Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Speeding up, slowing down

ANSWER HA · 6

Tests: speeding up means v and a have the same sign (not just $a > 0$); distance needs the turning points, with x found by integrating v (Unit 19). The tempting wrong answer in b. is "speeding up for $t > 2.5$, where $a > 0$", and in c. the displacement $|x(5)|$ for the distance.

a. $v = (t-1)(t-4) = 0$: at $t = \mathbf{1\ s}$ **and** $t = \mathbf{4\ s}$.

b. $a = 2t - 5$: negative before 2.5 s, positive after. v is positive before 1 s, negative from 1 s to 4 s, positive after 4 s.

Same signs (speeding up): $\mathbf{1 < t < 2.5}$ (both negative) and $\mathbf{4 < t \leq 5}$ (both positive).

Opposite signs (slowing down): $\mathbf{0 \leq t < 1}$ and $\mathbf{2.5 < t < 4}$.

c. $x = \frac{t^3}{3} - \frac{5t^2}{2} + 4t$ (the constant is 0, since $x = 0$ at $t = 0$).

$x(1) = \frac{11}{6}$, $x(4) = -\frac{8}{3}$, $x(5) = -\frac{5}{6}$.

Distance $= \frac{11}{6} + \left(\frac{11}{6} + \frac{8}{3}\right) + \left(-\frac{5}{6} + \frac{8}{3}\right) = \frac{11 + 27 + 11}{6} = \frac{49}{6} \approx \mathbf{8.17\ m}$ (the displacement is only $-\frac{5}{6}$ m).

MARKING SCHEME

- ✓ At rest at 1 s and 4 s.
- ✓ Speeding up on $1 < t < 2.5$ and $4 < t \leq 5$ (v and a the same sign).
- ✓ Slowing down on $0 \leq t < 1$ and $2.5 < t < 4$.
- ✓ $x = \frac{t^3}{3} - \frac{5t^2}{2} + 4t$.
- ✓ $x(1) = \frac{11}{6}$, $x(4) = -\frac{8}{3}$, $x(5) = -\frac{5}{6}$.
- ✓ Distance $\frac{49}{6} \approx 8.17$ m.

2 Velocity given by position

ANSWER A · 5

Tests: $a = v\frac{dv}{dx}$ when v is known in terms of x, and reading the motion from it. The tempting wrong answer is Sunil's, $a = \frac{dv}{dx}$.

a. $a = v\frac{dv}{dx} = \frac{1}{2}\frac{d}{dx}(v^2) = \frac{1}{2}(-18x) = \mathbf{-9}x$.

b. $v^2 \geq 0$ needs $9x^2 \leq 36$: it moves between $x = \mathbf{-2}$ **and** $x = \mathbf{2}$, where it comes to rest and turns back. v^2 is greatest at $x = 0$: speed **6 m/s**, and there the acceleration is **0** (fastest exactly where the acceleration vanishes).

c. $v = \sqrt{36 - 9x^2}$, $\frac{dv}{dx} = \frac{-9x}{\sqrt{36 - 9x^2}}$: at $x = 1$, $\frac{-9}{\sqrt{27}} \approx -1.73$, but $a = -9$ m/s^2. $\frac{dv}{dx}$ is how fast v changes per meter, not per second.

MARKING SCHEME

- ✓ $a = v\frac{dv}{dx} = \frac{1}{2}\frac{d}{dx}(v^2)$.
- ✓ $a = -9x$.
- ✓ Between $x = -2$ and $x = 2$.
- ✓ Fastest at $x = 0$, 6 m/s, acceleration 0.
- ✓ $\frac{dv}{dx} = -1.73$ at $x = 1$, not -9.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

3 A bus between two stops

ANSWER A · 6

Tests: planning a motion in stages from its velocity–time graph, and least time (the graph a triangle). The tempting wrong answer is to use one equation of motion for the whole trip, as if the acceleration were the same throughout.

a. The graph: a straight rise from 0 to 12 m/s, a level line, a steeper straight fall to 0.

Speeding up: $\frac{12}{1} = 12$ s and $\frac{12^2}{2\times 1} = 72$ m. Braking: $\frac{12}{2} = 6$ s and $\frac{12^2}{2\times 2} = 36$ m. At 12 m/s:

$600 - 72 - 36 = 492$ m in $\frac{492}{12} = 41$ s.

Time $= 12 + 41 + 6 =$ **59 s**. (Check: the trapezium's area is $\frac{1}{2}(59 + 41) \times 12 = 600$ m.)

b. Least time: speed up the whole time until braking must start; the graph is a triangle. With top speed V: $\frac{V^2}{2\times 1} + \frac{V^2}{2\times 2} = 600$, so $\frac{3V^2}{4} = 600$, $V^2 = 800$, $V = \mathbf{20\sqrt{2}} \approx \mathbf{28.3\ m/s}$. Time $= \frac{V}{1} + \frac{V}{2} = 1.5V \approx$ **42.4 s**.

MARKING SCHEME

- ✓ A v–t graph of a rise, a level part and a steeper fall.
- ✓ Speeding up 12 s and 72 m; braking 6 s and 36 m.
- ✓ 41 s at 12 m/s, total 59 s.
- ✓ Least time when the graph is a triangle: $\frac{V^2}{2} + \frac{V^2}{4} = 600$.
- ✓ $V = 20\sqrt{2} \approx 28.3$ m/s.
- ✓ Time $1.5V \approx 42.4$ s.

4 Rain on a roof

ANSWER HA · 6

Tests: two effects that pull opposite ways (a steeper roof is longer but gives a larger acceleration), worked out from $a = g\sin\theta$ and a least value. The tempting wrong answer is "the steeper, the sooner", true for slopes of the same height but not of the same width.

a. The roof's length is $l = \frac{b}{\cos\theta}$ and the acceleration down it $g\sin\theta$. From rest, $\frac{b}{\cos\theta} = \frac{1}{2}g\sin\theta\, t^2$, so $t^2 = \frac{2b}{g\sin\theta\cos\theta} = \frac{4b}{g\sin 2\theta}$.

b. t is least when $\sin 2\theta$ is greatest, 1, at $\theta = \mathbf{45}^\circ$: $t = \sqrt{\frac{4\times 4.9}{9.8}} = \sqrt{2} \approx$ **1.41 s**.

c. $\sin 60^\circ = \sin 120^\circ$, so both give $t = \sqrt{\frac{2}{\sin 60^\circ}} \approx 1.52$ s. Going steeper than 45°, the extra length of the roof costs more time than the larger acceleration saves.

MARKING SCHEME

- ✓ The length $\frac{b}{\cos\theta}$ and the acceleration $g\sin\theta$.
- ✓ $\frac{b}{\cos\theta} = \frac{1}{2}g\sin\theta\, t^2$.
- ✓ Simplified with $2\sin\theta\cos\theta = \sin 2\theta$.
- ✓ Least at 45° ($\sin 2\theta = 1$).
- ✓ $t = \sqrt{2} \approx 1.41$ s.
- ✓ $\sin 60^\circ = \sin 120^\circ$: the same time, 1.52 s; length and acceleration both grow with θ.

5 A stone past a window

ANSWER

Tests: a stage of a fall that does not start from rest, and choosing the unknown (the speed at the window's top). The tempting wrong answer takes the average speed, $\frac{2.0}{0.10} = 20$ m/s, as the speed at the top, which gives 20.4 m in b.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

a. Down positive, from the top of the window: $2.0 = 0.10u + \frac{1}{2}(9.8)(0.10)^2 = 0.10u + 0.049$, so $u = \mathbf{19.51\ m/s}$.

b. From rest to that speed: $u^2 = 2gd$, so $d = \frac{19.51^2}{2 \times 9.8} \approx \mathbf{19.4\ m}$ above the top of the window.

c. $v = 19.51 + 9.8 \times 0.10 \approx \mathbf{20.5\ m/s}$. (The average of 19.51 and 20.49 is 20 m/s, the window's average speed, as $s = \frac{1}{2}(u + v)t$ says.)

MARKING SCHEME

✓ $2.0 = 0.10u + 4.9(0.10)^2$.
✓ $u = 19.51$ m/s.
✓ $u^2 = 2gd$.
✓ $d \approx 19.4$ m.
✓ $v \approx 20.5$ m/s.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	0	16	12	**28**

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking