Unit 16 · Probability

ANSWER KEY · FOR TEACHERS

Full marks 35

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 3

1. sample space
2. equally likely
3. mutually exclusive
4. exhaustive
5. independent
6. relative frequency

Each carries 0.5 mark.

2 True or false

U · 3

i. ✓ **True**

ii. ✗ **False**: They are dependent: if one happens the other can't, so knowing one changes the other's chance to 0.

iii. ✓ **True**

iv. ✗ **False**: The two cases are not equally likely; the chance is found as a relative frequency from trials.

v. ✗ **False**: Only for mutually exclusive events; in general $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

vi. ✓ **True**

Each carries 0.5 mark.

3 Multiple choice

i. **a.** 0 — K · 1

No case is favorable: $\frac{0}{n} = 0$. 1 is a certain event's, and no probability is negative.

ii. **a.** $\frac{1}{2}$ — U · 1

The primes are 2, 3 and 5: $\frac{3}{6} = \frac{1}{2}$. $\frac{2}{3}$ counts 1 as a prime, which it is not.

iii. **a.** 0.8 — U · 1

Mutually exclusive: $P(A \cap B) = 0$, so $0.5 + 0.3 = 0.8$. 0.65 is the answer for independent events $(0.8 - 0.15)$; 0.15 is 0.5×0.3.

iv. **a.** $\frac{2}{13}$ — U · 1

4 kings and 4 queens, no card both: $\frac{8}{52} = \frac{2}{13} \cdot \frac{1}{169}$ multiplies the two chances, as if both had to happen.

v. **a.** $\frac{1}{4}$ — A · 1

Sum 4: 3 pairs; sum 8: 5; sum 12: 1. $\frac{9}{36} = \frac{1}{4} \cdot \frac{3}{11}$ counts the 11 sums 2 to 12 as equally likely cases, which they are not; $\frac{5}{36}$ is the sum 8 alone.

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vi. **a.** 0.5 HA · 1

$0.4 + p - 0.4p = 0.7$, so $0.6p = 0.3$ and $p = 0.5$. Or: both fail with $0.6(1 - p) = 0.3$. 0.3 treats the events as mutually exclusive.

4 Short answer

MARKING SCHEME U · 2

✓ 0.58 is an estimate from few trials; a fair coin's count varies from run to run.

✓ Many more trials are needed before the relative frequency is close to the probability.

5 Short questions

a. **ANSWER** A · 5

a. 366 days are 52 weeks and 2 days. The 2 extra days are one of 7 equally likely pairs (Sun–Mon, Mon–Tue, ..., Sat–Sun). A Monday or a Tuesday is in Sun–Mon, Mon–Tue and Tue–Wed: $P = \frac{3}{7}$.

(Adding $\frac{2}{7} + \frac{2}{7}$ counts Mon–Tue twice: $\frac{4}{7} - \frac{1}{7} = \frac{3}{7}$, the addition law.)

b. Multiples of 3: 10; of 5: 6; of both, that is of 15: 2. $P = \frac{10}{30} + \frac{6}{30} - \frac{2}{30} = \frac{14}{30} = \frac{7}{15}$.

MARKING SCHEME

✓ a. 366 days are 52 weeks and 2 extra days: 7 equally likely pairs.

✓ Sun–Mon, Mon–Tue, Tue–Wed: $\frac{3}{7}$.

✓ b. Multiples of 3: 10; of 5: 6.

✓ Of both (of 15): 2.

✓ $\frac{10 + 6 - 2}{30} = \frac{7}{15}$.

b. **ANSWER** A · 5

a. $0.8 \times 0.8 = 0.64$.

b. First only, or second only (mutually exclusive): $0.8 \times 0.2 + 0.2 \times 0.8 = 0.32$.

c. Both fail: $0.2 \times 0.2 = 0.04$.

d. $1 - 0.04 = 0.96$. (Check: $0.64 + 0.32 + 0.04 = 1$.)

MARKING SCHEME

✓ a. 0.64.

✓ b. The two cases 0.8×0.2 and 0.2×0.8.

✓ Added: 0.32.

✓ c. 0.04.

✓ d. 0.96.

6 Long question

ANSWER

a. (3, 6), (4, 5), (5, 4), (6, 3): $P = \frac{4}{36} = \frac{1}{9}$.

b. $P(A) = \frac{18}{36} = \frac{1}{2}$ (both dice odd or both even: 9 + 9 cases), $P(B) = \frac{1}{2}$. $A \cap B$ is both odd, 9 cases: $P(A \cap B) = \frac{9}{36} = \frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$. **Independent.**

c. $C = \{(4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)\}$, D the 6 doublets, $C \cap D = \{(5, 5), (6, 6)\}$.

$P(C \cup D) = \frac{6}{36} + \frac{6}{36} - \frac{2}{36} = \frac{10}{36} = \frac{5}{18}$.

$P(C \cap D) = \frac{2}{36} = \frac{1}{18}$, but $P(C)P(D) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$: **dependent**. A doublet makes a big sum more likely: 2 of the 6 doublets reach 10, a third of them, against a sixth of all throws.

MARKING SCHEME

✓ a. The 4 pairs with sum 9.

✓ $\frac{4}{36} = \frac{1}{9}$.

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A · 8

✓ b. $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{2}$.

✓ $A \cap B$ is both odd: $\frac{9}{36} = \frac{1}{4}$.

✓ $\frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$: independent.

✓ c. C (6 pairs), D (6 doublets), $C \cap D = \{(5, 5), (6, 6)\}$.

✓ $P(C \cup D) = \frac{10}{36} = \frac{5}{18}$.

✓ $\frac{1}{18} \neq \frac{1}{36}$: dependent.

7 Think

HA · 3

ANSWER

No. Her three cases are not equally likely. Listing the children in order, the equally likely cases are BB, BG, GB and GG, each $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. One of each is BG or GB: $\frac{2}{4} = \frac{1}{2}$. Her "one of each" is two cases, as (3, 5) and (5, 3) are with two dice.

MARKING SCHEME

✓ No: her three cases are not equally likely.

✓ The equally likely cases BB, BG, GB, GG, each $\frac{1}{4}$.

✓ One of each: $\frac{1}{2}$.

8 Activity: Bottle caps

(not marked)

WHAT STUDENTS SHOULD FIND

The two ways of landing are not equally likely (the cap's shape favors one), so there are no equally likely cases to count: only trials can estimate the chance. The relative frequency jumps about over the first few tosses and settles as the tosses grow; the whole class's hundreds of tosses give a steadier estimate than any one student's 50.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	4	8	19	4	**35**

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