Unit 23 · Dynamics

ANSWER KEY · FOR TEACHERS

Full marks 35

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 3

1. particle
2. displacement
3. distance
4. velocity
5. retardation
6. smooth

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: Only if it never turns back; when it does, the distance is larger (a ball thrown up and caught has displacement 0).

iii. ✗ **False**: Its velocity is zero there, but its acceleration is still $g = 9.8$ m/s^2 downward.

iv. ✗ **False**: Both have $a = g \sin\theta$, whatever the mass.

v. ✓ **True**

vi. ✓ **True**

Each carries 0.5 mark.

3 Multiple choice

i. **b.** $v\dfrac{dv}{dx}$ — K · 1

$a = \dfrac{dv}{dt} = \dfrac{dv}{dx}\cdot\dfrac{dx}{dt} = v\dfrac{dv}{dx}$, by the chain rule.

ii. **b.** 4 s — U · 1

$t = \dfrac{2u}{g} = \dfrac{2\times 19.6}{9.8} = 4$ s. 2 s is the time to the top only.

iii. **c.** 40 m — U · 1

From rest, $s = \dfrac{1}{2}at^2$ goes as t^2: twice the time, four times the distance. ($a = 5$ m/s^2, and $\dfrac{1}{2}\times 5\times 16 = 40$.) 30 m is what it covers in the 3rd and 4th seconds.

iv. **c.** 5 m/s^2 — U · 1

$0 = 20^2 - 2a\times 40$, so $a = \dfrac{400}{80} = 5$ m/s^2. 10 m/s^2 forgets the 2 in $2as$.

v. **d.** 10 m — A · 1

It stops at $t = 3$ and turns back. $x = 6t - t^2$ from its start: $x(3) = 9$, $x(4) = 8$. Distance $= 9 + 1 = 10$ m; 8 m is the displacement.

vi. **c.** 122.5 m — HA · 1

If the fall takes T s, $\dfrac{1}{2}gT^2 - \dfrac{1}{2}g(T-1)^2 = g\left(T - \dfrac{1}{2}\right) = 44.1$, so $T = 5$ s and $h = 4.9\times 25 = 122.5$ m. 99.2 m takes $T = 4.5$ s, the middle of the last second.

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4 Spot the error

MARKING SCHEME U · 2

✓ The mistake: $x(5) - x(0)$ is the displacement; the particle turns back at $t = 3$ ($v = 0$).

✓ Distance $= 9 + 4 = 13$ m.

5 Short questions

a. **ANSWER** A · 5

a. $v = 6t^2 - 30t + 36 = 6(t-2)(t-3) = 0$: at t **= 2 s and** t **= 3 s**.

b. $a = 12t - 30$: **-6 m/s²** at 2 s, **6 m/s²** at 3 s.

c. $x(0) = 5$, $x(2) = 16 - 60 + 72 + 5 = 33$, $x(3) = 54 - 135 + 108 + 5 = 32$. Distance $= 28 + 1 =$ **29 m** (the displacement is 27 m).

MARKING SCHEME

✓ $v = 6t^2 - 30t + 36 = 6(t-2)(t-3)$.

✓ At rest at $t = 2$ s and $t = 3$ s.

✓ $a = 12t - 30$: -6 m/s² and 6 m/s².

✓ $x(0) = 5$, $x(2) = 33$, $x(3) = 32$.

✓ Distance $= 28 + 1 = 29$ m.

b. **ANSWER** A · 5

a. With $s = ut + \frac{1}{2}at^2$:

From 1 s to 1.5 s: $\left(1.5u + \frac{1}{2}a(2.25)\right) - \left(u + \frac{1}{2}a\right) = 0.5u + 0.625a = 25$, so $u + 1.25a = 50$.

In the eleventh second (from 10 s to 11 s): $\left(11u + \frac{121}{2}a\right) - \left(10u + 50a\right) = u + 10.5a = 198$.

Subtracting: $9.25a = 148$, so a **$= 16$ m/s²**, and u **$= 30$ m/s**.

b. $v = u + at = 30 + 16 \times 11 =$ **206 m/s**.

MARKING SCHEME

✓ The half second: $0.5u + 0.625a = 25$.

✓ The eleventh second: $u + 10.5a = 198$.

✓ $a = 16$ m/s² and $u = 30$ m/s.

✓ b. $v = u + at$ with $t = 11$.

✓ $v = 206$ m/s.

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6 Long question

ANSWER A · 8

a. Along the plane the acceleration is $g \sin\theta$ (the weight's part along it; the rest is balanced by the plane). The plane's length is $l = \dfrac{h}{\sin\theta}$.

From rest, $l = \dfrac{1}{2}(g\sin\theta)t^2$, so $t^2 = \dfrac{2h}{g\sin^2\theta}$ and $t = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}}$.

$v^2 = 2(g\sin\theta)l = 2(g\sin\theta)\dfrac{h}{\sin\theta} = 2gh$, so $v = \sqrt{2gh}$.

b. $\sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{9.8}{9.8}} = 1$ s. At 30°: $t = \dfrac{1}{0.5} = \mathbf{2\ s}$. At 60°: $t = \dfrac{1}{\sin 60^\circ} = \dfrac{2}{\sqrt{3}} \approx \mathbf{1.15\ s}$. Speed at the foot of both: $\sqrt{2 \times 9.8 \times 4.9} = \mathbf{9.8\ m/s}$.

c. Dropped: $4.9 = 4.9t^2$, **1 s**. All three arrive at 9.8 m/s; the steeper the path, the sooner.

MARKING SCHEME

- ✓ a. The acceleration along the plane is $g \sin\theta$.
- ✓ The length is $\dfrac{h}{\sin\theta}$.
- ✓ $l = \dfrac{1}{2}(g\sin\theta)t^2$ gives $t = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}}$.
- ✓ $v^2 = 2(g\sin\theta)l = 2gh$.
- ✓ b. $\sqrt{\dfrac{2h}{g}} = 1$ s, so 2 s at 30°.
- ✓ $\dfrac{2}{\sqrt{3}} \approx 1.15$ s at 60°.
- ✓ 9.8 m/s at the foot of both.
- ✓ c. 1 s; the same speed, sooner on a steeper path.

7 Think

ANSWER HA · 3

Less. The 3.0 s is the fall **and** the sound's climb back, so the pebble fell for less than 3.0 s.

One correction: sound takes about $\dfrac{44.1}{340} \approx 0.13$ s to climb 44.1 m, so the fall took about $3.0 - 0.13 = 2.87$ s, and the depth is about $4.9 \times 2.87^2 \approx \mathbf{40\ m}$. (Solving $\sqrt{\dfrac{2h}{9.8}} + \dfrac{h}{340} = 3.0$ exactly gives 40.7 m.)

MARKING SCHEME

- ✓ Less: the 3.0 s includes the sound's time.
- ✓ The sound's time about 0.13 s, so the fall about 2.87 s.
- ✓ Depth about 40 m.

8 Activity: Measure g with a falling coin

(not marked)

WHAT STUDENTS SHOULD FIND

From 1 m the fall takes about 0.45 s, from 2 m about 0.64 s, so $\dfrac{2h}{t^2}$ comes out near 9.8 m/s² at every height. With a hand-held stopwatch the delay in pressing start and stop is a good part of a fall that lasts under a second, so the results scatter widely; slow-motion video, counting frames, gets within a few percent. A coin is heavy and small enough for air resistance to be ignored over 2 m.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	4	8	19	4	**35**

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