Unit 16 · Probability

HARDER PRACTICE QUESTIONS

Full marks 26

Write your answers on separate paper. Show your working. A calculator may be used.

1 Four throws or twenty-four [5]

A puzzle: which is more likely, at least one six in 4 throws of a die, or at least one double six in 24 throws of a pair of dice? One gambler argued: "The first is $4 \times \frac{1}{6} = \frac{2}{3}$ and the second $24 \times \frac{1}{36} = \frac{2}{3}$, so they are equally likely."

a. Show, with a simple case, that his method can't be right. [1]

b. Find both probabilities correctly, to three decimal places. [3]

c. Which is more likely? [1]

2 Three events from two coins [5]

Two fair coins are tossed. A: the first shows a head. B: the second shows a head. C: the two coins show different faces.

a. Show that A and B, A and C, and B and C are independent pairs. [3]

b. Are A, B and C independent, all three together? Explain what this means. [2]

3 Two students and one problem [5]

Pratiksha's chance of solving a problem is twice Aman's. Working alone, at least one of them solves it with probability 0.72.

a. Find each one's chance of solving it. [3]

b. Find the probability that exactly one of them solves it. [2]

4 Which seed to buy? [6]

A farmer will sow a ropani of maize at 3,900 spots and wants at most 1% of the spots to stay empty. Two seed lots are on sale (made-up figures): in a test of 200 seeds, 180 of lot A sprouted and 150 of lot B. Lot A costs Rs 6 for 100 seeds, lot B Rs 4 for 100 seeds. Take each test's relative frequency as each seed's chance, and assume the seeds in one spot sprout or fail independently.

a. With 2 seeds a spot, what is the lowest chance of sprouting a seed can have and still meet the farmer's aim? [2]

b. For each lot, find the least number of seeds a spot that meets the aim, and the cost of the seed for the ropani. [3]

c. Which lot should the farmer buy? [1]

5 What the numbers allow [5]

At a school, 70% of the students passed Mathematics and 60% passed Science (made-up figures). Let M and S be these events for a student picked at random.

a. Can M and S be mutually exclusive? Give a reason. [1]

b. Find the least and the greatest possible values of $P(M \cap S)$. [2]

c. In fact 45% passed both. Are M and S independent? What does your answer say about the students? [2]