Unit 4 · Dynamics

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Mechanics (Units 1–8 together): 26 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** an MCQ on forces in equilibrium; a 5-mark short question (state conservation of momentum, a jumbo jet's engines; or Hooke's law, Unit 8); the 8-mark velocity–time graph question's force, friction and distance
- **MCQ (1 mark)**
  - mangoes fall: inertia of rest
  - uniform velocity needs no force
  - mass from $\vec{F}$ and a; acceleration from three forces
  - third-law pairs equal
  - impulse is not a force; friction non-conservative
  - lift's apparent weight; ball on a wall: $2mu\cos\theta/t$
  - pulley's a; angle of repose $\tan^{-1}\mu$
  - acceleration on a rough floor, down a slope
- **Short (5 marks, in parts)**
  - first law, force, inertia; impulse–momentum theorem; china in straw
  - second law the real law; why $k = 1$; force to stop a body
  - couple and its work on a wheel
  - friction a necessary evil; angle of friction = angle of repose; μ from a stopping ball
  - verify the principle of moments; center of gravity of a loaded rod
  - conservation of momentum from the third law
- **Long (8 marks)**: pulley or lift derivations; masses over a pulley on a slope; a body on a slope with friction; a free-body diagram (chair pushed at 37°)

Key terms

Force: a push or pull that changes, or tends to change, a body's state of rest or motion; N.

Inertia: resistance to a change in motion; **mass** measures it; of rest, of motion, of direction.

Linear momentum: $\vec{p} = m\vec{v}$; kg m/s; vector.

Impulse: $\vec{F}\Delta t$ = change in momentum; area under the F–t graph; N s.

Conservation of momentum: with no external force, total momentum stays constant.

First law: a body stays at rest or in uniform motion unless a resultant force acts (law of inertia).

Second law: rate of change of momentum $\propto$ resultant force, in its direction.

Third law: to every action an equal and opposite reaction.

1 newton: force giving 1 kg an acceleration of 1 m/s^2.

Torque (moment): turning effect of a force, $\tau = rF\sin\theta$; N m.

Couple: two equal, opposite, parallel forces not in one line.

Equilibrium: $\sum\vec{F} = 0$ and $\sum\tau = 0$.

Limiting friction: the greatest static friction, just before sliding.

Coefficient of friction: $\mu = F/R$; no unit.

Angle of friction λ: between the resultant of F and R, and R, at the limit.

Angle of repose θ: least slope at which a body just begins to slide.

Formulas

$\vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$	$F = m(v-u)/t$ **Use:** second law
$\vec{F}\Delta t = m\vec{v} - m\vec{u}$	**Use:** impulse–momentum theorem
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$	no external force **Use:** conservation of momentum
$a = \frac{H-f}{m+m'}$	H horse's push, f friction **Use:** horse and cart
$a = \frac{(m_1-m_2)g}{m_1+m_2}$, $T = \frac{2m_1m_2g}{m_1+m_2}$	$m_1 > m_2$ **Use:** pulley
$R = m(g+a)$; $R = m(g-a)$	accelerating up; down **Use:** apparent weight in a lift
$\vec{\tau} = \vec{r}\times\vec{F}$, $\tau = rF\sin\theta$	**Use:** torque
$\tau = F \times d$	d arm of the couple **Use:** couple
$W = \tau\theta$	θ in radians **Use:** work by a couple
$F = \mu R$	$\mu_s > \mu_k$ **Use:** friction
$\tan\lambda = \tan\theta = \mu$	**Use:** angle of friction, of repose
$a = g(\sin\theta - \mu\cos\theta)$	**Use:** sliding down a rough slope

Inertia, momentum, impulse

- bus starts: fall back; brakes: fall forward; turns: lean out
- mangoes fall when shaken; long jumper's run-up
- jumping from a running bus: fall **forward**, in the bus's direction
- a fan stops: friction, not a broken first law
- longer stopping time → smaller force: straw round china, catching a ball with the hands drawn back, sand pit, air bags
- hard road tires the feet: same Δp in a shorter time

Newton's laws

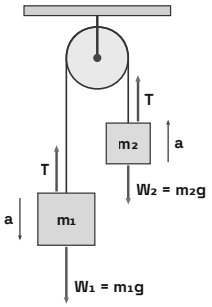
- **first law** gives the **meaning of force** (what changes motion) and of inertia
- **second law** ($F \propto (mv-mu)/t \to F = kma$; the newton chosen so $k = 1$)
- second law is the **real law**: $F = 0$ gives $a = 0$ (first law); with it, the third law gives momentum conservation
- **third law pairs**
  - act on **different bodies** → never cancel
  - same instant, same kind of force
  - book on a table: weight and reaction are not a pair (both on the book); angle between action and reaction 180°
- uses: swimming, rowing, rockets and jet engines (gas pushed back, plane pushed forward; no air needed), gun recoil, walking

Conservation of momentum from the third law

1. during contact Δt, A on B: $F_{12} = \frac{m_2v_2-m_2u_2}{\Delta t}$; B on A: $F_{21} = \frac{m_1v_1-m_1u_1}{\Delta t}$
2. third law $F_{12} = -F_{21}$
3. → $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Applications

- **horse and cart**: horse $H - T = ma$; cart $T - f = m'a$; add → a; the cart moves because the ground pushes the horse forward more than the cart pulls it back



Masses $m_1 > m_2$ over a light frictionless pulley; tension T in both parts.

- **pulley**: $m_1g - T = m_1a$; $T - m_2g = m_2a$; add → a; then $T = m_2(g+a)$
- **on a slope**: m_1 on a smooth slope θ, m_2 hanging: $a = \frac{(m_2-m_1\sin\theta)g}{m_1+m_2}$, $T = \frac{m_1m_2g(1+\sin\theta)}{m_1+m_2}$
- **lift**
  - rest or steady speed: $R = mg$
  - accelerating up: $m(g+a)$, heavier
  - accelerating down: $m(g-a)$, lighter
  - free fall: $R = 0$, weightless

Moments and equilibrium

- clockwise and anticlockwise moments; **principle of moments**: in equilibrium, the sum of clockwise moments = sum of anticlockwise moments about any point
- verify: meter rule on a knife edge, weights hung, balance, compare the two sums
- center of gravity of a loaded rod: take moments about one end, $x = \sum m_i x_i / \sum m_i$
- like parallel forces add; unlike subtract
- couple: $\tau = F(OL + OM) = F \times LM$, the same about any point; a couple turns, never moves a body along
- couple on a wheel: arm = **diameter** (forces at opposite ends); 100 N each on 0.5 m radius through 30°: $W = 100 \times 1 \times \pi/6 = 52.3$ J
- equilibrium: $\sum \vec{F} = 0$ (no linear acceleration) and $\sum \tau = 0$ (no turning)

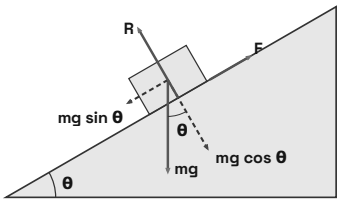
Friction

- cause: interlocking of bumps (old view); **adhesion** at tiny contact points (modern)
- static friction self-adjusts up to limiting; kinetic slightly less, nearly steady

Laws

1. opposes relative motion
2. depends on the surfaces' nature and roughness
3. acts along the surfaces
4. limiting friction independent of area
5. kinetic friction nearly independent of speed
6. limiting friction $\propto$ normal reaction

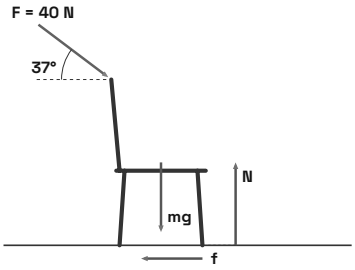
- rolling $\ll$ sliding: wheels, ball bearings; lubricants make surfaces slide easily
- **necessary**: walking, writing, holding, brakes, knots; **evil**: heat, wear, wasted fuel $\rightarrow$ reduce by oil, polish, bearings, streamlining
- friction is non-conservative



Block on a slope θ: $mg \sin\theta$ down it, $mg \cos\theta$ into it, R and F.

Angle of repose = angle of friction

1. $R = mg \cos\theta$; just sliding $F = mg \sin\theta$
2. $F/R = \tan\theta = \mu$
3. also $\tan\lambda = F/R = \mu \rightarrow \theta = \lambda$



Free-body diagram: weight mg down, normal force N up, push F at 37° below the horizontal, friction f back; $N = mg + F \sin 37°$.

- measure: raise a board until a block just slides; read θ; $\mu = \tan\theta$
- stopping on a level: $v^2 = 2\mu g s$ (14 m/s in 40 m: $\mu = 0.25$)

COMMON MISTAKES

✗ action and reaction cancel
✓ they act on **different** bodies.

✗ weight and the table's push are a third-law pair
✓ both act on the book; the pair is book on table, table on book.

✗ a moving body needs a force
✓ only to **change** its velocity.

✗ friction depends on area
✓ limiting friction doesn't.

✗ apparent weight less going up
✓ **more** while accelerating upward.

✗ couple's arm = radius
✓ the distance between the forces.

✗ impulse is a force
✓ force × time, a change of momentum.