Unit 19 · Anti-derivatives

WORKSHEET

Full marks 33

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 33

1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

anti-derivative	constant of integration	definite integral	derivative	integrand
lower limit	substitution	upper limit		

A function F with $F'(x) = f(x)$ is an _____ of f. In $\int f(x)\,dx = F(x) + C$, $f(x)$ is the _____ and C the _____. $\int_a^b f(x)\,dx$ is a _____, a number, and a is its _____.

Running the chain rule backwards gives integration by _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. $\int f(x)g(x)\,dx = \int f(x)\,dx \cdot \int g(x)\,dx$. ☐

ii. $\int \frac{1}{x^2}\,dx = \ln x^2 + C$. ☐

iii. $\sin^2 x + C$ and $-\frac{1}{2}\cos 2x + C$ are the same family of anti-derivatives. ☐

iv. If f is odd, $\int_{-a}^{a} f(x)\,dx = 0$. ☐

v. $\int_{-1}^{2} x\,dx = \frac{3}{2}$, so the area between $y = x$ and the X-axis from $x = -1$ to $x = 2$ is $\frac{3}{2}$. ☐

vi. In $\int_0^1 x(1 + x^2)^3\,dx$, putting $u = 1 + x^2$ gives $\frac{1}{2}\int_0^1 u^3\,du$. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. $\int \sec^2 x\,dx$ is: [1]

a. $\sec x + C$ **b.** $\tan x + C$

c. $\sec x \tan x + C$ **d.** $2\sec^2 x \tan x + C$

ii. $\int \cos(3x + 1)\,dx$ is: [1]

a. $\frac{1}{3}\sin(3x + 1) + C$ **b.** $\sin(3x + 1) + C$

c. $3\sin(3x + 1) + C$ **d.** $-\frac{1}{3}\sin(3x + 1) + C$

iii. $\int_{-2}^{2} (x^3 - 3x + 1)\,dx$ equals: [1]

a. 0 **b.** 2

c. 4 **d.** 8

iv. $\int x \sec^2 x\,dx$ is: [1]

a. $x \tan x - \ln|\sec x| + C$

b. $x \tan x + \ln|\sec x| + C$

c. $x \tan x + C$

d. $\frac{x^2}{2} \tan x + C$

v. The area bounded by the X-axis, the curve $y = 4x^3$ and the ordinates $x = 2$ and $x = 4$ is: [1]

a. 48 square units **b.** 240 square units

c. 256 square units **d.** 272 square units

vi. $\int_0^2 f(x)\,dx = 3$ and $\int_0^5 f(x)\,dx = 7$. Then $\int_2^5 [2f(x) + 1]\,dx$ is: [1]

a. 8 **b.** 9

c. 11 **d.** 17

4 Short questions [10]

Answer the following questions. Show your working.

a. Integrate by substitution: [5]

a. $\int \frac{2x + 3}{x^2 + 3x + 5}\,dx$ [2]

b. $\int x\sqrt{x - 1}\,dx$ [3]

b. a. Starting from the product rule, derive the formula for integration by parts, **[5]**

$\int uv\,dx = u\int v\,dx - \int \left(\frac{du}{dx}\int v\,dx \right) dx$. [3]

b. To integrate $\int xe^{-x}\,dx$ by parts, which function should be u? Explain what goes wrong with the other choice, then write the answer. [2]

5 Long question [8]

Answer the following question. Show your working.

The curve $y = x^2 - 4x + 3$ and the line $y = x - 1$.

a. Find the points where they meet, and sketch both on one pair of axes. [2]

b. Find the area of the region enclosed between the curve and the line. [3]

c. Evaluate $\int_0^4 (x^2 - 4x + 3)\,dx$, and find the area between the curve and the X-axis from $x = 0$ to $x = 4$. Why are the two numbers different? [3]

6 Think [3]

Sadikshya evaluated $\int_{-1}^{1} \frac{dx}{x^2} = \left[-\frac{1}{x} \right]_{-1}^{1} = -1 - 1 = -2$. Without integrating, explain why the answer can't be right, and find the step that went wrong.

7 Activity: An area by strips

(not marked)

On squared paper, draw $y = 4x - x^2$ from $x = 0$ to $x = 4$ (2 cm for one unit on both axes). Cut the region under it into 8 strips of width 0.5, and give each strip the curve's height at its middle. Fill in the table.

Middle of the strip, x	**0.25**	**0.75**	**1.25**	**1.75**	**2.25**	**2.75**	**3.25**	**3.75**
Height $4x - x^2$								
Strip area, height $\times$ 0.5								

Add the strip areas. Then work out $\int_0^4 (4x - x^2)\,dx$ and compare.