# Unit 6 · Circular Motion

STUDY NOTES

**WHAT NEB'S GRADE 11 EXAM ASKS**

- **CDC's grid (2077):** Mechanics (Units 1–8 together): 26 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** an 8-mark long question (or its alternative on Unit 7): a car on a circular track, the forces on it and its greatest safe speed
- **MCQ (1 mark)**
  - steady speed in a circle: KE same, momentum changes
  - fighter plane pulling out of a dive: air's force at the bottom
  - tension doubled → speed × $\sqrt{2}$
  - $a$ from revolutions, from $v$ and $r$; $\omega$ from $a$ and $r$
  - Earth's rotation at the equator: 0.034 m/s$^2$
  - speed doubled → force × 4
  - $\theta = s/r$; centrifugal force is fictitious
  - banking angle for a race track
- **Short (5 marks, in parts)**
  - linear vs circular motion; $f = 1/T$
  - net force though the speed is steady; accelerating at constant speed
  - banking angle, why needed, its value
  - conical pendulum's centripetal force; vs a simple pendulum
  - $\omega$ from revolutions a minute
- **Long (8 marks)**: derive $a = v^2/r$; conical pendulum (period, tension); vertical circle (tensions, $\sqrt{gr}$, $\sqrt{5gr}$); level and banked roads

## Key terms

**Circular motion**: motion along a circle; **uniform** when the speed is steady (velocity still changes direction).

**Angular displacement** $\theta$: angle turned at the center, $s/r$ radians.

**Angular velocity** $\omega$: rate of change of angular displacement, $d\theta/dt$; rad/s.

**Time period T**: time for one revolution; **frequency f**: revolutions per second, $f = 1/T$.

**Centripetal acceleration**: acceleration towards the center, $v^2/r$.

**Centripetal force**: the resultant of the real forces towards the center, $mv^2/r$; not a new force.

**Centrifugal force**: a pseudo-force felt only in the rotating frame, outward.

**Conical pendulum**: a bob going round a horizontal circle, its string tracing a cone.

**Critical velocity**: least speed at the top of a vertical circle that keeps the string tight, $\sqrt{gr}$.

**Banking**: raising a road's outer edge so the normal reaction supplies the centripetal force.

**Angle of banking**: the road's tilt, $\tan\theta = v^2/rg$.

## Formulas

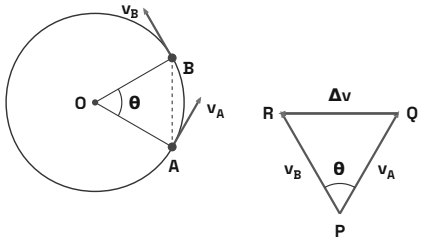
| | |
|---|---|
| $\theta = \frac{s}{r}, \omega = \frac{2\pi}{T} = 2\pi f$ | 1 rev = $2\pi$ rad<br>**Use:** angles, angular velocity |
| $v = r\omega; a_t = r\alpha$ | $\alpha$ angular acceleration<br>**Use:** linear and angular |
| $a = \frac{v^2}{r} = r\omega^2 = 4\pi^2 f^2 r$ | towards the center<br>**Use:** centripetal acceleration |
| $F = \frac{mv^2}{r} = mr\omega^2$ | **Use:** centripetal force |
| $a = \sqrt{a_c^2 + a_t^2}$ | **Use:** speed changing too |
| $\tan\theta = \frac{v^2}{rg}, \tau = 2\pi\sqrt{\frac{l\cos\theta}{g}}$ | $T = mg/\cos\theta$<br>**Use:** conical pendulum |
| $T = \frac{mv^2}{r} + mg\cos\theta$ | $\theta$ from the downward vertical<br>**Use:** vertical circle |
| $v_{top} = \sqrt{gr}, v_{bottom} = \sqrt{5gr}$ | **Use:** least speeds, vertical circle |
| $v_{max} = \sqrt{\mu gr}$ | **Use:** level bend |
| $\tan\theta = \frac{v^2}{rg}$ | **Use:** banked road, leaning cyclist |

## Uniform circular motion

- velocity along the **tangent**, always changing direction → accelerating, so a net force acts though the speed is steady
- KE and speed steady; momentum and velocity change; acceleration's size steady, direction changes

- string cut: the body flies off along the tangent
- examples: stone on a string (tension), Moon round the Earth (gravity), electron round a nucleus (electric force), car on a bend (friction)
- $f = 1/T$: $f$ turns a second, each takes $T$, so $fT = 1$

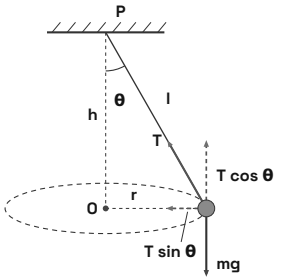
## $a = v^2/r$




A to B in $\Delta t$, radius turns $\theta$; PQ $= \vec{v}_A$, PR $= \vec{v}_B$, QR $= \Delta v$.

1. $\Delta\vec{v} = \vec{v}_B - \vec{v}_A =$ QR
2. triangles OAB and PQR similar (both isosceles, same $\theta$): $\frac{\Delta v}{v} = \frac{AB}{r}$
3. $\Delta t$ small: AB $\approx$ arc $= v\Delta t \to \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$
4. $\theta \to 0$: $\Delta v$ at 90° to v, so **towards the center**

## Conical pendulum

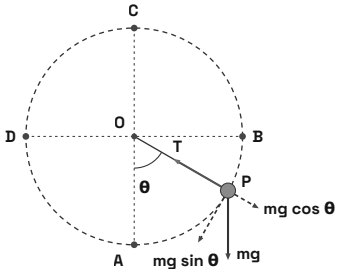



Bob on string $l$ from P, circle of radius $r$, angle $\theta$ to the vertical, height $h = l\cos\theta$.

1. vertical: $T\cos\theta = mg$
2. horizontal: $T\sin\theta = mv^2/r$
3. divide: $\tan\theta = v^2/rg$
4. $\tan\theta = r/h \to \omega^2 = g/h \to \tau = 2\pi\sqrt{h/g}$

- centripetal force: the tension's horizontal part $T\sin\theta$
- simple pendulum swings in a plane; conical goes round a horizontal circle

## Vertical circle

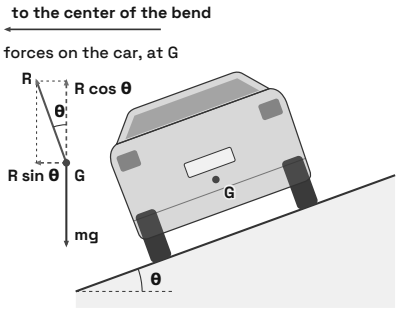



String at $\theta$ to the downward vertical at P; $mg\cos\theta$ along the string, $mg\sin\theta$ along the tangent.

- $T - mg\cos\theta = mv^2/r$
- bottom ($\theta = 0$): $T_{max} = mv^2/r + mg$ (string breaks here)
- side: $T = mv^2/r$; top: $T_{min} = mv^2/r - mg$
- top, $T = 0$: $v = \sqrt{gr}$
- energy from bottom to top (rise $2r$): $\frac{1}{2}mv_A^2 = \frac{1}{2}m(gr) + 2mgr \to v_A = \sqrt{5gr}$
- water in a bucket whirled overhead stays in above $\sqrt{gr}$
- plane pulling out of a dive: lift at the bottom $= mg + mv^2/r$

## Roads and cyclists

- **level bend**: friction gives $mv^2/r \le \mu mg \to v_{max} = \sqrt{\mu gr}$; faster → skids out; wet road → smaller $\mu$




Car on a road banked at $\theta$: R at $\theta$ to the vertical; $R\cos\theta = mg$, $R\sin\theta = mv^2/r$.

- **banked**: $R\cos\theta = mg$, $R\sin\theta = mv^2/r \to \tan\theta = v^2/rg$; no friction needed at that speed; wear and skidding less
- **cyclist** leans in by $\theta$: $N = mg$, friction $f = mv^2/r$, $\tan\theta = f/N = v^2/rg$
- same angle: railway tracks (outer rail higher), a plane banking to turn
- 50 km/h, 200 m: $\theta \approx 5.6°$

### COMMON MISTAKES

✗ uniform circular motion has no acceleration
✓ velocity changes direction: $a = v^2/r$.

✗ centripetal force is an extra force
✓ it is the resultant of real forces (tension, friction, gravity).

✗ centrifugal force acts on the body
✓ only a pseudo-force in the rotating frame.

✗ string cut: flies outward along the radius
✓ along the **tangent**.

✗ tension greatest at the top
✓ at the **bottom**.

✗ $\tan\theta = rg/v^2$
✓ $v^2/rg$.

✗ speed doubled → force doubled
✓ $F \propto v^2$: **× 4**.