Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Four throws or twenty-four

ANSWER HA · 5

Tests: "at least one" of independent events is 1 − the product of the chances of none; adding chances is the addition law used for events that are not mutually exclusive. The tempting wrong answer is his $\frac{2}{3}$ for both.

a. By his method, 6 throws would give at least one six with probability $6 \times \frac{1}{6} = 1$, certain, and 7 throws $\frac{7}{6} > 1$. But six throws can all miss. The sixes in different throws are not mutually exclusive (two throws can both be sixes), so their chances can't simply be added.

b. The throws are independent. No six in 4 throws: $\left(\frac{5}{6}\right)^4 = \frac{625}{1296}$, so at least one: $1 - \frac{625}{1296} = \frac{671}{1296} \approx 0.518$.

No double six in 24 throws: $\left(\frac{35}{36}\right)^{24} \approx 0.509$, so at least one: ≈ 0.491.

c. **At least one six in 4 throws** is slightly more likely (0.518 against 0.491): just better than even, while the other is just worse.

MARKING SCHEME

✓ a. His method gives probability 1 (or more) for 6 (or more) throws; the events can happen together.

✓ b. $1 - \left(\frac{5}{6}\right)^4$.

✓ ≈ 0.518.

✓ $1 - \left(\frac{35}{36}\right)^{24} \approx 0.491$.

✓ c. At least one six in 4 throws.

2 Three events from two coins

ANSWER HA · 5

Tests: independence as the multiplication law, checked for every group of events, not just in pairs; and that independence is not judged by whether events "seem connected". The tempting wrong answer: "independent in pairs, so independent", or "C depends on both coins, so it is dependent on each".
$S = \{HH, HT, TH, TT\}$, equally likely. $A = \{HH, HT\}$, $B = \{HH, TH\}$, $C = \{HT, TH\}$: each has probability $\frac{1}{2}$.

a. $A \cap B = \{HH\}$, $A \cap C = \{HT\}$, $B \cap C = \{TH\}$: each $\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$. So every pair is independent.

b. $A \cap B \cap C = \emptyset$: $P = 0$, but $P(A)P(B)P(C) = \frac{1}{8}$. The multiplication law fails for the group of three, so they are **not** independent. Knowing any one of them tells nothing about another, but knowing two decides the third: if both coins show heads, they can't differ.

MARKING SCHEME

✓ a. The sample space and $P(A) = P(B) = P(C) = \frac{1}{2}$.

✓ The three intersections, each one outcome.

✓ Each $\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$: independent in pairs.

✓ b. $P(A \cap B \cap C) = 0 \neq \frac{1}{8}$: not independent.

✓ Any two decide the third.

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3 Two students and one problem

ANSWER A · 5

Tests: "at least one" as 1 − both fail, turned into an equation, and a root that is not a probability. The tempting slips: $p + 2p = 0.72$ (adding the chances), or keeping $p = 1.2$.

a. Let Aman's chance be p, Pratiksha's $2p$; they work alone, so independently. Both fail: $(1 - p)(1 - 2p) = 1 - 0.72 = 0.28$.

$2p^2 - 3p + 1 = 0.28$, so $2p^2 - 3p + 0.72 = 0$: $p = \dfrac{3 \pm \sqrt{9 - 5.76}}{4} = \dfrac{3 \pm 1.8}{4}$, that is 1.2 or 0.3.

1.2 is more than 1, so not a probability (and $2p$ must be at most 1 too). **Aman 0.3, Pratiksha 0.6.**

b. Only Aman: $0.3 \times 0.4 = 0.12$; only Pratiksha: $0.7 \times 0.6 = 0.42$. Mutually exclusive, so add: **0.54**. (Check: at least one 0.72 minus both $0.3 \times 0.6 = 0.18$ is 0.54.)

MARKING SCHEME

- ✓ a. $(1 - p)(1 - 2p) = 0.28$.
- ✓ $p = 0.3$ or 1.2.
- ✓ 1.2 rejected (not a probability): Aman 0.3, Pratiksha 0.6.
- ✓ b. The two "only one" cases, 0.12 and 0.42.
- ✓ 0.54.

4 Which seed to buy?

ANSWER A · 6

Tests: "none sprouts" for independent seeds as a product, worked backwards to a needed chance and forwards to a count and a cost, so that the probability decides something. The tempting wrong answer: buying lot B because its seed is cheaper.

a. A spot is empty when both seeds fail: $(1 - p)^2 \leq 0.01$, so $1 - p \leq 0.1$: $p \geq 0.9$.

b. Lot A: $p = \dfrac{180}{200} = 0.9$, so 2 seeds are enough ($0.1^2 = 0.01$): $2 \times 3900 = 7800$ seeds, $78 \times 6 =$ **Rs 468**.

Lot B: $p = \dfrac{150}{200} = 0.75$, failing 0.25: $0.25^2 = 0.0625$, $0.25^3 \approx 0.0156$, $0.25^4 \approx 0.0039$: **4 seeds** a spot, $4 \times 3900 = 15600$ seeds, $156 \times 4 =$ **Rs 624**.

c. **Lot A**: dearer seed, but half as many seeds are needed, so it costs Rs 156 less for the ropani. (Lot A meets the aim only just, exactly 1% empty, and its 0.9 is an estimate from 200 seeds: a farmer should know the margin is thin.)

MARKING SCHEME

- ✓ a. $(1 - p)^2 \leq 0.01$.
- ✓ $p \geq 0.9$.
- ✓ b. Lot A: 2 seeds a spot, Rs 468.
- ✓ Lot B: $0.25^3 > 0.01 \geq 0.25^4$, so 4 seeds a spot.
- ✓ Lot B: Rs 624.
- ✓ c. Lot A, cheaper for the ropani.

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5 What the numbers allow

ANSWER

Tests: the addition law as a constraint ($P(M \cup S) \leq 1$), and independence judged by the multiplication law, not by the events "being about different subjects". The tempting wrong answers: "yes, mutually exclusive, they are different subjects" in a, and "independent, since the subjects are different" in c.

a. **No.** Then $P(M \cup S) = 0.7 + 0.6 = 1.3 > 1$, impossible.

b. $P(M \cap S) = 0.7 + 0.6 - P(M \cup S) \geq 1.3 - 1 = 0.3$ (when every student passed at least one). It can't be more than the smaller event: at most **0.6** (when everyone who passed Science also passed Mathematics).

c. If independent, $P(M \cap S)$ would be $0.7 \times 0.6 = 0.42$. It is 0.45, so they are **dependent**: a student who passed one subject is a little likelier to have passed the other (0.45 of the whole is more than the 0.42 that chance alone would give).

MARKING SCHEME

✓ a. No: $0.7 + 0.6 > 1$.
✓ b. Least 0.3, from $P(M \cup S) \leq 1$.
✓ Greatest 0.6, the smaller event.
✓ c. $0.7 \times 0.6 = 0.42 \neq 0.45$: dependent.
✓ Passing one goes with a slightly better chance of passing the other.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	0	11	15	**26**

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