Unit 23 · Dynamics

HARDER PRACTICE QUESTIONS

Full marks 28

Write your answers on separate paper. Show your working. Take $g = 9.8\ \text{m/s}^2$.

1 Speeding up, slowing down [6]

A particle starts from O and moves along a line with velocity $v = t^2 - 5t + 4$ m/s after t seconds.

a. When is it at rest? [1]

b. During the first 5 s, when is it speeding up and when is it slowing down? [2]

c. Find its position x at any time, and the distance it travels in the first 5 s. [3]

2 Velocity given by position [5]

A particle moves along a line so that its velocity v (m/s) and its position x (m) are always related by $v^2 = 36 - 9x^2$.

a. Show that its acceleration is $-9x$. [2]

b. Between which points does it move? Where is it fastest, and how fast is it there? What is its acceleration at that point? [2]

c. Sunil takes the acceleration to be $\dfrac{dv}{dx}$. Show that this gives a different value at $x = 1$, when the particle is moving in the positive direction. [1]

3 A bus between two stops [6]

A bus leaves one stop from rest and accelerates uniformly at $1\ \text{m/s}^2$ until it reaches 12 m/s. It keeps that speed, then brakes uniformly at $2\ \text{m/s}^2$ to stop exactly at the next stop, 600 m from the first. (The numbers are made up.)

a. Sketch its velocity–time graph and find the time between the stops. [3]

b. If the bus had no top speed, accelerating at $1\ \text{m/s}^2$ and braking at $2\ \text{m/s}^2$ as before, what is the least time it could take between the stops, and its greatest speed then? [3]

4 Rain on a roof [6]

A roof slopes down from its ridge to its edge, a horizontal distance b away. A raindrop slides from rest at the ridge straight down the roof, which is smooth and inclined at θ to the horizontal.

a. Show that the drop reaches the edge after a time t with $t^2 = \dfrac{4b}{g \sin 2\theta}$. [3]

b. For $b = 4.9$ m, which angle gets the rain off the roof soonest? Find that time. [2]

c. Show that roofs at 30° and at 60° take the same time, and explain why a steeper roof is not always quicker. [1]

5 A stone past a window [5]

A stone is dropped from rest from somewhere above a window. It takes 0.10 s to pass the window, which is 2.0 m tall.

a. Find its speed as it reaches the top of the window. [2]

b. How far above the top of the window was it dropped? [2]

c. How fast is it moving as it leaves the bottom of the window? [1]