Unit 7 · Gravitation

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Mechanics (Units 1–8 together): 26 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** the alternative 8-mark long question: gravitational potential, worked for the Juno probe near Jupiter
- **MCQ (1 mark)**
  - pendulum's period with height
  - g on a planet from M and R; radius 1% less → g 2% more
  - orbital speed independent of the satellite's mass
  - height for $g/2$ (2650 km); depth for $g/4$ (3R/4)
  - year if the Sun were half as far; period at 4 × the geostationary radius (8 days)
  - geostationary period 24 h; g zero at the center, greatest at the surface
  - weightless in a lift falling at g
- **Short (5 marks, in parts)**
  - why G is universal; $g = GM/R^2$; a satellite's acceleration at 2R
  - escape velocity: define, deduce, why it differs by planet
  - weightlessness; apparent weight falling or rising at 5 m/s^2
  - gravitational potential vs potential energy; center of mass and of gravity; a balanced rod
  - GPS: how it locates, how many satellites, other GNSS
- **Long (8 marks)**: g with height (36% less at 1600 km) and depth; orbital velocity and period; parking orbit's height; escape velocity (Mars from Earth's); potential energy $-GMm/r$

Key terms

Newton's law of gravitation: every two bodies attract with a force ∝ product of masses and ∝ 1/distance^2, along the line joining them.

G: force between two 1 kg masses 1 m apart; 6.67×10^{-11} N m^2/kg^2; universal: same for all bodies everywhere.

Gravity: the Earth's pull on a body (its weight); **gravitation**: the attraction between any two bodies.

Acceleration due to gravity g: acceleration of a body falling freely under the Earth's pull.

Inertial mass: $m = F/a$; **gravitational mass**: from the pull of gravity on it; equal.

Gravitational field strength: force on unit mass, $E = F/m = g$; N/kg.

Gravitational potential: work done bringing unit mass from infinity to the point, without acceleration; $-GM/r$; J/kg.

Gravitational potential energy: work done bringing mass m from infinity; $-GMm/r$.

Center of mass: point where the whole mass can be taken to be.

Center of gravity: point through which the resultant weight acts.

Orbital velocity: speed needed to circle the Earth at a given height.

Escape velocity: least speed from the surface to escape the Earth's pull for ever.

Geostationary satellite: period 24 h, over the equator, west to east; stays above one place.

Weightlessness: no supporting force on a body, in free fall.

Formulas

$F = G\frac{m_1 m_2}{r^2}$	**Use:** law of gravitation
$g = \frac{GM}{R^2}; M = \frac{gR^2}{G}$	$g = \frac{4}{3}\pi G\rho R$ **Use:** g; weighing the Earth
$E = \frac{GM}{r^2}, E = -\frac{dV}{dr}$	**Use:** field strength
$V = -\frac{GM}{r}, U = -\frac{GMm}{r} = mV$	zero at infinity **Use:** potential, potential energy
$g' = g\frac{R^2}{(R+h)^2} \approx g\left(1 - \frac{2h}{R}\right)$	$h \ll R$ **Use:** height
$g' = g\left(1 - \frac{d}{R}\right)$	zero at the center **Use:** depth

$v = \sqrt{\frac{GM}{r}} = R\sqrt{\frac{g}{R+h}}$	$r = R + h$ **Use:** orbital velocity
$T = 2\pi\sqrt{\frac{r^3}{GM}}$	$T^2 \propto r^3$ **Use:** period
$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$	Earth 11.2 km/s **Use:** escape velocity
$K = \frac{GMm}{2r}, E = -\frac{GMm}{2r}$	$E = U/2 = -K$ **Use:** satellite's energy
$X = \frac{\sum m_i x_i}{\sum m_i}$	**Use:** center of mass

g, the field and the potential

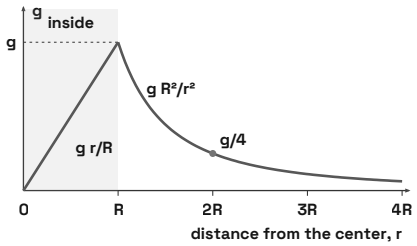
- $mg = GMm/R^2 \to g = GM/R^2$: **independent of the falling body's mass**
- Cavendish measured G $\to M = gR^2/G \approx 6 \times 10^{24}$ kg
- g on a planet: $g_p = g\frac{M_p}{M}\left(\frac{R}{R_p}\right)^2$
- field lines point **inward**; $E = g$

$V = -GM/r$

1. at distance x, hold unit mass back with GM/x^2 outward
2. $dW = \frac{GM}{x^2}dx$
3. $W = \int_\infty^r \frac{GM}{x^2}dx = -\frac{GM}{r}$
4. mass m: $U = -GMm/r$

- negative: the field does work pulling it in; a bound body has negative energy
- potential is per unit mass; potential energy is for the mass m

Variation of g



g against distance from the center: rises in a straight line to its greatest at the surface, then falls as $1/r^2$.

Height: divide $GM/(R+h)^2$ by GM/R^2

- $g/2$ at $h = (\sqrt{2} - 1)R \approx 2650$ km

Depth (uniform Earth): only the ball of radius $r = R - d$ inside pulls

1. $M' = \frac{4}{3}\pi r^3\rho \to g' = \frac{4}{3}\pi G\rho r$
2. surface: $g = \frac{4}{3}\pi G\rho R$
3. divide: $g' = g\frac{r}{R} = g(1 - d/R)$; $g/4$ at $d = 3R/4$; zero at the center

Latitude: less at the equator: bulge (farther from the center) and spin ($R\omega^2 = 0.034$ m/s^2 used as centripetal force); ≈ 9.78 m/s^2 at the equator, 9.83 at the poles

- pendulum clocks lose on mountains (g smaller, T longer)

Center of mass, stability

- center of gravity = center of mass where g is the same over the body
- stable: low C.G., wide base, the C.G.'s vertical line within the base
- balanced rod: moments about the wedge, the rod's own weight at its middle

Satellites

- **orbital velocity**: $\frac{GMm}{r^2} = \frac{mv^2}{r} \to v = \sqrt{GM/r}$; near the surface $\sqrt{gR} \approx 7.9$ km/s
- **period**: $T = 2\pi r/v$; near the surface ≈ 85 min; $T^2 \propto r^3$ (4 × radius → 8 × period)
- **height for a period**: $h = \left(\frac{gR^2T^2}{4\pi^2}\right)^{1/3} - R$
- **geostationary** (parking orbit): 24 h, equator, west to east, $\approx 36\,000$ km up, 3.1 km/s; TV, weather
- **energy**: $K = GMm/2r$, $U = -GMm/r$, $E = -GMm/2r$
- **escape velocity**
  1. work to take m from R to infinity: $\int_R^\infty \frac{GMm}{x^2}dx = \frac{GMm}{R}$
  2. $\frac{1}{2}mv_e^2 = \frac{GMm}{R} \to v_e = \sqrt{2gR} = 11.2$ km/s
  3. depends on the planet's M and R, not on the body's mass
- $v_e = \sqrt{2}\times$ orbital velocity near the surface
- Moon has no air: v_e only 2.4 km/s, so its gas molecules (hundreds of m/s, some faster) slowly leaked away
- **weightlessness**: astronauts and the station fall together round the Earth; gravity is still about 90% of g there
- lift accelerating down at a: $m(g - a)$; up: $m(g + a)$

GPS

- satellites send their position and the exact time; the receiver finds each distance from the signal's travel time × c (trilateration)
- **four** satellites: three for position, one for the receiver's clock
- segments: space (satellites), control (ground stations), user (receivers)
- others: GLONASS (Russia), Galileo (EU), BeiDou (China), NavIC (India): together GNSS

COMMON MISTAKES

✗ weight in orbit is zero as no gravity
✓ gravity acts; the body is in **free fall**.

✗ heavier bodies fall faster
✓ g doesn't depend on the body's mass.

✗ g greatest at the center
✓ zero there; greatest at the **surface**.

✗ potential is positive
✓ zero at infinity, **negative** elsewhere.

✗ satellite's orbital speed depends on its mass
✓ only on M and r.

✗ $g' = g(1 - h/R)$ above the surface
✓ $g(1 - 2h/R)$ above; $(1 - d/R)$ below.