Unit 20 · Numerical Computation

WORKSHEET

Full marks 33

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 33

1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

algebraic	**derivative**	**intermediate value theorem**	**iteration**	**root**	**tangent**	**tolerance**	**transcendental**

A number that makes $f(x) = 0$ true is a _____ of the equation. $x = e^{-x}$ is a _____ equation, since it has an exponential function of x in it. If f is continuous on $[a, b]$ and $f(a), f(b)$ have opposite signs, the _____ says there is a root between a and b. Each step of a numerical method is an _____. Newton–Raphson follows the _____ at x_n down to the X-axis. We stop when the approximate error is below a set _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. If $f(a)$ and $f(b)$ have the same sign, $f(x) = 0$ has no root between a and b. ☐

ii. Bisection needs only the signs of f, not its derivative. ☐

iii. Each Newton–Raphson step halves the error. ☐

iv. If $f'(x_n) = 0$, the tangent at x_n never meets the X-axis, and Newton–Raphson can't take the next step. ☐

v. After 10 bisection steps from an interval of width 1, the error is at most 0.1. ☐

vi. $x \log_{10} x = 1.2$ is an algebraic equation. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. The Newton–Raphson formula is: [1]

a. $x_{n+1} = x_n + \dfrac{f(x_n)}{f'(x_n)}$ **b.** $x_{n+1} = x_n - \dfrac{f'(x_n)}{f(x_n)}$

c. $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ **d.** $x_{n+1} = \dfrac{x_n + f(x_n)}{2}$

ii. Bisection for $x^3 - x - 1 = 0$ starts on $[1, 2]$. After the first step the root is in: [1]

a. $[1, 1.5]$ **b.** $[1.5, 2]$

c. $[1, 2]$ **d.** $[1.25, 1.5]$

iii. Two successive approximations to a root are $x_3 = 0.5671$ and $x_4 = 0.5672$. The relative approximate error of x_4 is about: **[1]**

a. 0.0001%
b. 0.01%
c. 0.0176%
d. 1.76%

iv. One step of Newton–Raphson for $x^3 - 2x - 5 = 0$ from $x_0 = 2$ gives: **[1]**

a. 2.1
b. 1.9
c. 2.083
d. 2.5

v. For $x^3 + x - 4 = 0$, the root is in $[1, 4]$. The least number of bisection steps that makes the error less than 0.01 is: **[1]**

a. 7
b. 8
c. 9
d. 300

vi. Bisection starts on $[0, 1]$ with $f(0) < 0 < f(1)$. Its first four midpoints are 0.5, 0.25, 0.375 and 0.4375. The signs of $f(0.5)$, $f(0.25)$ and $f(0.375)$ were: **[1]**

a. $+, -, -$
b. $-, +, +$
c. $+, +, -$
d. $-, -, +$

4 Short questions **[10]**

Answer the following questions. Show your working.

a. Show that $e^x = 3x$ has a root between 0 and 1, and find it correct to two decimal places by bisection. **[5]**

Answer: _____

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https://science-playground.com/learn/grade-11-math/numerical-computation

b. a. The tangent to $y = f(x)$ at $(x_n, f(x_n))$ meets the X-axis at x_{n+1}. Derive $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, and say when it can't be used. [3] **[5]**

b. Explain why the error after n steps of bisection is at most $\frac{b-a}{2^n}$, and find how many steps from an interval of width 1 make it less than 0.001. [2]

5 Long question **[8]**

Answer the following question. Show your working.

The equation $xe^x = 2$.

a. Show that it has a root in (0, 1). [1]

b. Do four steps of bisection. Give the interval the root is in, and the most the last midpoint can be from the root. [3]

c. Find the root correct to four decimal places by Newton–Raphson from $x_0 = 1$. [3]

d. How many more bisection steps, from the interval of b., would make the error less than 0.0005? [1]

Answer: _____

6 Think [3]

Without finding it, explain why $x^3 + x - 3 = 0$ has exactly one real root, and give an interval of width 1 that holds it.

7 Activity: Watch the digits double (not marked)

Find $\sqrt{2}$ with a calculator and Newton–Raphson on $f(x) = x^2 - 2$, whose rule is $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n} \right)$. Enter 1, then type (Ans + 2 ÷ Ans) ÷ 2 and press = again and again. Fill in the table, writing every digit the calculator shows.

n	x_n	**correct decimal places ($\sqrt{2}$ = 1.41421356237…)**
0	1	
1		
2		
3		
4		

How does the number of correct places grow from one step to the next?