Unit 3 · Kinematics

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Mechanics (Units 1–8 together): 26 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** an MCQ on a long jumper as a projectile, and an 8-mark long question on a velocity–time graph (acceleration, then force, friction and distance with Unit 4)
- **MCQ (1 mark)**
  - average speed over thirds of a distance
  - $S_2 = 3S_1$ in equal times from rest
  - $s \propto t \to a = 0$
  - minute hand, half a lap: displacement vs distance
  - bullet losing half its speed in a block
  - trains on parallel tracks
  - dropped together → land together
  - distance, displacement, deceleration from graphs
- **Short (5 marks, in parts)**
  - average speed and velocity; laps of a track
  - v and a from $x(t)$ by differentiating
  - why a graph can't be right
  - parabolic path (horizontal); range at 30° → at 45°
  - javelin at 32–36°; same range at θ and $90° - \theta$
  - bomb released early; fired and dropped together
- **Long (8 marks)**
  - derive T, H, R at an angle; $R_{\max} = 4H$
  - parabolic path; equations of motion from a graph
  - sketch a run's v–t graph (speed up, steady, slow down); areas give the distance

Key terms

Rest and motion: no change, or change, of position with time **relative to** a chosen frame.

Distance: length of the path; scalar; never negative.

Displacement: change of position, start to end, straight; vector.

Speed: distance ÷ time; **velocity**: displacement ÷ time.

Instantaneous velocity: $v = ds/dt$, slope of the s–t graph.

Acceleration: rate of change of velocity, $a = dv/dt$; m/s^2.

Uniform acceleration: equal changes of velocity in equal times.

Relative velocity of A with respect to B: velocity of A as seen from B, $\vec{v}_A - \vec{v}_B$.

Projectile: a body moving under gravity alone; its path is its **trajectory**.

Rectilinear motion: along a straight line, the simplest motion.

Formulas

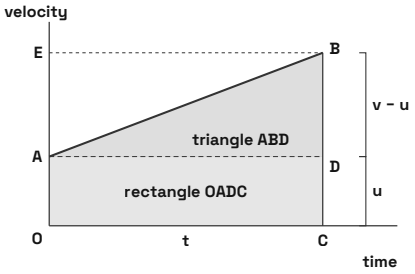
$v = u + at$	u start, v end velocity **Use:** uniform acceleration
$s = ut + \frac{1}{2}at^2$	**Use:** displacement after t
$v^2 = u^2 + 2as$	**Use:** no time given
$s = \frac{1}{2}(u + v)t$	**Use:** average velocity × time
$s_{n\text{th}} = u + \frac{a}{2}(2n - 1)$	**Use:** in the nth second
under gravity: $a = g$	up negative if down positive **Use:** free fall
$y = \frac{g}{2u^2}x^2,\ T = \sqrt{\frac{2h}{g}},\ R = u\sqrt{\frac{2h}{g}}$	from height h, y down **Use:** fired horizontally
$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$	**Use:** path at angle θ
$T = \frac{2u\sin\theta}{g},\ H = \frac{u^2\sin^2\theta}{2g}$	**Use:** time of flight, greatest height
$R = \frac{u^2\sin 2\theta}{g}$	$R_{\max} = u^2/g$ at 45° **Use:** range
$v = \sqrt{a^2 + b^2 - 2ab\cos\theta}$	velocities a, b at θ **Use:** relative velocity

Graphs

- s–t: slope = velocity; straight line = uniform velocity; curve = acceleration
- v–t: slope = acceleration; **area under it = displacement** (below the axis negative); distance = areas without signs

A v–t graph is always

- one connected line, one value at each instant
- forward in time, never turning back
- never vertical (that would be infinite acceleration)
- straight for uniform acceleration, curved when the acceleration changes



OA = u, BC = v, OC = t, BD = $v - u$. Slope of AB = a; area OABC = s.

Equations from the graph

1. slope: $a = \frac{BD}{AD} = \frac{v-u}{t} \rightarrow v = u + at$
2. trapezium: $s = \frac{1}{2}(u + v)t$
3. rectangle + triangle: $s = ut + \frac{1}{2}t(v - u) = ut + \frac{1}{2}at^2$
4. put $t = (v - u)/a$ in $2 \rightarrow v^2 = u^2 + 2as$
5. nth second: $s_n - s_{n-1} \rightarrow u + \frac{a}{2}(2n - 1)$

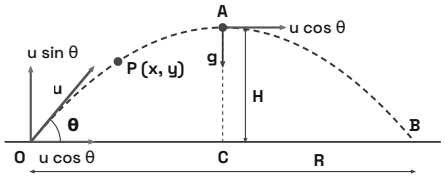
- by calculus: integrate $dv = a\,dt$, $ds = v\,dt$
- slowing: $a \rightarrow -a$
- from rest, distances in successive equal times 1 : 3 : 5 …; so $S_2 = 3S_1$
- speed changes, so acceleration and velocity need not point the same way; zero velocity can have acceleration (top of a throw)

Relative velocity

- same direction: subtract (15 − 10 = 5 m/s); opposite: add
- at an angle: reverse $\vec{v}_B$, add by the parallelogram law
- swimmer: $\vec{v}_{\text{ground}} = \vec{v}_{\text{water}} + \vec{v}_{\text{river}}$
- rain falling at v_r, person walking at v_p: rain seems to come at $\tan\theta = v_p/v_r$ from the vertical → hold the umbrella **forward** by θ; $v = \sqrt{v_r^2 + v_p^2}$

Projectiles

- two independent motions: horizontal $a_x = 0$ (steady $u\cos\theta$); vertical $a_y = g$ down



Fired from O at u, angle θ: $u\cos\theta$ across, $u\sin\theta$ up; at P, x across, y up.

Fired horizontally (height h)

1. $x = ut \rightarrow t = x/u$
2. $y = \frac{1}{2}gt^2 = \frac{g}{2u^2}x^2$: $y = kx^2$, a **parabola**
3. $T = \sqrt{2h/g}$: same as dropped → **land together**
4. at landing $v = \sqrt{u^2 + 2gh}$; $\tan\alpha = gt/u$ below the horizontal

Fired at θ

1. $x = (u\cos\theta)t$; $y = (u\sin\theta)t - \frac{1}{2}gt^2$; remove $t \rightarrow y = bx - cx^2$, a parabola
2. back to $y = 0$: $T = 2u\sin\theta/g$
3. top: $v_y = 0 \rightarrow 0 = u^2\sin^2\theta - 2gH \rightarrow H = u^2\sin^2\theta/2g$
4. $R = u\cos\theta \times T = \frac{u^2\sin 2\theta}{g}$
5. $\sin 2\theta = 1 \rightarrow \theta = 45°$, $R_{\max} = u^2/g$

- 45°: $H = u^2/4g \rightarrow R_{\max} = 4H$; thrown straight up $H = u^2/2g \rightarrow R_{\max} = 2H$
- θ and $90° - \theta$: same range ($\sin(180° - 2\theta) = \sin 2\theta$); the higher stays up longer
- least speed for a range: launch at 45°
- range at 30° is 2 km → at 45°: $2/\sin 60° = 4/\sqrt{3}$ km
- $v = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$
- **bomb** released early: it keeps the plane's forward speed while falling
- **javelin** at 32–36°: released about 2 m up, glides in air, thrown faster at a lower angle
- height and range depend only on speed and angle (and g), not mass

COMMON MISTAKES

✗ average speed = average of speeds
✓ total distance ÷ total time (10, 20, 60 km/h over thirds → 18 km/h).

✗ displacement = distance
✓ equal only in one direction without turning back.

✗ half a lap of radius 25 m in 25 s: average velocity π m/s
✓ 2r/t = **2 m/s**.

✗ zero velocity means zero acceleration
✓ at the top of a throw $a = g$.

✗ a horizontally fired body lands later
✓ **same time** as a dropped one.

✗ $R_{\max} = 2H$ and $4H$ clash
✓ 4H at 45°; 2H against a throw straight up.

✗ area under the s–t graph is displacement
✓ area under the **v–t** graph.