Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 When the resultant turns square

ANSWER HA · 6

Tests: turning a condition on the resultant's direction into an equation, and asking when it can hold. The tempting wrong start is to write the new resultant as $\sqrt{P^2 + 4Q^2}$, as if P and $2Q$ were at right angles; it is the new **resultant** that is at right angles to P. In c. the step to see is that the new resultant is the old one plus one more Q, so R, the new resultant and Q make a triangle.

a. The new resultant is at right angles to P when $\tan\alpha$ has no value, that is, when its part along P is zero: $P + 2Q\cos\theta = 0$, so $\cos\theta = -\dfrac{P}{2Q}$.

Then $R^2 = P^2 + Q^2 + 2PQ\cos\theta = P^2 + Q^2 + 2PQ\left(-\dfrac{P}{2Q}\right) = P^2 + Q^2 - P^2 = Q^2$, so $R = Q$.

b. $\cos\theta = -\dfrac{P}{2Q}$ needs $\dfrac{P}{2Q} \le 1$. If $P = 2Q$, $\theta = 180°$ and the doubled Q cancels P exactly, leaving no resultant at all; so a resultant at right angles to P needs $P < 2Q$.

c. Call the new resultant R': $R'^2 = P^2 + 4Q^2 + 4PQ\cos\theta$. The forces P and $2Q$ are P, Q and one more Q, so R' is R and Q added head to tail: R, Q and R' form a triangle, with the right angle between R and R'. By Pythagoras, $Q^2 = R^2 + R'^2$:

$Q^2 = 2P^2 + 5Q^2 + 6PQ\cos\theta$, so $\cos\theta = -\dfrac{2P^2 + 4Q^2}{6PQ} = -\dfrac{P^2 + 2Q^2}{3PQ}$.

This is negative, so it can happen only if $\cos\theta > -1$: $P^2 + 2Q^2 < 3PQ$, that is $(P - Q)(P - 2Q) < 0$, so $Q < P < 2Q$. (At $P = Q$ or $P = 2Q$, $\theta = 180°$ and one of the two resultants is zero.) For example $P = 3$, $Q = 2$: $\cos\theta = -\dfrac{17}{18}$, $\theta \approx 160.8°$.

MARKING SCHEME

- ✓ Perpendicular to P: $P + 2Q\cos\theta = 0$.
- ✓ Put into the parallelogram law: $R^2 = Q^2$, so $R = Q$.
- ✓ $|\cos\theta| \le 1$ and $P = 2Q$ leaves no resultant: $P < 2Q$.
- ✓ R, Q and the new resultant form a right triangle with Q the hypotenuse (or the two resultants' parts multiplied and added to zero).
- ✓ $Q^2 = R^2 + R'^2$ worked out to $\cos\theta = -\dfrac{P^2 + 2Q^2}{3PQ}$.
- ✓ $\cos\theta > -1$ gives $(P - Q)(P - 2Q) < 0$: $Q < P < 2Q$.

2 Lifting with two slings

ANSWER

Tests: resolving along two lines that are not at right angles, and seeing how the shares change as one line moves. The tempting wrong answers are that the two slings share W equally, that the flatter sling (pulling more sideways) carries more, and in c. that a sling nearer the vertical is always safer.

a. W makes α with the left sling and β with the right, on either side of it, so (each part carries the sine of the **other** angle):

left $= \dfrac{W\sin\beta}{\sin(\alpha + \beta)} = \dfrac{W\sin 60°}{\sin 90°} \approx \mathbf{0.87W}$, right $= \dfrac{W\sin 30°}{\sin 90°} = \mathbf{0.5W}$.

The steeper sling, nearer the vertical, carries more. (The two add up to 1.37W, more than W: only as vectors do they make W.)

b. Left pull $= \dfrac{W\sin 60°}{\sin(\alpha + 60°)}$, and $\sin(\alpha + 60°) \le 1$, so it is at least $W\sin 60°$. It is least when $\alpha + 60° = 90°$: $\alpha = \mathbf{30°}$, the slings at right angles.

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c. Left: $\frac{W \sin 60^\circ}{\sin(\alpha + 60^\circ)} \leq 0.9W$ needs $\sin(\alpha + 60^\circ) \geq \frac{0.866}{0.9} = 0.962$, so $74.2^\circ \leq \alpha + 60^\circ \leq 105.8^\circ$: $14.2^\circ \leq \alpha \leq 45.8^\circ$. Right: $\frac{W \sin \alpha}{\sin(\alpha + 60^\circ)}$ is at most about $0.74W$ in that range, safe. So the lift is safe for about $\mathbf{14}^\circ \leq \alpha \leq \mathbf{46}^\circ$. Too near the vertical, the left sling takes nearly all the load (all of it at $\alpha = 0^\circ$); too wide, the two slings pull against each other. **A · 5**

MARKING SCHEME

- ✓ Pulls $\frac{W \sin \beta}{\sin(\alpha + \beta)}$ and $\frac{W \sin \alpha}{\sin(\alpha + \beta)}$: 0.87W and 0.5W.
- ✓ The steeper sling carries more.
- ✓ $\sin(\alpha + 60^\circ) \leq 1$, so at least $W \sin 60^\circ$; least at $\alpha = 30^\circ$.
- ✓ $\sin(\alpha + 60^\circ) \geq 0.962$.
- ✓ $14.2^\circ \leq \alpha \leq 45.8^\circ$, the right sling checked (at most 0.74W).

3 Holding a box on a slope

ANSWER **A · 5**

Tests: choosing the directions to resolve along (along and across the slope), and seeing that a force not along the slope has a part into it. The tempting wrong answer in b. is $200 \sin 30^\circ = 100$ N again, as if any push only had to match the weight's part down the slope.

Resolve along the slope and at right angles to it; the box is in equilibrium, so each sum is zero.

a. Along the slope: $P = 200 \sin 30^\circ = \mathbf{100\ N}$. Across it: $N = 200 \cos 30^\circ = 100\sqrt{3} \approx \mathbf{173\ N}$.

b. A horizontal push makes 30° with the slope, so only $P \cos 30^\circ$ acts up the slope: $P \cos 30^\circ = 200 \sin 30^\circ$, $P = 200 \tan 30^\circ = \frac{200}{\sqrt{3}} \approx \mathbf{115\ N}$.

c. Across the slope, the push also has a part $P \sin 30^\circ \approx 57.7$ N into the slope: $N = 200 \cos 30^\circ + P \sin 30^\circ = \frac{200}{\cos 30^\circ} \approx \mathbf{231\ N}$. Part of the horizontal push is spent pressing the box into the slope, which is why it must be bigger than the push along the slope.

MARKING SCHEME

- ✓ Along the slope: $P = 100$ N.
- ✓ $N = 100\sqrt{3} \approx 173$ N.
- ✓ Horizontal: $P \cos 30^\circ = 200 \sin 30^\circ$.
- ✓ $P = \frac{200}{\sqrt{3}} \approx 115$ N.
- ✓ $N \approx 231$ N, bigger because the horizontal push has a part $P \sin 30^\circ$ into the slope.

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4 A force from the resultant it must give

ANSWER A · 5

Tests: using X and Y backwards, from what the resultant must be to an unknown force, and reading which of two angles really works. The tempting wrong answers are to take only the calculator's $\cos^{-1}$, or to keep both angles of $\cos\theta = -0.4$ without checking that the resultant then points north and not south.

a. Due north means no part towards the east: $10 + 25\cos\theta = 0$, so $\cos\theta = -0.4$: $\theta \approx 113.6°$ or $\theta \approx 246.4°$.

Towards the north the parts add to $20 + 25\sin\theta$. At $113.6°$, $\sin\theta \approx 0.917$: $20 + 22.9 = 42.9$ N. At $246.4°$, $\sin\theta \approx -0.917$: $20 - 22.9 = -2.9$ N, a resultant of 2.9 N due **south**.

So $\theta \approx$ **113.6**°, and the resultant is about **42.9 N due north**.

b. $\cos\theta = -\dfrac{10}{F}$ needs $F \geq 10$; then $F\sin\theta = \pm\sqrt{F^2 - 100}$ and the northward sum is $20 \pm \sqrt{F^2 - 100}$. With + it always points north. With − it points north only while $\sqrt{F^2 - 100} < 20$, $F^2 < 500$. At $F = 10$ the two directions are one (due west), and at $F = 10\sqrt{5}$ the second gives a zero resultant (equilibrium). So two directions for $\mathbf{10 < F < 10\sqrt{5}} \approx$ **22.4 N**; a 25 N force, as in a., has only one.

MARKING SCHEME

- ✓ $10 + 25\cos\theta = 0$: both angles, 113.6° and 246.4°.
- ✓ The northward sum for each: 42.9 N and −2.9 N.
- ✓ $\theta \approx 113.6°$, resultant 42.9 N due north (246.4° gives 2.9 N south).
- ✓ Northward sum $20 \pm \sqrt{F^2 - 100}$, with $F \geq 10$.
- ✓ Two directions for $10 < F < 10\sqrt{5}$ N.

5 The raft on the canal

ANSWER HA · 6

Tests: least and most with resolution; a sideways part that must be cancelled, and two quantities (Dawa's pull and the pull along the canal) that move together. The tempting wrong answer is that Dawa must pull 300 N, the same as Nirmala, or that the least pull is the most useful.

a. Across the canal, Nirmala's part is $300\sin 30° = 150$ N; Dawa's part across must cancel it: $Q\sin\beta = 150$, where β is his rope's angle with the bank. $Q = \dfrac{150}{\sin\beta}$ is least when $\sin\beta = 1$: **150 N, straight across the canal** ($\beta = 90°$). The resultant along the canal is then only Nirmala's part, $300\cos 30° \approx$ **260 N**.

b. $\beta = 45°$: $Q = \dfrac{150}{\sin 45°} \approx$ **212 N**; resultant $259.8 + 212.1\cos 45° = 259.8 + 150 \approx$ **410 N**. $\beta = 30°$: $Q =$ **300 N**; resultant $259.8 + 259.8 \approx$ **520 N**.

c. Not if the aim is to move the raft. The least pull only cancels the sideways part and adds nothing along the canal. Pulling at a smaller angle costs more effort but adds $Q\cos\beta$ along the canal: at 45°, 62 N more pull than the least gives 150 N more along the canal.

MARKING SCHEME

- ✓ Nirmala's part across the canal: 150 N.
- ✓ Least Q: 150 N, straight across ($\sin\beta = 1$).
- ✓ The resultant then 260 N along the canal.
- ✓ At 45°: 212 N and about 410 N.
- ✓ At 30°: 300 N and about 520 N.
- ✓ The least pull adds nothing along the canal; a slanting rope pulls more but moves the raft harder.

6 Testing the law in the laboratory

ANSWER

Tests: using the law to predict, then comparing measured data with the prediction and judging the size and pattern of the differences. The tempting wrong answer is to accept or reject the law from one reading, or to expect a measurement to hit the prediction exactly.

a. The two upper pulls must have a resultant of 5 (×100 g) upward: $25 = 9 + 16 + 24\cos\theta$, so $\cos\theta = 0$: **90**°.

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b. $88°$: $\sqrt{25 + 24\cos 88°} \approx$ **5.08**. $96°$: $\sqrt{25 + 24\cos 96°} \approx$ **4.74**.

c. The claim is wrong. The readings fall on both sides of $90°$ ($88°$ to $96°$, average $92°$), and the diagonals come within about 5% of 5, scattered rather than all off one way: this is friction in the pulleys and the error of judging an angle, not a failure of the law. A wrong law would give results off by the same kind of amount in the same direction every time.

MARKING SCHEME

- ✓ $90°$, from $25 = 9 + 16 + 24\cos\theta$.
- ✓ $88°$: about 5.08.
- ✓ $96°$: about 4.74.
- ✓ The readings scatter on both sides of $90°$ and within about 5%: measurement error and friction, not a wrong law.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	0	19	12	**31**

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