Unit 18 · Derivatives

WORKSHEET

Full marks 33

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 33

1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

concave upward	**local maximum**	**monotonic**	**normal**	**point of inflection**	**secant**
stationary point	**tangent**				

A line through two points of a curve is a _____. As the second point slides to the first, it turns into the _____, whose slope is the derivative there. A point where $f'(x) = 0$ is a _____. If f' changes from $+$ to $-$ at c, f has a _____ at c. Where $f''(x) > 0$, the curve is _____, bending like a cup. A point where the concavity changes is a _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. A function that is continuous at a is differentiable at a. ☐

ii. If $f'(x) > 0$ on an interval, f is increasing on it. ☐

iii. $(uv)' = u'v'$. ☐

iv. If $f''(c) = 0$, the curve has a point of inflection at c. ☐

v. $\frac{d}{dx}(a^x) = a^x \ln a$. ☐

vi. For $x = f(t), y = g(t)$, $\frac{d^2y}{dx^2} = \frac{d^2y/dt^2}{d^2x/dt^2}$. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. $\frac{d}{dx}(\operatorname{cosec} x)$ is: [1]

a. $\operatorname{cosec} x \cot x$ **b.** $-\operatorname{cosec} x \cot x$

c. $-\operatorname{cosec}^2 x$ **d.** $\sec x \tan x$

ii. $\frac{d}{dx}\sqrt{1+x^2}$ is: [1]

a. $\frac{1}{2\sqrt{1+x^2}}$ **b.** $\frac{x}{\sqrt{1+x^2}}$

c. $2x\sqrt{1+x^2}$ **d.** $\frac{x}{1+x^2}$

iii. If $x^2 - y^2 = 9$, then $\frac{dy}{dx}$ is: [1]

a. $\frac{x}{y}$ **b.** $-\frac{x}{y}$

c. $\frac{y}{x}$ **d.** $-\frac{y}{x}$

iv. If $y = e^{3x}$, then $\frac{d^4y}{dx^4}$ is: [1]

a. e^{3x} **b.** $12e^{3x}$

c. $81e^{3x}$ **d.** $3^4\, e^x$

v. The greatest value of $f(x) = xe^{-x}$ is: [1]

a. 1 **b.** $\frac{1}{e}$

c. e **d.** 0

vi. $f(x) = x^3 + ax^2 + 3x + 1$ is increasing for every x exactly when: [1]

a. $a \geq 0$ **b.** $-3 \leq a \leq 3$

c. $-\sqrt{3} \leq a \leq \sqrt{3}$ **d.** $a = 0$ only

4 Short questions [10]

Answer the following questions. Show your working.

a. Find each derivative. [5]

a. From the definition, find the derivative of $\sin 3x$. [2]

b. Differentiate $y = \tan^{-1}\left(\frac{1+x}{1-x}\right)$, for $|x| < 1$. [3]

Answer: _____

b. A curve is given by $x = 2t + 1, y = t^3 - 3t$. **[5]**

a. Find $\frac{dy}{dx}$. [1]

b. Find the points where the tangent is parallel to the X-axis. [2]

c. Find $\frac{d^2y}{dx^2}$, and say whether the curve is concave up or down at $t = 1$. [2]

Answer: _____

5 Long question **[8]**

Answer the following question. Show your working.

$f(x) = x^4 - 8x^2 + 3$.

a. Find the intervals in which f is increasing and decreasing. [2]

b. Find its local maximum and minimum values. [2]

c. Find the intervals of concavity and the points of inflection. [2]

d. Find the greatest and least values of f on $[-1, 3]$. [2]

Answer: _____

6 Think [3]

Gyanu has 60 m of fencing to make a rectangular pen for goats against a long straight wall, so the fence makes only three sides. Ankit says: "The best rectangle from a fixed length is always a square, so make it 20 m by 20 m." Is he right? Find the best pen.

7 Activity: Slopes with a ruler (not marked)

On squared paper, draw $y = x^2$ from $x = 0$ to $x = 3$, with 2 cm for 1 unit across and 1 cm for 1 unit up. At $x = 1$, 2 and 3, lay a ruler along the curve so that it just touches it, draw the tangent, and measure its slope as rise ÷ run in the graph's units.

x	1	2	3
slope you measured			
$f'(x) = 2x$			

How close are your slopes to $2x$? Where was the tangent hardest to draw?