Unit 21 · Numerical Integration

WORKSHEET

Full marks 35

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 35

1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

chord	**error**	**even**	**numerical integration**	**odd**	**ordinates**	**parabola**	**tangent**

Finding a definite integral from the heights of a curve at a few points, without an anti-derivative, is _____. The heights of the curve at the cuts between the strips are the _____. On each strip the trapezoidal rule replaces the curve by a _____; over each pair of strips Simpson's rule puts a _____ through three ordinates, so it needs an _____ number of strips. The exact value minus the approximate value is the _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. With $n = 6$ strips there are 7 ordinates. ☐

ii. In Simpson's rule every ordinate between the first and the last is multiplied by 4. ☐

iii. The trapezoidal rule always gives an answer that is too big. ☐

iv. Simpson's rule with $n = 2$ gives the exact value of $\int_0^2 x^2\,dx$. ☐

v. Halving the width of the strips halves the error of the trapezoidal rule. ☐

vi. The trapezoidal rule gives the average of the left-height and right-height rectangle sums. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. In Simpson's rule, the bracket of weighted ordinates is multiplied by: [1]

a. h **b.** $\frac{h}{2}$

c. $\frac{h}{3}$ **d.** $\frac{h}{6}$

ii. $\int_2^5 f(x)\,dx$ is to be found from 7 equally spaced ordinates. The width of each strip is: [1]

a. $\frac{3}{7}$ **b.** $\frac{1}{2}$

c. $\frac{3}{8}$ **d.** 3

iii. In Simpson's rule with $n = 6$, the ordinates $y_0, y_1, \ldots, y_6$ are multiplied by: [1]

a. 1, 4, 2, 4, 2, 4, 1 **b.** 1, 2, 4, 2, 4, 2, 1

c. 1, 4, 4, 4, 4, 4, 1 **d.** 1, 2, 2, 2, 2, 2, 1

iv. The trapezoidal rule, with any number of strips, is used on $\int_0^{\pi/2} \sin x\,dx$. Its answer is: [1]

a. too big, because $\sin x$ is increasing on $[0, \frac{\pi}{2}]$

b. too small, because $\sin x$ bends downward there and every chord lies under the curve

c. exact, because $\sin x$ has no turning point inside the interval

d. too big or too small, depending on n

v. A curve has ordinates 1, 3, 4, 4 and 2 at $x = 0, 1, 2, 3, 4$. Simpson's rule gives the area under it as: [1]

a. 11 **b.** 12.5

c. 13 **d.** 39

vi. T_2 and T_4 are trapezoidal answers for the same integral with 2 and 4 strips. Then $\frac{4T_4 - T_2}{3}$ is: [1]

a. the exact value **b.** Simpson's rule with 4 strips

c. Simpson's rule with 2 strips **d.** the trapezoidal rule with 8 strips

4 Spot the error [2]

Find the mistake in the working and correct it.

A curve has the ordinates 1, 2, 3, 3 and 2 at $x = 0, 0.5, 1, 1.5, 2$. Kiran finds the area under it by the trapezoidal rule:

"5 ordinates, so $h = \frac{2}{5} = 0.4$, and the area is $\frac{0.4}{2}\left[\,(1+2) + 2(2+3+3)\,\right] = 0.2 \times 19 = 3.8$."

5 Short questions [10]

Answer the following questions. Show your working.

a. A plot of land lies between a straight boundary 60 m long and a stream. The distances (offsets) from the boundary to the stream, every 10 m along it, are 12, 15, 18, 17, 20, 16 and 14 m. Find the plot's area by Simpson's rule, in square meters and in ropani (1 ropani = 5476 square feet = 508.74 m^2 = 16 aana). [5]

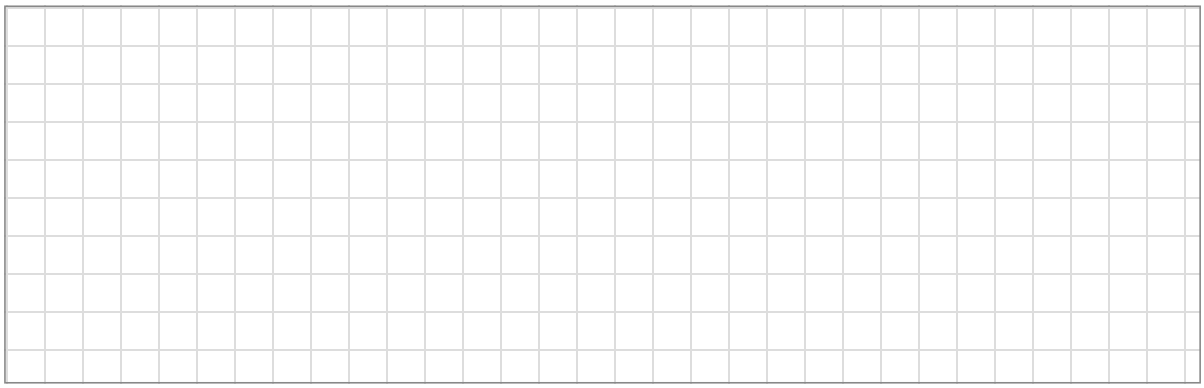
Answer:

b. Sushma measures the width of a pond every 3 m along its length of 15 m. The widths are 0, 5.2, 7.8, 8.4, 6.6 and 0 m. (The numbers are made up.) **[5]**

a. Find the pond's area by the trapezoidal rule. [2]

b. Why can't Simpson's rule be used on all six widths? [1]

c. Find the area by Simpson's rule on the first four strips, with the last strip taken as a trapezium. [2]



Answer: _____

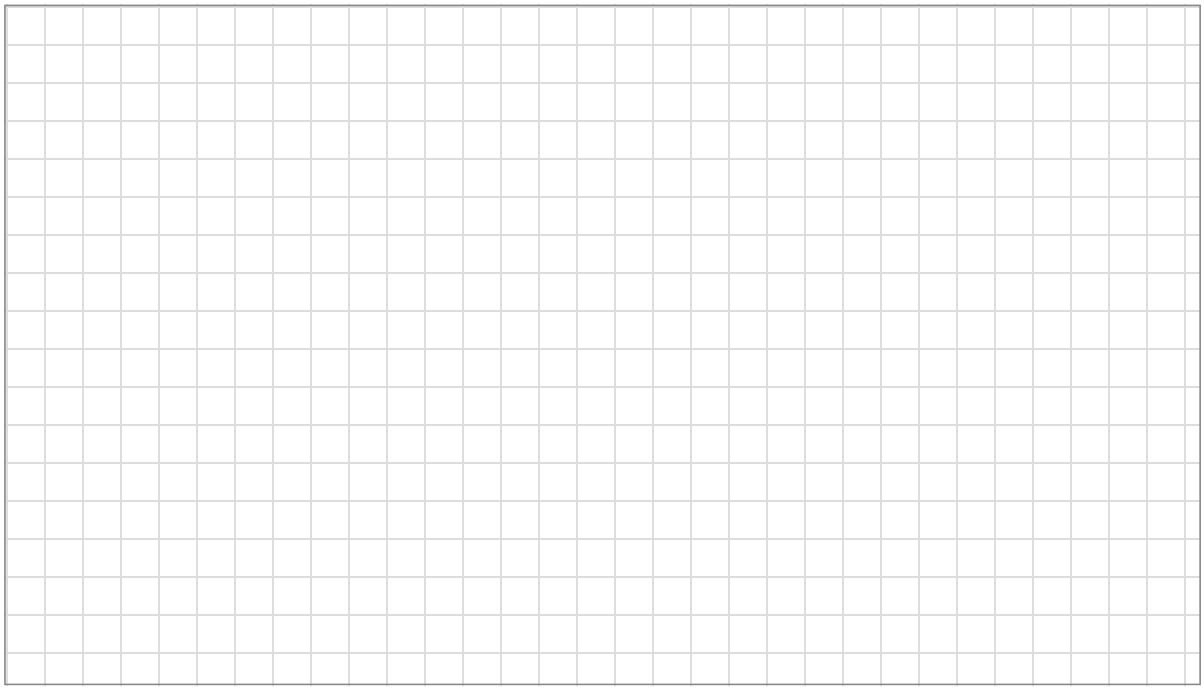
6 Long question **[8]**

Answer the following question. Show your working.

a. A parabola $y = Ax^2 + Bx + C$ passes through $(-h, y_0)$, $(0, y_1)$ and (h, y_2). Show that the area under it from $x = -h$ to $x = h$ is $\frac{h}{3}(y_0 + 4y_1 + y_2)$. [3]

b. Evaluate $\int_0^{1.5} e^{-x}\,dx$ with $n = 6$ by Simpson's rule and by the trapezoidal rule, taking the ordinates to four decimal places. [3]

c. Find the exact value of the integral and the error of each answer. Explain why the trapezoidal answer is too big. [2]



Answer: _____

7 Think [3]

Bishal says: "Simpson's rule is always more accurate than the trapezoidal rule." Test his claim on $\int_0^4 |x-2|\,dx$, whose graph is a V with its corner at (2, 0), using $n = 2$ for both rules. Explain what you find.

8 Activity: The area of a plate (not marked)

You need: a round plate, 1 cm squared paper and a ruler. Draw round the plate on the paper and measure its diameter. Rule a diameter across the circle and, every 2 cm along it, measure the width of the circle at right angles to it (at the two ends the width is 0). Stop the table at your plate's diameter.

Distance along the diameter (cm)	Width across (cm)
0	0
2	
4	
6	
8	
10	
12	
14	
16	
18	
20	

Area by the trapezoidal rule: _____ cm^2. By Simpson's rule: _____ cm^2. By πr^2: _____ cm^2.
Which rule came closer? (If the number of strips is odd, use Simpson's rule on all but the last strip and add the last as a trapezium.)