Unit 22 · Statics

ANSWER KEY · FOR TEACHERS

Full marks 35

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 3

1. line of action
2. concurrent
3. resultant
4. composition
5. resolution
6. equilibrium

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: It can be smaller: two equal forces at 150° give $2P\cos 75° \approx 0.52P$, and opposite forces give $|P - Q|$.

iii. ✓ **True**

iv. ✗ **False**: They can: 20 N resolved along lines at 30° and 45° to it gives 14.64 N and 10.35 N, 24.99 N in all; only as vectors do they make 20 N.

v. ✗ **False**: The part down the slope is $W\sin\theta$; $W\cos\theta$ is the part into the slope.

vi. ✓ **True**

Each carries 0.5 mark.

3 Multiple choice

i. **a.** $F\cos\theta$, $F\sin\theta$ — K · 1

The part along a line is F times the cosine of the angle with that line: θ with OX, $90° - \theta$ with OY, and $\cos(90° - \theta) = \sin\theta$.

ii. **d.** 2 N — U · 1

$|7 - 4| \le R \le 7 + 4$: every value from 3 N to 11 N is possible, 2 N is not.

iii. **c.** 120° — U · 1

$2P\cos\frac{\theta}{2} = P$ gives $\cos\frac{\theta}{2} = \frac{1}{2}$, $\frac{\theta}{2} = 60°$, $\theta = 120°$. At 60° the resultant is $P\sqrt{3}$.

iv. **a.** 10 N — U · 1

The weight makes 30° with the perpendicular to the slope, so its part down the slope is $20\sin 30° = 10$ N. $10\sqrt{3} = 20\cos 30°$ is the part into the slope.

v. **a.** 7 N — A · 1

$R^2 = 9 + 25 + 2 \cdot 3 \cdot 5 \cdot \frac{1}{2} = 49$. $\sqrt{19}$ uses the minus sign, $\sqrt{34}$ forgets the $2PQ\cos\theta$ term, 8 N adds them as numbers.

vi. **b.** $\sqrt{Q^2 - P^2}$ — HA · 1

At right angles to P: $\tan\alpha$ is infinite, so $P + Q\cos\theta = 0$. Then $R^2 = P^2 + Q^2 + 2P(-P) = Q^2 - P^2$. On the figure, Q is the hypotenuse of a right triangle with legs P and R.

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4 Spot the error

MARKING SCHEME U · 2

✓ The mistake: with X and Y both negative the resultant is in the third quadrant, not the first.

✓ $\alpha = 180° + 53.1° = 233.1°$.

5 Short questions

a. **ANSWER** A · 5

a. $R^2 = 36 + 100 + 2 \cdot 6 \cdot 10 \cos 120° = 136 - 60 = 76$, so $R = \sqrt{76} = 2\sqrt{19} \approx$ **8.72 N**.

$\tan \alpha = \dfrac{10 \sin 120°}{6 + 10 \cos 120°} = \dfrac{5\sqrt{3}}{1} \approx 8.66$, so $\alpha \approx$ **83.4° from the 6 N force**.

b. Reversed, the 6 N force makes $180° - 120° = 60°$ with the 10 N force:
$R^2 = 136 + 2 \cdot 6 \cdot 10 \cos 60° = 196$, $R =$ **14 N**.

MARKING SCHEME

✓ $R^2 = 36 + 100 + 120 \cos 120° = 76$.

✓ $R = 2\sqrt{19} \approx 8.72$ N.

✓ $\tan \alpha = 5\sqrt{3}$, $\alpha \approx 83.4°$ from the 6 N force.

✓ b. The angle between them becomes 60°.

✓ $R = 14$ N.

b. **ANSWER** A · 5

a. $X = 5 - 15 = -10$ N; $Y = 10 - 20 = -10$ N.

b. $R = \sqrt{100 + 100} = 10\sqrt{2} \approx$ **14.1 N**. $\tan \alpha = \dfrac{-10}{-10} = 1$, with both negative: the third quadrant, $\alpha =$ $180° + 45° =$ **225° from the east**, that is, towards the south-west.

c. A force equal and opposite to the resultant: **$10\sqrt{2}$ N towards the north-east** (45°).

MARKING SCHEME

✓ $X = -10$ N.

✓ $Y = -10$ N.

✓ $R = 10\sqrt{2} \approx 14.1$ N.

✓ Direction 225° from the east (south-west), the quadrant read from the signs.

✓ $10\sqrt{2}$ N towards the north-east.

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6 Long question

ANSWER A · 8

a. Let $c = \cos\theta$.

i. $R^2 = P^2 + Q^2 + 2PQc$

ii. $4R^2 = P^2 + 4Q^2 + 4PQc$ (Q doubled)

iii. $4R^2 = P^2 + Q^2 - 2PQc$ (Q reversed)

$4 \times$ (i) $-$ (ii): $0 = 3P^2 + 4PQc$, so $PQc = -\frac{3}{4}P^2$.

$4 \times$ (i) $-$ (iii): $0 = 3P^2 + 3Q^2 + 10PQc = 3P^2 + 3Q^2 - \frac{15}{2}P^2$, so $Q^2 = \frac{3}{2}P^2$.

Then (i): $R^2 = P^2 + \frac{3}{2}P^2 - \frac{3}{2}P^2 = P^2$.

So $P^2 : Q^2 : R^2 = 1 : \frac{3}{2} : 1 = 2 : 3 : 2$. ✓

b. $\cos\theta = \dfrac{-\frac{3}{4}P^2}{PQ} = -\dfrac{3P}{4Q} = -\dfrac{3}{4}\sqrt{\dfrac{2}{3}} = -\dfrac{\sqrt{6}}{4} \approx -0.612$, so $\theta \approx \mathbf{127.8}°$.

MARKING SCHEME

✓ The parallelogram law for Q: $R^2 = P^2 + Q^2 + 2PQ\cos\theta$.

✓ For $2Q$: $4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$.

✓ For $-Q$: $4R^2 = P^2 + Q^2 - 2PQ\cos\theta$ (the sign of the cosine term changed).

✓ Eliminating: $PQ\cos\theta = -\frac{3}{4}P^2$.

✓ $Q^2 = \frac{3}{2}P^2$ and $R^2 = P^2$, so $P^2 : Q^2 : R^2 = 2 : 3 : 2$.

✓ b. $\cos\theta = -\dfrac{3P}{4Q}$.

✓ $= -\dfrac{3}{4}\sqrt{\dfrac{2}{3}} = -\dfrac{\sqrt{6}}{4} \approx -0.612$.

✓ $\theta \approx 127.8°$.

7 Think

ANSWER HA · 3

At the shirt, the two halves of the line pull with T each, at θ above the horizontal on either side. Their horizontal parts $T\cos\theta$ cancel; their upward parts must hold the weight: $2T\sin\theta = 10$, so $T = \dfrac{10}{2\sin\theta}$.

$\theta = 10°$: $T = \dfrac{10}{2 \times 0.1736} \approx \mathbf{28.8\ N}$. $\theta = 2°$: $T = \dfrac{10}{2 \times 0.0349} \approx \mathbf{143\ N}$.

No. As θ goes to 0°, $\sin\theta$ goes to 0 and T grows without limit: a perfectly straight line has no upward part at all to hold the shirt.

MARKING SCHEME

✓ Vertical parts: $2T\sin\theta = 10$ (the horizontal parts cancel).

✓ $T \approx 28.8$ N at 10° and ≈ 143 N at 2°.

✓ Never straight: as $\theta \to 0°$, T grows without limit; a level line has no upward part.

8 Activity: Two strings, one bag

(not marked)

WHAT STUDENTS SHOULD FIND

The reading grows as the strings spread: about $\frac{W}{2}$ at 0°, $0.58W$ at 60°, $0.71W$ at 90° and the whole of W at 120°, each within a few percent of the formula (angles judged by eye are the main error). Each string's upward part must hold half the weight, $T\cos\frac{\theta}{2} = \frac{W}{2}$, so the wider the angle, the harder each string pulls.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	4	8	19	4	**35**

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