Unit 22 · Statics

HARDER PRACTICE QUESTIONS

Full marks 31

Write your answers on separate paper. Show your working.

1 When the resultant turns square [6]

Two forces P and Q act at a point with an angle θ between them, and their resultant is R. When Q is doubled (same direction), the new resultant is at right angles to P.

a. Prove that $R = Q$. [2]

b. Show that this can happen only if $P < 2Q$. [1]

c. Suppose instead that doubling Q turns the resultant through a right angle: the new resultant is at right angles to R. Show that then $\cos\theta = -\dfrac{P^2 + 2Q^2}{3PQ}$, and that this can happen only if $Q < P < 2Q$. [3]

2 Lifting with two slings [5]

A crane's hook holds a load of weight W by two slings, the left one at α to the vertical and the right one at β, on the other side. The pull in each sling is equal and opposite to the part of W along that sling when W is resolved along the two sling lines.

a. For $\alpha = 30^\circ$ and $\beta = 60^\circ$, find the pull in each sling. Which carries more, the steeper sling or the flatter one? [2]

b. Keep $\beta = 60^\circ$. Show that, whatever α is, the left sling pulls with at least $W \sin 60^\circ \approx 0.87W$. For which α is its pull least? [1]

c. Each sling is safe up to a pull of $0.9W$. With $\beta = 60^\circ$, for which angles α is the lift safe? [2]

3 Holding a box on a slope [5]

A box of weight 200 N rests on a smooth slope at 30° to the horizontal. The slope pushes on it only at right angles to its surface (the reaction N). Gopal holds the box still with a push P.

a. Find P if he pushes along the slope, and the reaction N then. [2]

b. Find P if he pushes horizontally, towards the slope. [2]

c. Find N for the horizontal push, and say why it differs from a. [1]

4 A force from the resultant it must give [5]

Forces of 10 N towards the east and 20 N towards the north act at a point, with a third force F at an angle θ to the east (measured from the east towards the north).

a. Take $F = 25$ N. Find the direction θ that makes the resultant of the three point due north, and the resultant then. [3]

b. For which values of F are there two different directions θ that make the resultant point due north? [2]

5 The raft on the canal [6]

A raft floats on a straight irrigation canal. Nirmala, on one bank, pulls it with a rope at 300 N, the rope making 30° with the bank. Dawa, on the other bank, pulls with a second rope so that the raft moves straight along the canal, that is, the resultant of the two pulls is along the bank.

a. Find the least pull Dawa can use, and its direction. What is then the resultant along the canal? [3]

b. Find Dawa's pull, and the resultant, if his rope makes 45° with the bank (pointing ahead, as Nirmala's does), and if it makes 30°. [2]

c. Dawa says: "The least pull is the best choice." Is he right? [1]

6 Testing the law in the laboratory [4]

In the unit's project, a class hangs 300 g and 400 g over two pulleys and 500 g straight down, all from one knot. Four groups measure the angle between the two upper strings: 88°, 91°, 93° and 96°. (The readings are made up.)

a. What angle does the parallelogram law predict? [1]

b. For the 88° and 96° readings, find the diagonal of the parallelogram of the two upper pulls (in units of 100 g). [2]

c. One group says: "Nobody got exactly the predicted angle, so the law is wrong." Judge the claim. [1]