \frac{1}{2}kx^2$ → $x = 2mg/k$, twice the slow extension
- wire under its own weight: average load $mg/2$ → $mgL/2AY$
- catapult: elastic PE → stone's KE

Poisson's ratio

- stretched: longer and thinner; cork ≈ 0, glass 0.2–0.25, steel ≈ 0.3, copper 0.34, rubber ≈ 0.5 (volume hardly changes); cork fits a bottle neck: hardly widens when pushed in
- volume change: $\varepsilon - 2\sigma\varepsilon$; $\sigma = 0.2$, $\varepsilon = 4 \times 10^{-3}$ → 0.24%

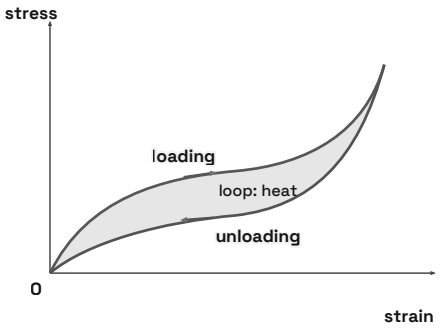
Stress–strain curve



Mild steel: O–A proportional, B elastic limit, C yield, C–D strain hardening, D ultimate stress, E fracture.

- O–A: Hooke's law, slope = Y; A proportional limit
- A–B: still elastic, not straight; B elastic limit
- B–C: **yield**, stretches with no extra stress; permanent set
- C–D: plastic flow, **strain hardening**; D ultimate (tensile) stress
- D–E: **necking**, then fracture at E
- **ductile**: long plastic region (copper, mild steel, gold) → wires, sheets
- **brittle**: breaks near the elastic limit (glass, cast iron, concrete)
- steepest line on a stress–strain graph = largest Y = stiffest

After-effect, hysteresis, fatigue



Rubber: loading curve above the unloading curve; the loop's area is the energy turned to heat.

- after-effect: quartz and phosphor bronze recover at once (galvanometer suspensions); glass slowly
- hysteresis: **tires warm up** (squashed and released each turn); rubber mounts absorb vibration
- fatigue: a paper clip bent to and fro snaps; rails, bridges and aircraft inspected for fatigue cracks

COMMON MISTAKES

✗ rubber is more elastic than steel
✓ steel: it takes far more stress for the same strain.

✗ Hooke's law holds up to breaking
✓ only to the **proportional limit**.

✗ strain in m
✓ a ratio: **no unit**.

✗ extension under a suddenly released load = mg/k
✓ $2mg/k$.

✗ area in Y as πd^2
✓ $\pi d^2/4$ (d diameter).