Unit 17 · Limits and Continuity

WORKSHEET

Full marks 34

Name _____ Class _____ Roll no. _____ Date _____ Marks _____ / 34

1 Key words [3]

Fill in the blanks with words from the box. Two words in the box are not needed. Each blank carries 0.5 mark.

asymptote	continuous	indeterminate form	jump	left-hand limit	oscillating	removable	sandwich theorem

As x closes in on a from below, the number that $f(x)$ closes in on is the _____. A form such as $\frac{0}{0}$ or 1^{∞}, whose value depends on the functions and not on the form, is an _____. f is _____ at a when its limit there equals $f(a)$. A hole in a graph, mended by defining $f(a)$ as the limit, is a _____ discontinuity; a step from one height to another, which no value of $f(a)$ can mend, is a _____ discontinuity. If $g(x) \le f(x) \le h(x)$ near a and g and h close in on the same number, f closes in on it too: this is the _____.

2 True or false [3]

Write ✓ for a true statement and ✗ for a false one. Correct the false ones.

i. If $f(a)$ is not defined, $\lim_{x \to a} f(x)$ can't exist. ☐

ii. 1^{∞} is an indeterminate form. ☐

iii. $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ whether θ is in degrees or in radians. ☐

iv. No value of $f(a)$ can remove a jump discontinuity at a. ☐

v. $\infty - \infty = 0$. ☐

vi. $|x|$ is continuous at $x = 0$. ☐

3 Multiple choice [6]

Choose the correct option for each of the following questions.

i. Which of these is an indeterminate form? [1]

a. $\frac{0}{\infty}$ **b.** 0^0

c. $\infty \times \infty$ **d.** 0^{∞}

ii. $\lim_{x \to 0} \frac{\sin 5x}{\tan 2x}$ is: [1]

a. 1 **b.** $\frac{5}{2}$

c. $\frac{2}{5}$ **d.** 10

iii. $\lim_{x \to 0} \dfrac{3^x - 1}{x}$ is: [1]

a. 1 **b.** 3

c. $\ln 3$ **d.** $\dfrac{1}{\ln 3}$

iv. $f(x) = \dfrac{x^2 - 5x + 6}{x - 3}$ for $x \neq 3$, and $f(3) = 2$. At $x = 3$, f has: [1]

a. no discontinuity **b.** a removable discontinuity

c. a jump discontinuity **d.** an infinite discontinuity

v. $\lim_{x \to 0} \left(1 + \dfrac{x}{2}\right)^{3/x}$ is: [1]

a. 1 **b.** e^6

c. $e^{2/3}$ **d.** $e^{3/2}$

vi. Which of these functions is continuous at $x = 0$? ($[x]$ is the greatest integer function.) [1]

a. $f(x) = [x]$

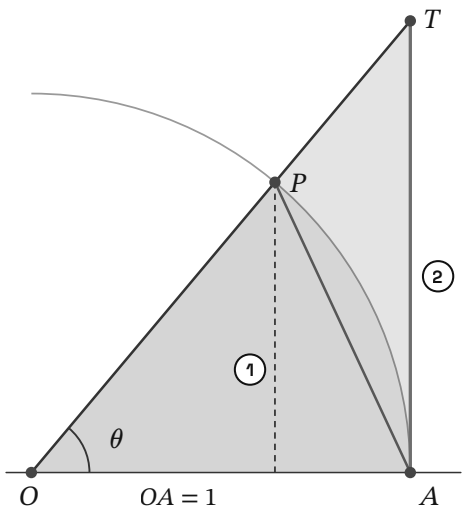
b. $f(x) = \dfrac{\sin x}{x}$ for $x \neq 0$, and $f(0) = 0$

c. $f(x) = x[x]$

d. $f(x) = \dfrac{|x|}{x}$ for $x \neq 0$, and $f(0) = 0$

4 Label the drawing [1]

The drawing is the one the unit uses to prove $\lim_{\theta \to 0} \dfrac{\sin \theta}{\theta} = 1$: a circle of radius 1, an angle θ at its center O, and the tangent at A. Write the length that belongs at each number. Each carries 0.5 mark.



1. _____ **2.** _____

5 Short questions [10]

Answer the following questions. Show your working.

a. Evaluate: [5]

a. $\lim_{x \to 2} \dfrac{x^4 - 16}{x^3 - 8}$ [2]

b. $\lim_{x \to 0} \dfrac{\cos 2x - \cos 4x}{x^2}$ [3]

Answer: _____

b. [5]

a. Find the points of discontinuity of $f(x) = \dfrac{x^2 + x - 6}{x^2 - 4}$, and name the type of each. [3]

b. A function is given in two pieces:

$g(x)$	**when**
$kx + 1$	$x \leq 2$
$x^2 - k$	$x > 2$

Find k so that g is continuous at $x = 2$. [2]

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6 Long question [8]

Answer the following question. Show your working.

Use the drawing in the labelling question, with $0 < \theta < \frac{\pi}{2}$.

a. Compare the areas of triangle OAP, sector OAP and triangle OAT, and show that $\cos\theta < \frac{\sin\theta}{\theta} < 1$. Hence show that $\lim_{\theta \to 0^+} \frac{\sin\theta}{\theta} = 1$. [3]

b. Why is the left-hand limit the same? [1]

c. $f(x) = \frac{1 - \cos 4x}{x \sin x}$ for $x \neq 0$, and $f(0) = k$. Find k so that f is continuous at $x = 0$. [2]

d. Evaluate $\lim_{x \to \pi} \frac{\sin x}{\pi - x}$. [2]

7 Think [3]

Make up a function f, given in pieces, with $f(0) = 0$, that is continuous everywhere except at $x = 1$, where it has a removable discontinuity, and at $x = 2$, where it has a jump of 1. Show that your function does all of this.

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8 Activity: Closing in on ln 2

(not marked)

Work out $\frac{2^x - 1}{x}$ on a calculator for each x in the table, to six decimal places.

x	$\frac{2^x - 1}{x}$
0.1	
0.01	
0.001	
−0.001	
−0.01	
−0.1	

What number do the values close in on from each side? Compare it with ln 2 on your calculator.