Unit 2 · Vectors

STUDY NOTES

WHAT NEB'S GRADE 11 EXAM ASKS

- **CDC's grid (2077):** Mechanics (Units 1–8 together): 26 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **MCQ (1 mark)**
  - resultant of two equal vectors; of 4 N and 3 N
  - cross product of parallel vectors
  - position vector of (3, 2, 5)
  - which is a vector (field, not current)
  - fly across a room: $\sqrt{a^2 + b^2 + c^2}$
  - equal components → 45°
  - angle between $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$
  - parallel vectors' λ; projection of A on B
  - A perpendicular to B and C → along $\vec{B} \times \vec{C}$
- **Short (5 marks, in parts)**
  - define negative, component, unit vector; a unit vector along $\hat{i} + \hat{j}$
  - state triangle, parallelogram, polygon laws
  - $|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B} \rightarrow 45°$; angle of $2\hat{i} + 3\hat{j}$
  - $\hat{i} \times \hat{j}$ vs $\hat{j} \times \hat{i}$; $\vec{A} + \vec{B} = 0$?
  - is anything with direction a vector?; area by $\vec{A} \times \vec{B}$
  - equal vectors at 120°; derive magnitude and direction of the resultant

Key terms

Scalar: magnitude only (mass, time, speed, work, energy).

Vector: magnitude **and** direction, **and** adds by the triangle law (displacement, velocity, force); current has a direction but is a scalar.

Unit vector: magnitude 1, direction only; $\hat{n} = \vec{A}/A$.

Position vector: from the origin to P(x, y, z): $x\hat{i} + y\hat{j} + z\hat{k}$.

Equal vectors: same magnitude and direction.

Negative vector: same magnitude, opposite direction.

Parallel / antiparallel: same way / opposite ways.

Collinear: along one line or parallel lines; **coplanar**: in one plane.

Null vector: zero magnitude, no particular direction; $\vec{A} + (-\vec{A})$.

Resolution: splitting a vector into components; **rectangular** when at right angles.

Formulas

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$	θ angle between P and Q **Use:** parallelogram law
$\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$	α from P **Use:** direction of R
$A_x = A\cos\theta, A_y = A\sin\theta$	θ from the x-axis **Use:** components
$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$	$\tan\theta = A_y/A_x$ **Use:** magnitude, direction
$\vec{A} \cdot \vec{B} = AB\cos\theta$	$= A_xB_x + A_yB_y + A_zB_z$, a scalar **Use:** dot product, work $\vec{F} \cdot \vec{s}$
$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$	$\hat{n}$ by the right-hand rule **Use:** cross product

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$	**Use:** angle between vectors
$\dfrac{\vec{A}\cdot\vec{B}}{B}$	**Use:** projection of A on B
$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	**Use:** cross product from components

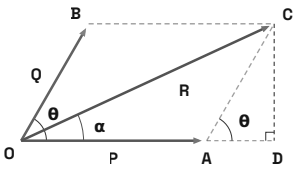
Representing vectors

- an arrow: length to scale = magnitude; way it points = direction; tail and head
- written $\vec{A}$ or bold **A**; magnitude $|\vec{A}|$ or A
- moved without turning: the same vector
- $\hat{i}, \hat{j}, \hat{k}$: unit vectors along x, y, z

Adding vectors

Triangle law: two vectors as two sides of a triangle taken in order, head to tail → the third side, from the first's tail to the second's head, is the resultant

Parallelogram law: two vectors from one point as adjacent sides of a parallelogram → the diagonal from that point is the resultant



$\vec{P}$ = OA, $\vec{Q}$ = OB at angle θ; R = OC; CD perpendicular to OA extended.

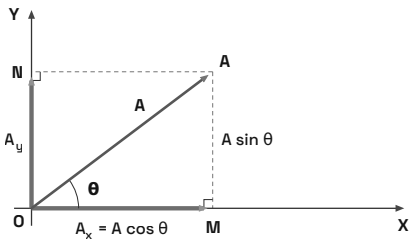
Derivation

1. AD $= Q\cos\theta$, CD $= Q\sin\theta$
2. $OC^2 = (OA + AD)^2 + CD^2$
3. $R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2 = P^2 + Q^2 + 2PQ\cos\theta$
4. $\tan\alpha = \frac{CD}{OD} = \frac{Q\sin\theta}{P+Q\cos\theta}$

- $\theta = 0°$: $R = P + Q$, largest; $\theta = 90°$: $R = \sqrt{P^2 + Q^2}$; $\theta = 180°$: $R = |P - Q|$, smallest, along the larger
- so $|P - Q| \le R \le P + Q$: 4 N and 3 N give 1 N to 7 N
- equal vectors at 120°: $R = \sqrt{P^2 + P^2 - P^2} = P$
- two equal vectors: $R = 2P\cos\frac{\theta}{2}$, $\alpha = \frac{\theta}{2}$

Polygon law: many vectors head to tail; the side closing the polygon from the first tail to the last head is the resultant; a closed polygon → resultant zero

Subtraction: $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$



$\vec{A}$ at θ to the x-axis: $A_x = A\cos\theta$ along OX, $A_y = A\sin\theta$ along OY.

- add many vectors: add their x parts and y parts, then $R = \sqrt{R_x^2 + R_y^2}$
- equal components → 45° to the x-axis

Products

Dot product (a scalar)

- $\theta = 90° \to 0$: **orthogonality** test
- $\vec{A}\cdot\vec{A} = A^2$; commutative, distributive
- $\hat{i}\cdot\hat{i} = 1, \hat{i}\cdot\hat{j} = 0$

Cross product (a vector, perpendicular to both)

- $\theta = 0°$ or $180° \to \vec{0}$: **parallelism** test
- $\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$ (not commutative); distributive
- $|\vec{A}\times\vec{B}| = AB\sin\theta$ = area of the parallelogram (base A, height $B\sin\theta$)
- $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$; reversed order → minus; $\hat{i}\times\hat{i} = \vec{0}$
- $|\vec{A}\times\vec{B}| = \vec{A}\cdot\vec{B} \to \tan\theta = 1, \theta = 45°$
- uses: torque $\vec{r}\times\vec{F}$, work $\vec{F}\cdot\vec{s}$

COMMON MISTAKES

✗ anything with a direction is a vector
✓ it must also add by the triangle law (current doesn't).

✗ resultant of 4 N and 3 N is 7 N
✓ anything from 1 N to 7 N, by the angle.

✗ $\hat{i}\times\hat{j} = \hat{j}\times\hat{i}$
✓ $\hat{j}\times\hat{i} = -\hat{k}$.

✗ cross product of parallel vectors is a unit vector
✓ the **null vector**.

✗ α measured from Q
✓ from **P**, the vector the angle starts at.

✗ unit vector of $2\hat{i} + \hat{j} - 2\hat{k}$ is itself
✓ divide by its magnitude 3.